Geosystemics: a systemic view of the Earth's magnetic field and possibilities for an imminent geomagnetic transition

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Abstract

Geosytemics is a way to see and study the Earth in its wholeness, together with the eventual couplings among the subsystems composing our planet. This paper will provide this view for the Earth's magnetic field, reviewing most of the results obtained in our recent works. The main tools used by geosystemics are some nonlinear quantities, such as some kinds of entropy. Through them, it is possible to: a) establish the chaoticity and ergodicity of the recent geomagnetic field in a direct and simple way; b) indentify the most extreme events in its history, as the most rapid and the slowest ones, i.e. jerks and polarity changes (reversals or excursions). In particular, regarding the latter phenomena, with the help of these entropic concepts and together with the use of the theory of critical transitions, some clues can be given for a possible imminent change of the geomagnetic field dynamical regime.

Key words: Geosytemics; Geomagnetic field; Chaos, Ergodicity; Global Transition.

1. Introduction

Earth is an ever-changing planet. This statement is obvious if we take into due account some important processes happening in our planet, i.e., growing population and, consequently, pollution; climate change; biodiversity and resources reduction; greater weakness of the present society against disasters caused by geohazards such as earthquakes, volcanic eruptions, hurricanes, etcetera. Our planet is a complex system constituted by numerous subsystems interacting each other (Skinner and Porter, 1995). For this reason, the understanding of our planet in its whole complexity is a challenging task.

Here, we will deepen the study of an important property of Earth, i.e., the geomagnetic field, and will then try to prove or to reject the hypothesis that this field is going toward a global transition. In addition, we will try to find whether and how other processes interact with this field. More in detail, recently a lot of interest has been dedicated to how the present geomagnetic field is quite distinct from the field of the recent past. Gubbins (1987) found that the southern hemisphere gives the largest contribution to the present decrease in the dipole moment, which is directly related to the intensification and southward movement of a pair of patches of reverse flux under South Africa: this state could eventually lead to a polarity change in terms of a geomagnetic reversal or excursion. Through an inversion of MAGSAT and Ørsted satellite magnetic data, Hulot et al. (2002) confirmed Gubbins (1987) results, identifying a reverse magnetic flux under the southern hemisphere. Rajaram et al. (2002) and De Santis (2007) found a rapid fall of the recent geomagnetic field in Antarctica. Jackson (2003) identified some intense equatorial flux spots on the top of the Earth's core, as manifestation of a high variability in the core. Bloxham et al. (2002) explained the frequency increase of the number of jerks, which are very rapid variations in the change of the slope of the secular variation with time scale of around 1 year, as likely due to an increased excitation of torsional waves towards the end of the last century. Gubbins et al. (2006) evidenced that the recent geomagnetic field increased its rate of decay from 1840 by about 5% per century even though they attributed this fact to an erratic aspect of the present field. De Santis et al. (2004) proposed that the present geomagnetic field could be in a chaotic state next to a geomagnetic reversal or excursion, with significant possible implications in the biosphere, in the atmosphere and in some other components of the Earth system (Constable and Korte, 2006). All the above papers express some evidence for an irregular, likely chaotic, state of the present geomagnetic field, with some possibilities for an imminent change of magnetic polarity.

A problem that we can meet when we investigate in detail some aspects of the geomagnetic field, concerns with the conventional multipolar approach, i.e. dipolar /non-dipolar representation of the geomagnetic field potential: this approach may not provide clear information regarding the dynamical and configurational properties of the whole field, because the huge dipolar contribution mostly obscures the results taken from the other multipoles. Therefore, other kinds of analyses must be exploited by following a more holistic approach to the problem. This approach looks at the field for its wholeness rather than at each specific and minute part of it (e.g., De Santis, 2009, 2014). This is exactly what we will do in this paper: we will try to use the concepts of ***Geosystemics*** with the purpose of having a more complete view of the geomagnetic field system. To do it, we will establish and compare the variation of the Shannon Information and some other quantities in order to affirm that the present geomagnetic field behaviour is consistent with a possible current planetary transition in terms of a significant change of its main characteristics (energy, dipole moment, related core dynamics, etc.) possibly going toward an excursion or even a reversal.

This article is a review of our recent contributions to the understanding of the recent geomagnetic field. In the next section we will introduce the concept of Geosystemics, and then we will define some useful mathematical tools like the Shannon Entropy (and Information) and its application to the geomagnetic field. We will also illustrate a new technique for the detection of geomagnetic jerks. Then, we will study in detail an important feature of the geomagnetic field, i.e., the South Atlantic Anomaly (SAA) and will relate this feature with another physical quantity, the global sea level (GSL), strictly connected with climate.

2. Geosystemics

The ***Geosystemics*** studies the Earth system from a holistic point of view, looking with particular attention at self-regulation phenomena and relations among the parts composing the Earth, together with the possible trends of change or persistence of the specific system or sub-system under study (De Santis, 2009, 2014). This approach puts at its centre the concepts of entropy and information content: to characterise the world, not only energy and matter are important, but also information (Bekenstein, 2003). In particular self-regulation, nonlinear coupling, emergent behaviour, irreversibility have to be taken into the due account since these are important constituents of the planet, so they must be matter of study for Geosystemics. In particular, the information exchanged and the increased entropy allow us to better understand those irreversible processes occurring in the Earth's interior. Geosystemics has the objective to observe, study, represent and interpret those aspects of geophysics that determine the structural characteristics and dynamics of our planet and the complex interactions of the elements that compose it. Some universal nonlinear tools are fundamental for Geosystemics: among many, we will focus on information and entropy. It will be also important an approach based on multi-scale/parameter/platform observations in order to cover and monitor the particular sub-system of Earth under study as much as possible. Although this latter aspect will not be considered in this review, it is a fundamental issue of geosystemics, because there is no better way to understand the behaviour of a complex system than looking at it from as many perspectives as possible.

3. Shannon Entropy and Shannon Information

The concept of Shannon Entropy H(t) (Shannon, 1948) is an important tool which can be used for the space-time characterization of a dynamical system. In the case of a system characterized by N possible independent states, this entropy is defined in a certain time t as follows:

$$H(t) = -\sum_{n=1}^{N} p_n(t) \cdot \log p_n(t) \qquad (1)$$

where $p_n(t)$ represents the probability of the system to be at the *n*-th state. For convenience, we impose $\sum_n p_n = 1$ and $\log p_n = 0$ if $p_n = 0$ to remove the corresponding singularity.

In literature we can find a wide number of physical interpretations of the Shannon Entropy. Among these, we choose the simplest one: it is a non-negative measure of our ignorance about the state of the system of concern. The Shannon Entropy has a great importance in studying and interpreting the behaviour of complex systems like Earth in general, and geomagnetic field, in particular. On the other hand, we find in literature also the Shannon Information, $I(t)$, which is simply related to H(t) as $I(t)$=- H(t). It is a negative quantity that measures our knowledge on the state of the system when we know only the distribution of probability *p(t)* (Beck and Schlögl, 1993). In practice, this quantity denotes our decreasing ability to predict the evolution of the system into the future.

4. Shannon Information and Entropy of the geomagnetic field

The Shannon Information has been already applied to the present (De Santis et al., 2004) and recent past (De Santis and Qamili, 2010) geomagnetic field, **B**(t), that can be defined at and above the Earth's surface as the negative gradient of a scalar potential V(t). In turn, this potential can be expressed, at a given time *t,* by a spherical harmonic expansion in space characterized by a set of Gauss coefficients, $g_n^m(t)$, $h_n^m(t)$, with n=1, … N degrees and m=0, ... n orders of the potential field expansion. We can define the Shannon Information $I(t)$ of the geomagnetic field as:

$$I(t) = -H(t) = \sum_{n=1}^{N} p_n(t) \cdot \log p_n(t) \qquad (2)$$

$p_n(t)$ is the probability of having a particular n-th multipole rather than another, and is calculated as (De Santis et al., 2004):

$$p_n = \frac{\langle B_n^2 \rangle}{\langle B^2 \rangle} = \frac{(n+1)\left(\frac{a}{r}\right)^{2n+4} \sum_{m=0}^{n} (c_n^m)^2}{\sum_{n'=1}^{N} (n'+1)\left(\frac{a}{r}\right)^{2n'+4} \sum_{m=0}^{n'} (c_{n'}^m)^2} \qquad (3)$$

In this formula $(c_n^m)^2 = (g_n^m)^2 + (h_n^m)^2$ and a= 6371.2 km. $<B^2>$ and $<B_n^2>$ are the mean squared amplitudes over the sphere with radius r of the total field and of the field due to the *n-th* multipole, respectively (Lowes, 1966). Although the brackets <…> denote strictly spatial averages of the squared field strength over the terrestrial sphere, actually they are also time averages because any global model of the geomagnetic field, for the way it is constructed, is a smoothed averaged model in time and space. Therefore, the definition of the Shannon Information (2) with the probabilities (3) assumes an ergodic geomagnetic field: in the next section we will confirm this property of the field. It is important to underline the fact that the time behaviour of the Shannon Information for the geomagnetic field can help us in understanding a possible chaotic scenario for the dynamical system that generates and sustains the field, in terms of geodynamo models (Chillingworth and Holmes, 1980). The slow temporal variation of the geomagnetic field with time scales from years to thousand years, is called secular variation (SV): mathematically, it is defined as the time derivative of the field. Therefore, we can introduce also an analogous Shannon Information for the SV i.e., I(SV) (we relax the dependence with time, that is implicit) where the corresponding probability $\tilde{p}_n$ will be similar to that of eq. (3) but $(c_n^m)^2$ will be replaced by $(\dot{c}_n^m)^2 = (\dot{g}_n^m)^2 + (\dot{h}_n^m)^2$:

$$\tilde{p}_n = \frac{\langle \dot{B}_n^2 \rangle}{\langle \dot{B}^2 \rangle} = \frac{(n+1)\left(\frac{a}{r}\right)^{2n+4} \sum_{m=0}^{n} (\dot{c}_n^m)^2}{\sum_{n'=1}^{N} (n'+1)\left(\frac{a}{r}\right)^{2n'+4} \sum_{m=0}^{n'} (\dot{c}_{n'}^m)^2} \tag{4}$$

De Santis et al. (2004) showed that the recent geomagnetic field is in the particular situation that $p_n \approx \tilde{p}_n$, which is a necessary (but not sufficient) condition for a geomagnetic reversal or excursion.

In Figure 1 some synthetic examples are given for different Shannon Entropy H values (De Santis and Qamili, 2008). These configurations are given in terms of the normalised Shannon Entropy H^* (with value between 0 and 1) as:

$$H^* = \frac{H}{H_{\max}} = \frac{I}{I_{\min}} \tag{5}$$

with $H_{\max}$=-$I_{\min}$= log N. In this figure, the example with entropy H^*=0.3 represents the real case of the present geomagnetic field deduced from IGRF-11 model at 2010 (Finlay et al., 2010), while the others are synthetic cases. From these examples it is clear that the Shannon Entropy provides a way to measure the degree of complexity of the field spatial configuration: the higher the Shannon Entropy (or the lower Shannon Information), the more all probabilities are equally possible, and, then, the more complex the derived spatial configuration of the field will be. The interpretation of this is that when the Shannon Information is low there is a lower degree of organization for the system under study (see for instance the recent review by Balasis et al., 2013). For this reason the corresponding Shannon Entropy is also called spatial or configuration entropy (e.g., Rodrìguez-Iturbe et al., 1998). Of course, when we apply these concepts to the real geomagnetic field, we recognise that the value of the information quantity must be referred to a specific reference radial distance, r. For instance, while the normalised entropy of the present real geomagnetic field is around 0.3 at the Earth's surface, it becomes 0.8 at the core mantle boundary (CMB). This is normal because the field is more complex going toward the sources, and the increase of entropy denotes an increase of complexity.

The temporal trends of I(t) of the real geomagnetic field for the last 7000 years, at Earth's Surface and at CMB, by using CALS7K (Korte and Constable, 2005), CALS3K (Korte et al., 2009) and IGRF-11 (Finlay et al., 2010) global models, are shown in Figure 2 (De Santis and Qamili, 2010). All these global models are based on a spherical harmonic representation of the field. The trends in this figure show also the estimated error bars associated to CALS7K moving from a maximum of 33% at 5000 BC to 3% at 1950 AD. In order to allow the "contact" of the I(t) of CALS7K at the CMB with that of IGRF-11, here we show the possible effect of the spectral damping typical of CALS7K model (at the Earth's surface I(t) is practically unaffected). Our results indicate that the present Shannon Information is much lower than the Shannon Information of the past, i.e., the present field is much more chaotic than the field of the past.

When a process is ergodic and chaotic, I(t) can be related with the Kolmogorov entropy or K-entropy that represents the rate of loss of information, by (Wales, 1991):

$$K = -\frac{dI}{dt} \tag{6}$$

This quantity measures the degree of unpredictability of the future evolution of the system between successive points on the trajectory in the phase space (Beck and Schlögl, 1993; Buchner and Zebrowski, 1998). An alternative definition of the K-entropy can be given also as the sum of all positive Lyapunov exponents of the dynamical system (Schuster and Jung, 2005). A practical consequence is that after a characteristic time τ=1/K, the system's behaviour can no longer be predicted. Figure 3 shows the K-entropy of the geomagnetic field as derived from equation (6) from

5000 BC to present at Earth's Surface and at CMB. Here, the K-entropy is derived as a linear fit to each 100-year interval at Earth's surface and at CMB; each single IGRF-11 value is given in the plots as a star. The K-entropy of the present field is rather high with respect to the past (De Santis and Qamili, 2010). Although we cannot exclude that some marginal contribution to this feature could be due to insufficient data in the past when compared with the present "abundance", we think that the effect is mostly real, because our definition of the K-entropy is based on the dynamical variations of the Shannon Information of global models with realistic spatial power spectra, so with a reliable repartition between dipolar and non-dipolar parts.

5. Ergodicity of the recent geomagnetic field

In a general case, the information quantities introduced in the previous section by the equations (3) and (6), should be estimated in the phase space. However, if we prove that, apart from being chaotic, the geomagnetic field is also ergodic, i.e., time average of the original signal is equal to the density average in the phase space (Eckmann and Ruelle, 1985), then the phase space reconstruction will not be necessary, and we can perform all the analysis in the time domain. In that case, if we want to investigate the nonlinearities present in a system, we can simply perform a nonlinear forecasting approach in the time domain. Taking into account the chaotic properties of the geomagnetic field, any small change ε of the initial orbit in the phase space propagates exponentially with time, i.e., $\varepsilon(t) = \varepsilon_0 \exp(K \cdot t)$ where K is the above defined K-entropy.

Let us consider a generic measure ρ of the dynamical system moving in the phase space Ω. For every continuous function φ, a dynamics f is called ergodic if it has the same behaviour averaged over time as averaged over phase space and the space average is weighted by the invariant measure

ρ (Eckmann and Ruelle, 1985). Under general assumptions, this can be expressed mathematically with the following equation:

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T \varphi\left[f^t(x_0)\right]dt = \int \rho(dx)\varphi(x) \tag{7}$$

This means that if the system is ergodic, after a certain time evolution, the system is no longer dependent on its initial state x_0 (Egolf, 2000). In the case of the geomagnetic field, the invariant measures are the K-Entropy and its inverse value $<\tau>$ $=1/K$, i.e., the limiting mean time of prediction.

Considering all the global geomagnetic models present in literature, we could have large errors if we extrapolate the geomagnetic field outside their typical time of validity (De Santis et al., 2011, 2013a). These errors can be estimated from a comparison between the predicted and definitive part of each model. More precisely these errors can be calculated by means of Gauss coefficients from the formula (Maus et al., 2008):

$$\varepsilon = \sqrt{\sum_{n=1}^{N}(n+1)\sum_{m=0}^{n}\left[\left(c_n^m\right)_{pred}-\left(c_n^m\right)_{def}\right]^2} \tag{8}$$

An example of the trend of these errors is given in figure 4, where one 10-year segment from 1965 to 1975 and seven 5-year segments from 1975 to 2010 taken from IGRF-11 global model have been considered. De Santis et al. (2011) have analyzed also other global models like CHAOS (Olsen et al, 2014), CM4 (Sabaka et al. , 2004), GUFM1 (Jackson et al., 2000), WMM (Maus et al., 2010), POMME (Maus et al., 2005), obtaining the same results (but they are not shown here). Since the IGRF-11 model gives a constant predictive field, in this analysis we have used the CM4 model as predictive part in order to avoid this problem. For visual convenience, we have imposed the same initial time. This means that each exponential growth will have an offset of $-\varepsilon_0$. In this case the formula for the calculation of the errors, substituting the K-entropy with its inverse τ, will be:

$$\varepsilon(t) = \varepsilon_0 \exp(t/\tau) - \varepsilon_0 = \varepsilon_0 \left(\exp(t/\tau) - 1\right) \tag{9}$$

In this figure, for each segment we indicate the corresponding τ values together with their associated errors. As result, all the segments show a clear exponential growth with characteristic mean time $<\tau>=5.9\pm2.3$ years. We can conclude that the exponential temporal divergence of the errors between several couples of predictive and definitive global geomagnetic models supports a chaotic state of the present geomagnetic field with no reliable prediction after around 6 years (see De Santis et al., 2011). This result confirms the nonlinear analysis performed in the phase space by De Santis et al. (2002) and has direct consequences in repeating magnetic surveys and updating global and regional models of the geomagnetic field (De Santis et al., 2013a). The total agreement of these analyses made in the phase space and in time domain, confirms the ergodicity of the geomagnetic field (De Santis et al., 2011).

6. Jerks as chaotic fluctuations of the geomagnetic field

Geomagnetic jerks have been generally identified in geomagnetic observatory time series (e.g. Courtillot et al., 1978; Mandea et al., 2010). The fact that some timescales of jerk occurrences are overlapping with those of the solar activity (e.g. sunspots cycle of almost 11 years) complicates the clear identification of jerks in the geomagnetic field time series. A better alternative was presented recently. In this section we will describe the results obtained by Qamili et al. (2013) who extended the nonlinear forecasting approach in the time domain, over the last 400 years, period covered by

GUFM1 (Jackson et al., 2000) global model with the objective to re-interpret the geomagnetic jerks. We analyze the temporal behaviour of the differences between predicted and definitive values of the geomagnetic field calculated from this model in order to find periods more or less chaotic than others. The *predicted*/*definitive* comparison is made over successive 10-years segments, moving at steps of 1 year. The problem is that GUFM1 model does not give a predictive field. This part was calculated by extrapolating the prior 10-year secular variation, into the subsequent 10 years and compared them with the real GUFM1 field values for the same period of time. Each of the analyzed segments shows an evident exponential growth with characteristic time of predictability $\langle \tau \rangle \approx 6 \pm 2.5$ years. The temporal fluctuations of τ value around its mean linear trend from 1600 to 1980 are shown in Figure 5. What is clear from this figure is that the past field is less predictive than the recent one, because the number of accentuated negative fluctuations of $\langle \tau \rangle$ increases with time. A simple explanation of this result could be the progressive improvement of the data quality used to build the GUFM1 model. Around this general trend we have found some interesting fluctuations, i.e., periods where the geomagnetic field is more chaotic (smaller time of predictability τ) and also periods where the field is less chaotic (greater time of predictability τ). Checking carefully all the epochs where the τ-value becomes suddenly lower with respect to the values that surround it (a sort of V-shape in the temporal behaviour of τ), we find that most of these epochs corresponds to already known geomagnetic jerks (epochs evidenced by arrows but considering an uncertainty of a few years for each event), detected by other authors (Mandea et al., 2010 and references therein). But not all the chaotic fluctuations correspond to already known geomagnetic jerks. This could be because the techniques introduced till now for the identification of geomagnetic jerks, where most of them are applied to direct measurements, had not been able to detect all these features produced by the geomagnetic field. For this reason, here, we detect a number of new (undetected till now) geomagnetic jerks (events evidenced by blue arrows in Figure 5). As conclusion, we can say that geomagnetic jerks appear in those epochs where the geomagnetic field is more chaotic. It is interesting to notice that the more recent field is characterised by more frequent jerks. During the applied analysis we have identified also some short periods where the field appears less chaotic than usual but this aspect will need more investigation to understand the corresponding origin.

7. Toward a global geomagnetic transition?

Some previous (e.g., Gubbins, 1987; Hulot et al., 2002; De Santis 2007) and more recent (De Santis et al., 2013b) results show that it is very important to investigate the geomagnetic field of the southern hemisphere, since it contributes more to the overall decaying trend of the geomagnetic field. Could this be considered as a symptom that the Earth's magnetic field is going toward a global transition? We will see that this hypothesis is fostered by the presence of one of the most important features of the present geomagnetic field, i.e., the South Atlantic Anomaly (SAA; figure with H*=0.3 in Figure 5). We follow here the reasoning of the recent article by De Santis et al. (2013b).

The SAA is a significant depression in the total intensity of the present geomagnetic field that has been persisting at least for the last 400 years. It is generally interpreted as the Earth's surface expression of a magnetic vortex present in the outer core, as a component of a strong reversed magnetic flux (Olson and Amit, 2006). During the last 400 years, the SAA has changed in space and in time. If we consider its extension from 1590 to present using GUFM1 and IGRF-11 models (we have considered the extent of the 32000 nT isoline because it is the lowest value in the oldest epoch), we obtain the trend shown as a thick curve in Figure 6. The continuous and accelerating growth of this anomaly is evident, especially during the last 250 years. We could ask whether this acceleration happens just by chance or not. Fig. 7 shows the real acceleration of the SAA in the last 400 years (red curve) compared with 10,000 simulations (blue curves) where all SAA increments have been randomly shuffled. Green curves represent the maximum acceleration (lower green curve) and deceleration (upper green curve). The real acceleration of the SAA stands clearly at the lower limit of the possibilities (maximum acceleration), supporting the case that the present situation is not occurring just by chance.

De Santis et al. (2012) found that also another, apparently unrelated quantity, the global sea level rise (GSL; Jevrejeva et al., 2008) has followed the same growing trend during the last three centuries (thin curve in Figure 6). To assess a real correlation between the two time series, some statistical tests have been performed, i.e., Spearman correlation test (Davis, 1986) and Kullback-Leibler Entropy (Kullback and Leibler, 1951). The results taken from the statistics, (both with or without a trend removal) confirm the high correlation between SAA extension area and GSL (see De Santis et al., 2012, 2013b). Although correlation does not always mean causation, we should consider this possibility as a serious hypothesis. In that case, what physical mechanism could be behind the observed correlation? De Santis et al. (2012) propose three possible mechanisms (two external and one internal):

1. an increase of the SAA area facilitates the entrance of charged particles from space. As result we have a warmer atmosphere, which implies a consequent melting of major ice caps (Antarctica and Greenland) that finally causes a global increase of sea level;

2. a possible reduction of the ozone layer in the upper stratosphere over the South Atlantic region can modify the radiative flux at the top of the atmosphere and hence can cause changes in the weather and climate patterns, including cloud coverage;

3. both SAA and GSL time variations could share the same common internal cause, i.e., a convective dynamism in the outer core causes a variation of the magnetic field and an elastic deformation at the Earth's surface (Greff-Lefftz et al., 2004).

An interesting question concerns with the best temporal function that fits the SAA surface area change in time. We will see that this function follows the typical behaviour in time of critical systems, i.e. those complex systems approaching a critical transition.

The deformation (or energy release) y(t) of a material that approaches a failure satisfies the following empirical equation (Voight, 1989):

$$\ddot{y} = a\dot{y}^{\alpha} \tag{12}$$

where a and α are two empirical constants. The latter is an exponent that measures the degree of nonlinearity and normally takes values between 1 and 2. We can extend the concept of the failure of a material to critical systems approaching their tipping point, i.e. the time when the system undergoes a dramatic (usually abrupt) change of its dynamical properties. In this way, eq. (12) assumes a more universal importance. Indeed, the solutions of equation (12) have been largely applied for the prediction of different critical systems like volcanic eruptions (Voight, 1988), earthquake main failure (Bufe and Varnes, 1993), financial crashes (Sornette, 2003), magnetic storms (Balasis et al., 2011), etcetera. Integrating equation (12) for $\alpha \neq 1$, we obtain a first-order equation whose solution takes the form of a power-law increase with time:

$$y = A + B\left(t_c - t\right)^p \tag{13}$$

where t_c represents the time to failure or critical time of the system under study, $p=[(2-\alpha)/(\alpha-1)]$ is a power-law exponent (usually less than 1); $A>0$ and $B<0$ are parameters to be found from the experimental data. Vandewalle et al. (1998) introduced a logarithmic function in alternative of the power-law form, i.e., $y = A + B\ln\left(t_c - t\right)$. Sornette and Sammis (1995) propose a more generalised solution which is decorated by a log-periodic function. Thus we can write:

$$y(t) = A + B\ln(t_c - t)\cdot\left\{1 + D\cdot\cos[2\pi f\ln(t_c - t) + \phi]\right\} \tag{14}$$

D is the magnitude of the log-periodic fluctuations around the acceleration growth, f is the frequency of the fluctuations, ϕ is the phase shift.

We applied equation (14) over SAA (Figure 8) and GSL (Figure 9) trends (De Santis et al., 2013b). The equation (14) fits very well both the considered time series, with very high correlation coefficient r (in both cases r>0.98). Since original GSL data set is very noisy (especially the oldest values) before the fit we averaged the data every 5 years (black circles in Fig.9). Regarding the estimation of the critical time t_c, we find practically the same time, i.e., $t_c \approx 2034\pm3$ yrs for SAA and $t_c \approx 2033\pm11$ yrs for GSL (estimated errors are just statistical, as they could be even larger than indicated; see the end of this section). Also the D and f parameters are very similar in both SAA and GSL fits, indicating that the fluctuations affect the acceleration in almost the same way in both physical quantities. In practice, the A parameter of the SAA fit corresponds to the value of the SAA area at the critical time (actually at t_c-*1* year), that, in this case, will reach more than 50% of the whole Earth surface. Alternative fits with functions with a comparable number of coefficients (such as, for instance, a 5-degree polynomial in time,) are equally possible, however they are much more unrealistic outside the data they use. All these results suggest that the same trend of these quantities is not a mere coincidence, and, probably, both these systems are behaving as dynamical systems close to a critical point.

However some words of caution are necessary. Any fit made with the function of (14) is rather instable when the critical point is far, so it needs as more data as possible before to provide some stable result: in general, the fit converges to a stable result as the data approach the critical transition (e.g. see Sornette et al., 2004 for landslide predictions). To have some quantitative idea of the stability (or instability) of this kind of analysis, we apply the fit in subsequent segments of the SAA dataset, in particular from 1590 to 1960, then from 1590 to 1965, and so on, till the last epoch of 2010, and for every fit we keep note of the prediction , t_{pred}, of the critical transition. This means that in every analysis we truncate the data at some time $t_{max} < t_c$ and use only the data up to t_{max}. We choose the SAA data instead of GSL data because the latter are much noisier and reducing the points of the fit would provide very unstable results. We can then consider difference $\Delta t = t_{pred} - t_{max}$, i.e. between prediction (t_{pred}) and epoch (t_{max}) at which the prediction is made for the last 50 years (Fig. 10). A linear fit over Δt would predict Δt=0 at around 2060, indicating that the most recent prediction (made at t_{max}=2010) of $t_c \approx 2034$ yrs (or, alternatively, the error of ±3 yrs) is probably underestimated. However, in the last 15 years Δt almost stopped to decrease, so also this linear prediction must be taken with some caution (Fig. 10). What we can affirm is that the analysed time series has been so far behaving as a critical system, but we will need more time and more data before to completely confirm this result. This is also due to the fact that a critical system reveals its criticality as it approaches more the critical point. It is for this reason that log-periodic functions are called “sloppy” functions (e.g. Brée et al., 2013).

8. Conclusions

In this paper we made an overview of the complex characteristics of the present Earth’s magnetic field by means of the Geosystemics approach, reviewing most of the results obtained in our recent works. More precisely, useful tools like Entropy and Information have been applied to the present and to the recent past geomagnetic field in order to derive important information regarding the corresponding dynamical system originating in the outer terrestrial core. Moreover, the temporal evolution of an important feature of the present geomagnetic field, the South Atlantic Anomaly, has been deeply investigated. Since the main objective of Geosystemics is to study the Earth system from the holistic point of view, together with the SAA we have also analysed another physical quantity, the GSL, and studied its possible correlation with the geomagnetic field. After all these analysis we can conclude that the present geomagnetic field is more chaotic than the one of the past. In addition, there are many intriguing aspects that encourage us to suggest that the present geomagnetic field is rather special and a possible imminent change of geomagnetic polarity could be not so unexpected. We find the interesting result that both SAA and GSL can be described by a critical system evolution, with similar critical time that can be interpreted as "the point of no return" for both the whole geomagnetic field and GSL. We do not interpret the critical time as the exact moment of a geomagnetic reversal, because the typical diffusion time of the Earth's core would require a few thousand years, we rather consider t_c as the time when the irreversible process, that will drive the magnetic field to change its polarity, will start. To make a simple figurative analogy, consider the case in which we play with a ball near a deep well: if the ball falls into the well, the critical point will be the moment at which its centre of gravity is beyond the border of the well and not the time the ball will touch the bottom. About the exact time of the critical transition we warned about its uncertainty, mostly due to the instability of the fit of expression (14) over any experimental data. For this reason, further investigation is still demanding in the near future to confirm or confute the found results. In particular, the recent ESA Swarm mission (Olsen and Haagmans, 2006) of three twin satellites (launched on 22 November 2013) with precise magnetic sensors aboard will be an unprecedented occasion to verify the present results with great accuracy.

Acknowledgements

We thank three referees (Vladimir Kossobokov and two anonymous) for their useful comments that improved the quality of the paper. The first author (ADS) would like to thank also the organisers of the conference "Mathematics and Geosciences: Global and Local Perspectives", in particular Maria Luisa Osete, for the kind invitation for a keynote presentation and for her enthusiastic encouragement. The simulations shown in Figure 7 (together with the figure) have been kindly performed by Francisco Javier Pavón-Carrasco using *Matlab*© routines.

References

Balasis, G., C. Papadimitriou, I. A. Daglis, A. Anastasiadis, L. Athanasopoulou, and K. Eftaxias (2011), Signatures of discrete scale invariance in Dst time series, Geophys. Res. Lett., 38, L13103, doi:10.1029/2011GL048019.

Balasis, G., R. V. Donner, S. M. Potirakis, J. Runge, C. Papadimitriou, I. A. Daglis, K. Eftaxias, and J. Kurths, Statistical mechanics and information-theoretic perspectives on complexity in the Earth system, Entropy, 15 (11), 4844–4888; doi:10.3390/e15114844, 2013.

Beck, C., and Schlogl, F., Thermodynamics of Chaotic Systems (Cambridge University Press, Cambridge 1993) p. 306.

Bekenstein, J.D. (2003), Information in the Holographic Universe, Scientific American, 289, 2, August 2003, p. 61.

Bloxham, J., Zatman, S., and Dumberry, M. (2002), The origin of geomagnetic jerks, Nature 420 (6911), 65–68.

Brée D., Challet D. and Peirano P.P. (2013) Prediction accuracy and sloppiness of log-periodic functions, Quantitative Finance, 13, 275-280.

Buchner, T., and Zebrowski, J. (1998), Local entropies as a measure of ordering in discrete maps, Chaos Soliton. Fract. 9 (1/2), 19–28.

Bufe, C.G. and Varnes, D. J. (1993), Predictive modelling of the seismic cycle of the Greater San Francisco Bay region, J. Geophys. Res. 98, 9871–9883.

Chillingworth, D.R.J. and Holmes, P.J. (1980), Dynamical systems and models for the reversals of the Earth's magnetic field, Math. Geol. 12, 41-59.

Constable, C.G., and Korte, M. (2006), Is Earth's magnetic field reversing?, Earth Planet. Sci. Lett. 246, 1–16.

Courtillot, V., J. Ducruix, and J.L. LeMouël (1978), Sur une accélération récente de la variation séculaire du champ magnétique terrestre, C. R. Acad. Sci. Paris Ser. D, 287, 1095 – 1098.

Davis, J.C. (1986), Statistics and Data Analysis in Geology. Wiley & Son, New York.

De Santis, A. (2007), How persistent is the present trend of the geomagnetic field to decay and, possibly, to reverse?, Phys. Earth Planet. Int. 162, 217–226.

De Santis, A. (2009), Geosystemics, Proceedings of WSEAS Conference on Geology and Seismology, GES'09, Feb. 2009 Cambridge, 36-40.

De Santis, A. (2014), Geosystemics, entropy and criticality of earthquakes: a vision of our planet and a key of access, in "Nonlinear phenomena in Complex Systems: from Nano to Macro Scale" ed. E. Stanley and D. Matrasulov, NATO Science for Peace and Security Series – C: Environmental Security, 3-20.

De Santis, A., and Qamili, E. (2008), Are we going towards a global planetary magnetic change? 1st WSEAS International Conference on Environmental and Geological Science and Engineering (EG'08), 149–152.

De Santis, A., and Qamili, E. (2010), Shannon Information of the geomagnetic field for the past 7000 years, Nonlinear Proc. Geoph. 17, 77–84.

De Santis, A., Barraclough, D.R., and Tozzi, R. (2002), Nonlinear variability of the Recent Geomagnetic Field, Fractals 10, 297–303.

De Santis, A., Tozzi, R., and Gaya-Piquè, L.R. (2004), Information content and K-Entropy of the present geomagnetic field, Earth Planet. Science Lett. 218, 269–275.

De Santis, A., Qamili, E., and Cianchini, G. (2011), Ergodicity of the recent geomagnetic field, Phys. Earth Plan. Int. 186, 103–110.

De Santis, A., Qamili, E., Spada, G., and Gasperini, P. (2012), Geomagnetic South Atlantic Anomaly and global sea level rise: a direct connection?, J. Atmos. Sol. Terr. Phys. 74, 129–135.

De Santis A., Qamili E. and Cianchini G. (2013a), Repeat-station surveys: implications from chaos and ergodicity of the recent geomagnetic field, Annals of Geophysics 56 (1), R0103, doi:10.4401/ag-5491.

De Santis A., Qamili E. and Wu L.X. (2013b), Toward a possible next geomagnetic transition ? Nat. Haz. Earth Syst. Sc. 13, 3395-3403.

Eckmann, J.-P., and Ruelle, D. (1985), Ergodic theory of chaos and strange attractors, Part I, Rev. Mod. Phys. 57(0.3), 617–654.

Egolf, D. (2000), Equilibrium regained: from nonequilibrium chaos to statistical mechanics, Science 287, 101–104.

Finlay, C.C., et al. (2010), International Geomagnetic Reference Field: The eleventh generation, Geophys. J. Int. 183, 1216–1230, doi:10.1111/j.1365-246X.2010.04804.x.

Greff-Lefftz, M., Pais, M.A., Le Mouel, J.-L. (2004), Surface gravitational field and topography changes induced by the Earth's fluid core motions. Journal of Geodesy 78, 386–392.

Gubbins, D. (1987), Mechanism for geomagnetic polarity reversals, Nature 326, 167–169.

Gubbins, D., Jones, A.L., and Finlay, C.C. (2006), Fall in Earth’s Magnetic Field is erratic, Science 312, 900–902.

Hulot, G., Eymin, C., Langlais, B., Mandea, M., and Olsen, N. (2002), Small-scale structure of the geodynamo inferred from Ørsted and Magsat satellite data, Nature 416, 620–623.

Jackson, A. (2003), Intense equatorial flux spots on the surface of Earth's core, Nature 424, 760-763.

Jackson, A., Jonkers, A.R.T. and Walker, M.R. (2000), Four centuries of geomagnetic secular variation from historical records, Phil. Trans. R. Soc. Lond. A, 358, 957–990.

Jevrejeva, S., Moore, J.C., Grinsted, A., and Woodworth, P.L. (2008), Recent global sea level acceleration started over 200 years ago?, Geophys. Res. Lett. 35, L08715, doi:10.1029/2008GL033611.

Korte, M. and Constable, C.G. (2005), Continuous geomagnetic field models for the past 7 millennia: 2. CALS7K, Geochem. Geophy. Geosy. 6, Q02H16, doi:10.1029/2004GC000801.

Korte, M., Donadini, F. and Constable, C.G. (2009), Geomagnetic field for 0–3 ka: 2. A new series of time-varying global models, Geochem. Geophy. Geosy. 10, Q06008, doi:10.1029/2008GC002297.

Kullback, S., and Leibler, R.A. (1951), On information and sufficiency. The Annals of Mathematical Statistics 22 (1), 79–86.

Lowes, F. J. (1966), Mean-square values on sphere of spherical harmonic vector fields, J. Geophys. Res. 71, 2179.

Mandea, M., Holme, R., Pais, A., Pinheiro, K., Jackson, A., and Verbanac, G. (2010), Geomagnetic Jerks: Rapid Core Field Variations and Core Dynamics, Space Sci Rev. 155, 147–175.

Maus S., Lühr H., Balasis G., Rother M. and Mandea M. (2005), Introducing POMME, the POtsdam Magnetic Model of the Earth, in "Earth Observation with CHAMP", ed. by C. Reigber, H. Lühr, P. Schwintzer and J. Wickert, pp. 293-298, Springer, New York.

Maus, S., Silva, L., and Hulot, G. (2008), Can core-surface flow models be used to improve the forecast of the Earth's main magnetic field? J. Geophys. Res. 113, B08102, doi:10.1029/2007JB005199.

Maus, S., Macmillan S., McLean S., Hamilton B., Thomson A., Nair M., and Rollins C. (2010) The US/UK World Magnetic Model for 2010-2015, NOAA Technical Report NESDIS/NGDC.

Olsen, N., and Haagmans, R., (Guest Editors) (2006) Swarm—The Earth's Magnetic Field and Environment Explorers. Special Issue, Earth Planets Space, 58, 349–496.

Olsen, N., Lühr, H., Finlay, C.C., Sabaka, T.J., Michaelis, I., Rauberg, J., and Toffner-Clausen, L. (2014) The CHAOS-4 geomagnetic field model. Geophys. J. Int., Doi: 10.1093/gji/ggu033.

Olson, P., and Amit, H. (2006), Changes in earth's dipole, Naturwissenschaften 93, 519-542.

Qamili, E., De Santis, A., Isac, A., Mandea, M., and Duka, B. (2013), Geomagnetic jerks as chaotic fluctuations of the Earth's magnetic field, Geochem. Geophys. Geosyst. 14, 839-850, doi:10.1029/2012GC004398.

Rajaram, G., Arun, T. Dhar, A., and Patil, G. (2002), Rapid decrease in total magnetic field F at Antarctic stations - its relationship to core-mantle features, Antarctic Science 14, 61-68.

Rodriguez-Iturbe, I., D'Odorico, P., and Rinaldo, A. (1998), Configuration entropy of fractal landscapes, Geophys. Res. Lett. 25(7), 1015–1018.

Sabaka T.J., Olsen N., Purucker M.E. (2004) Extending comprehensive models of the Earth's magnetic field with Ørsted and CHAMP data, Geophys. J. Int., 159:521-547.

Schuster H.G., and Just W. (2005) Deterministic chaos. An introduction. Wiley-VCH Verlag GmbH & Co.KGaA, Weinbein, pp. 287.

Shannon, C. (1948), A mathematical theory of communication, Bell System Technical Journal 27, 379–423, 623–656.

Skinner, B.J., and Porter, S.C., The blue planet: an introduction to earth system science (Wiley and Sons, 1995).

Sornette, D., Why stock markets crash. Critical events in complex financial systems (Princeton Univ. Press, Oxford, 2003).

Sornette, D., and Sammis, C. (1995) Complex critical exponents from renormalization group theory of earthquakes: implications for earthquake predictions, J. Phys. I France 5, 607–619.

Sornette D. Helmstetter A. Andersen J.V. Gluzman S., Grasso J.-R. and Pisarenko V.F. (2004) Towards Landslide Predictions: Two Case Studies. Physica A 338, 605-632

Vandewalle, N., Ausolos, M., Boveraus, P., and Minguet, A. (1998), How the financial crash of October 1997 could have been predicted, Eur. Phys. J. B., 4, 139–141.

Voight, B. (1988), A method for prediction of volcanic eruptions, Nature 332, 125–130.

Voight, B. (1989), A relation to describe rate-dependent material failure, Science 243, 200–203.

Wales, D.J. (1991), Calculating the rate of loss of information from chaotic time series by forecasting, Nature 350, 485–488.

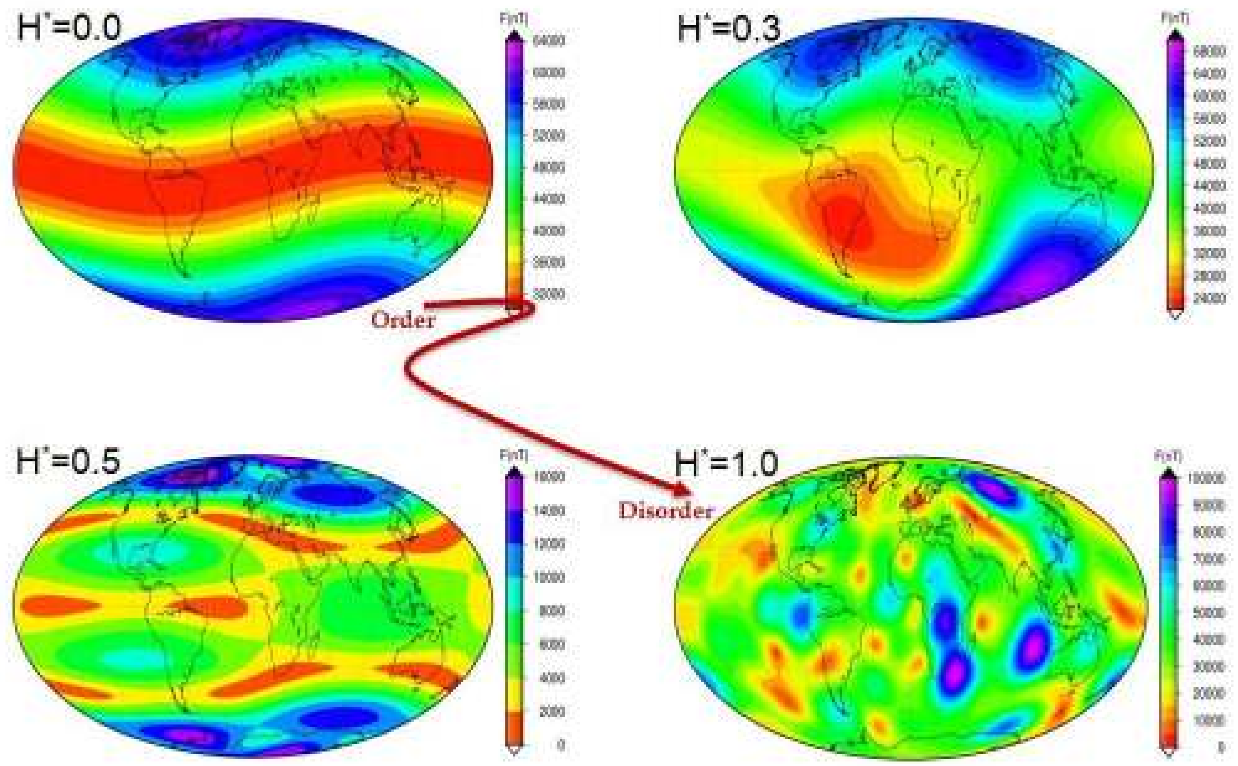

Figure 1. Four geomagnetic total intensity configurations with different increasing (decreasing) normalised Shannon Entropy (Information), H^* from 0 to 1 (from 0 to -1). The upper right case is real and represents the present geomagnetic field, while the other three cases are synthetic examples (from De Santis and Qamili, 2008).

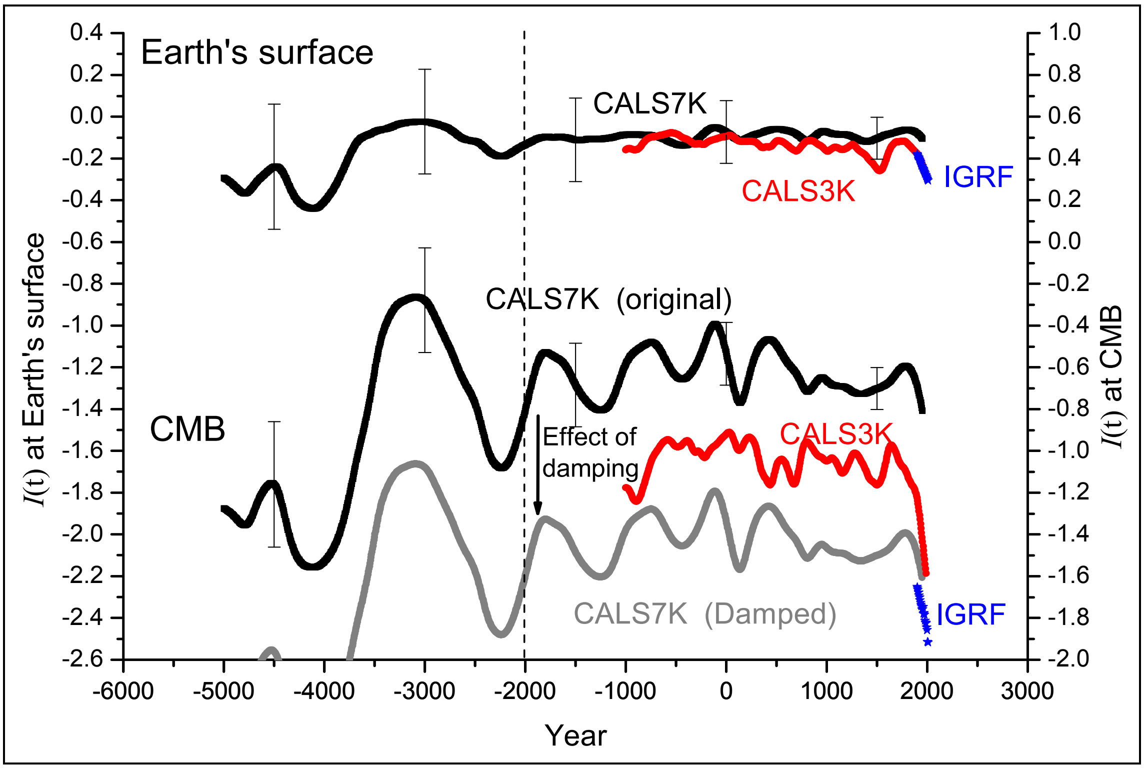

Figure 2. Shannon Information I(t) of the geomagnetic field from 5000 BC to present from CALS7K (black), CALS3K (red) and IGRF-11 (blue) models, at the Earth's surface and at the CMB. For visual convenience, the estimated error vertical bars are shown every 1500 years for CALS7K only (from De Santis and Qamili, 2010).

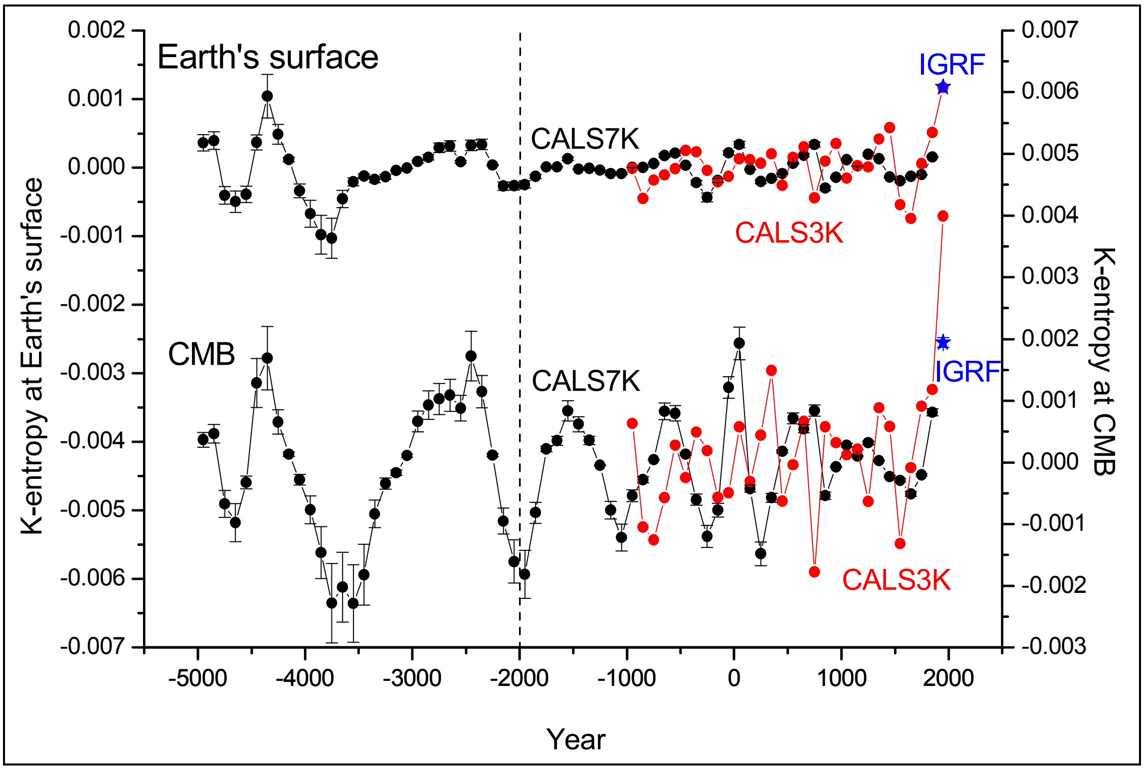

Figure 3. K-entropy of the field from 5000 BC to present from CALS7K (black), CALS3K (red) and IGRF-11 (blue) models, at the Earth's surface and at the CMB. The present value of K-entropy of the field based on the IGRF-11 model is the highest over all the investigated period (from De Santis and Qamili, 2010).

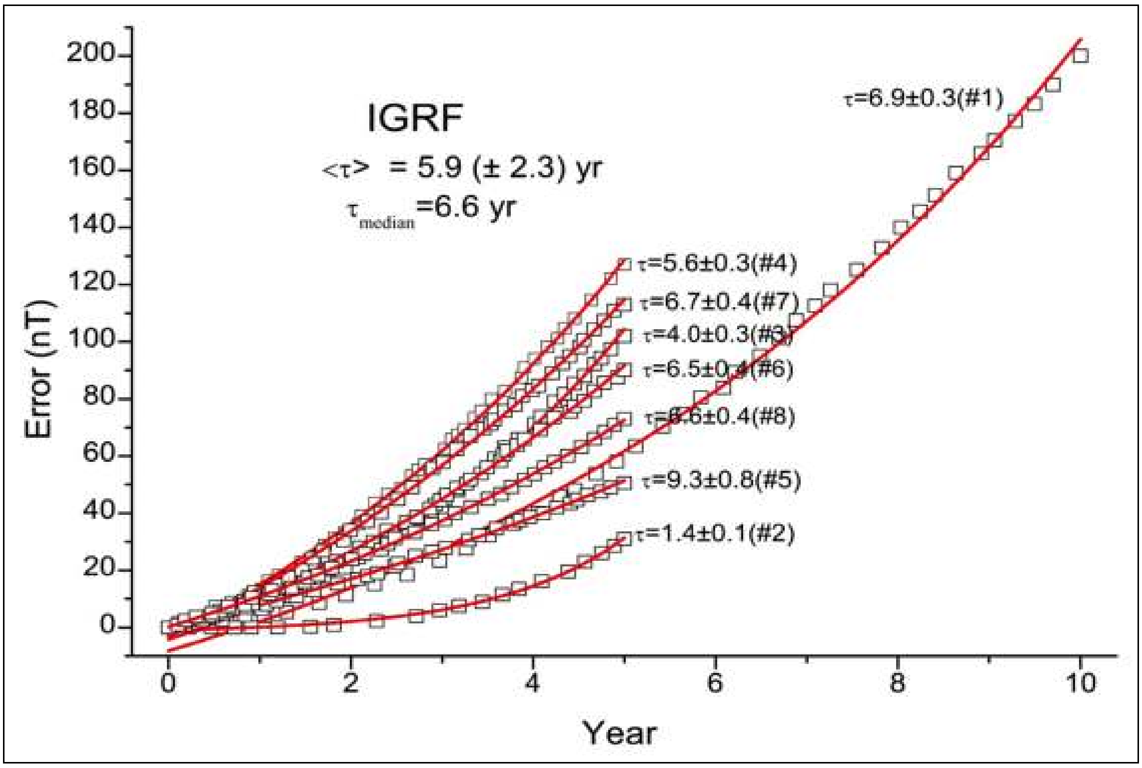

Figure 4. Time evolution of errors between the predicted and observed global IGRF-11 model (from De Santis et al. 2011). The exponential increase in time is a symptom of a chaotic geomagnetic field.

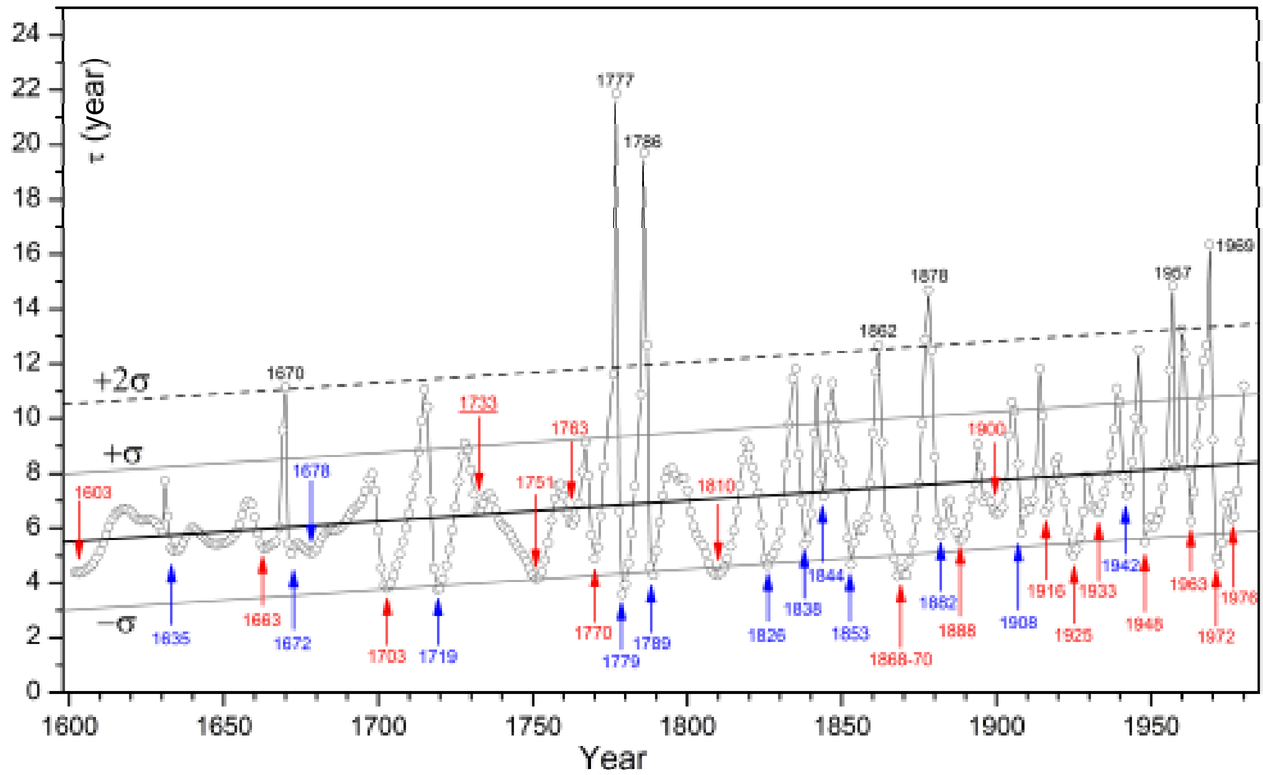

Figure 5. Estimation of the time of predictability <τ> every year over the period 1600–1980 from GUFM1 model. The epochs of already noted geomagnetic jerks are indicated by red arrows and those for which new possible events are suggested by blue arrows. The mean trend is indicated by the best fit line across the data points: the linear increase in time can be ascribed by a better quality of data and model with time (from Qamili et al., 2013). The most recent times are characterised by more frequent jerks.

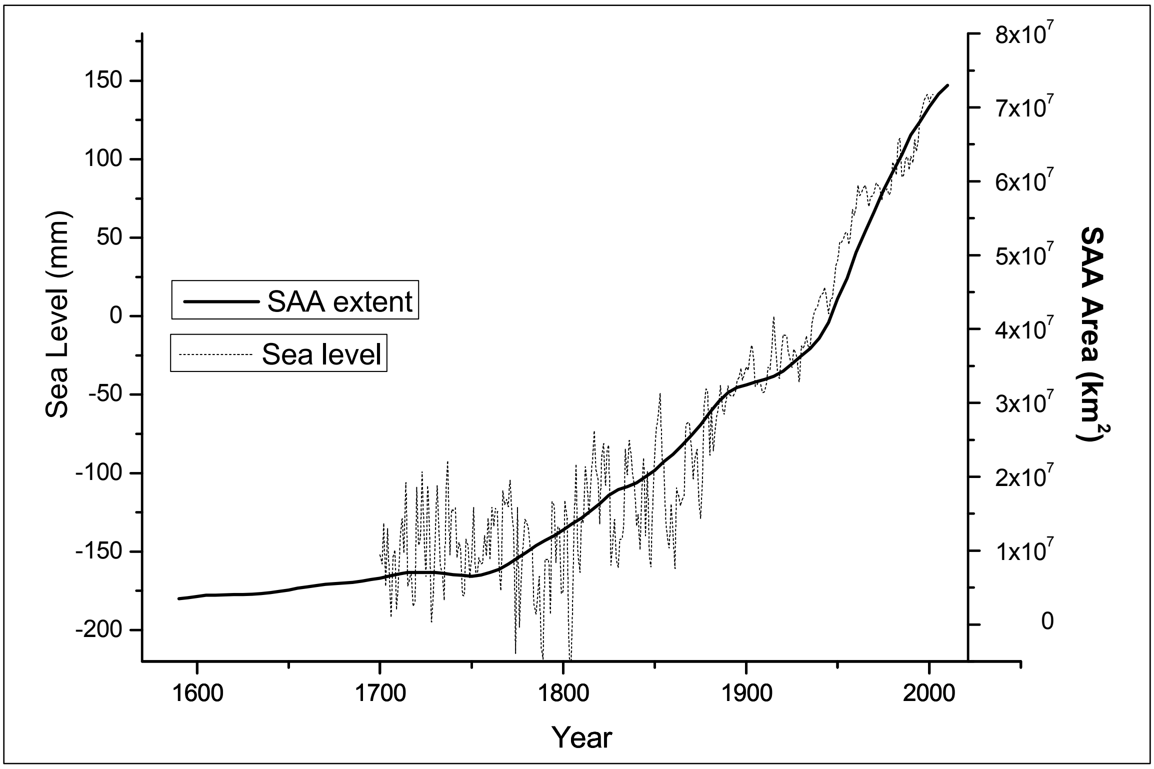

Figure 6. Extent of the SAA surface area obtained from GUFM1 and IGRF-11 (1590-2010) from 1590 to 2010, together with the global sea level rise (original data set) from 1700 to 2002 (redrawn from De Santis et al., 2012).

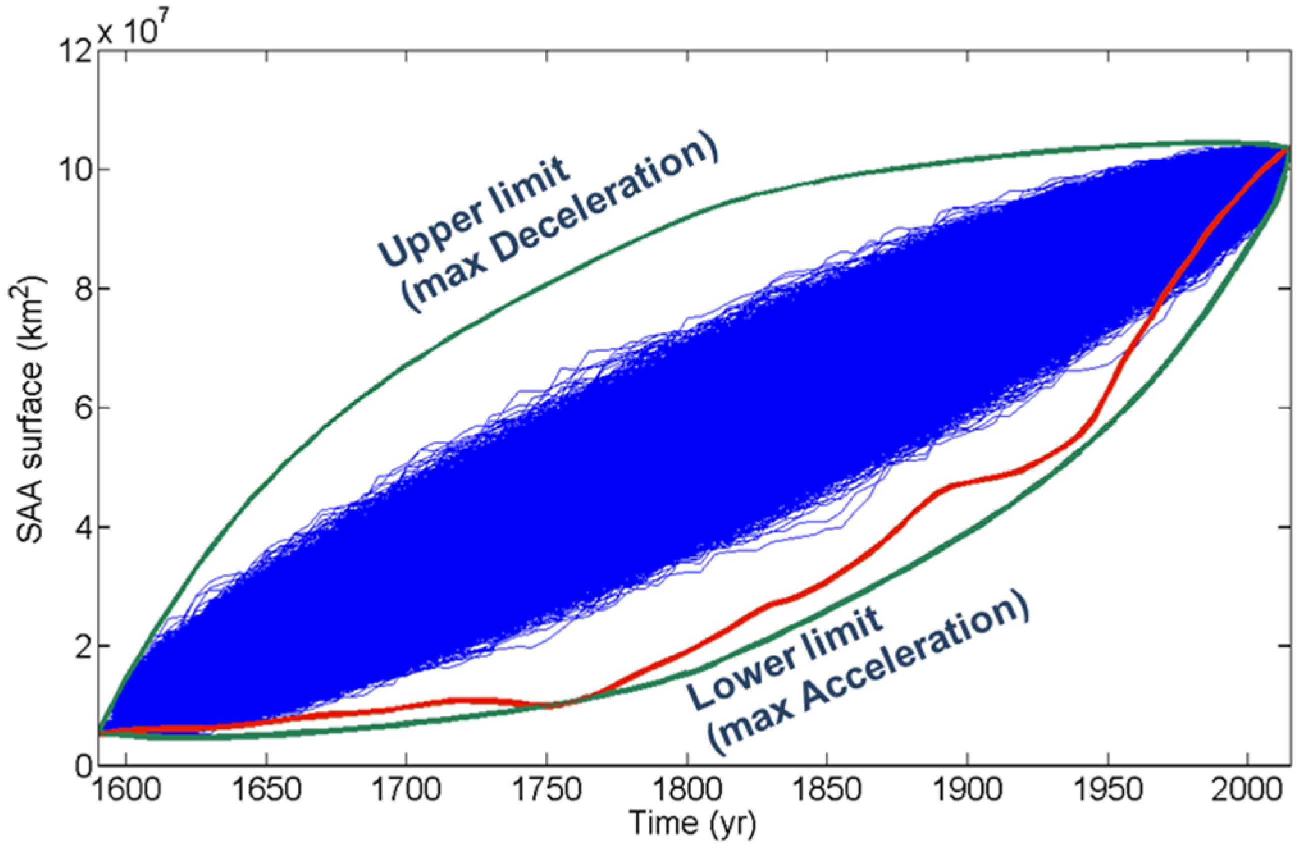

Figure 7. The real acceleration of the SAA in the last 400 years (red curve) compared with simulated data (blue curves) where all SAA increments have been randomly shuffled (using Matlab routines). Green curves represent the maximum acceleration (lower green curve) and deceleration (upper green curve).

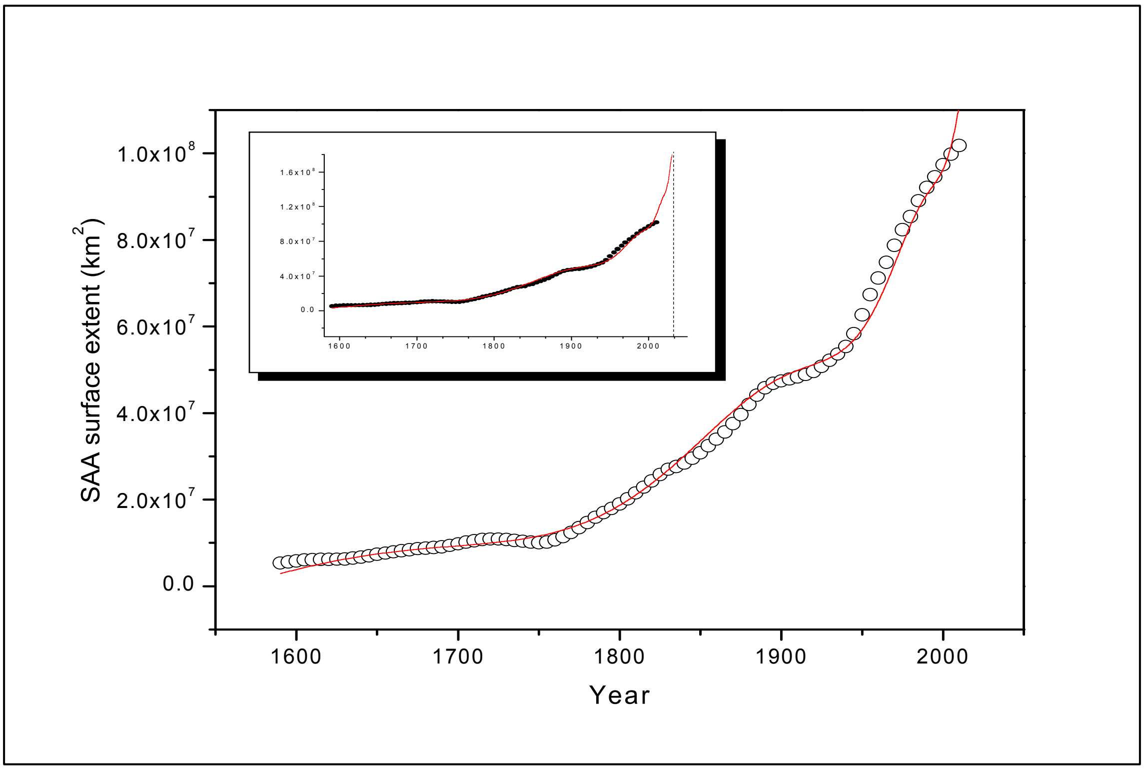

Figure 8. Extent of the SAA surface area over the last 400 years and the best nonlinear fit with the function indicated in the text as eq. (14). The "critical time" $t_c \approx 2034 \pm 3$ years, where the curve will approach a singularity. This time could represent the time of no return for a great change of the geomagnetic field, possibly going toward a reversal or excursion (redrawn from De Santis et al., 2013b).

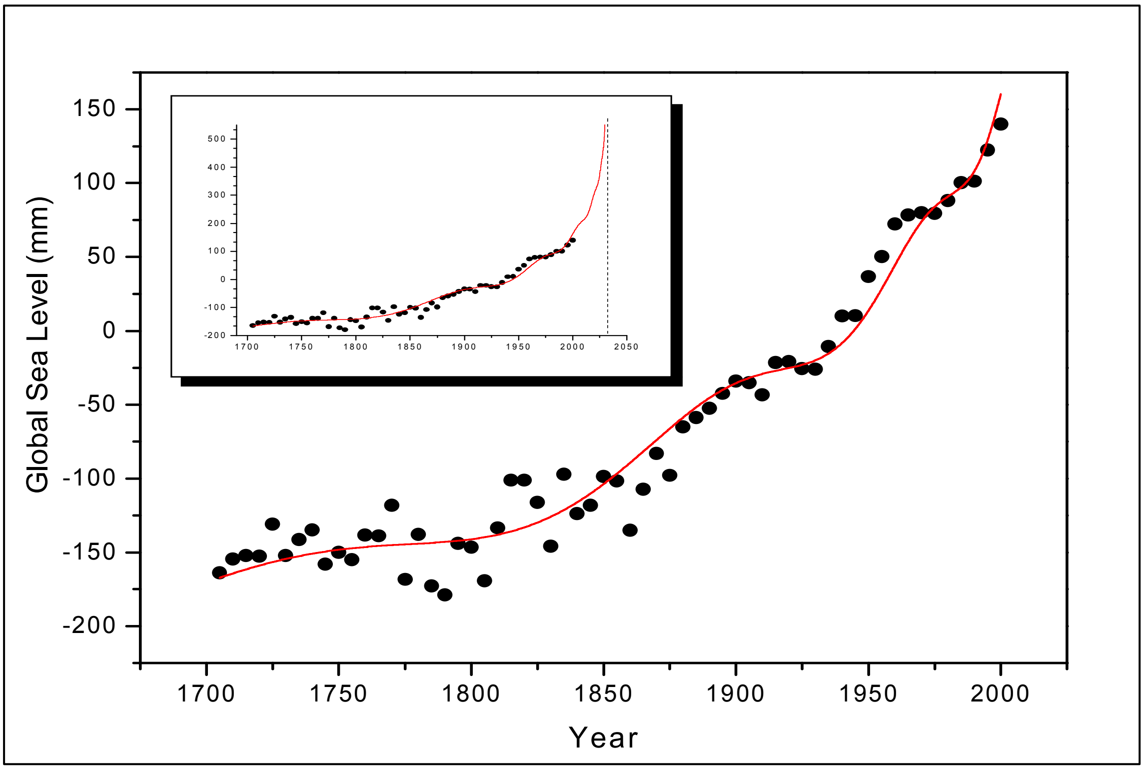

Figure 9. Global Sea Level rise (averaged every 5 years) and its best log-periodic fit with the critical time $t_c \approx 2033 \pm 11$ years (redrawn from De Santis et al., 2013b).

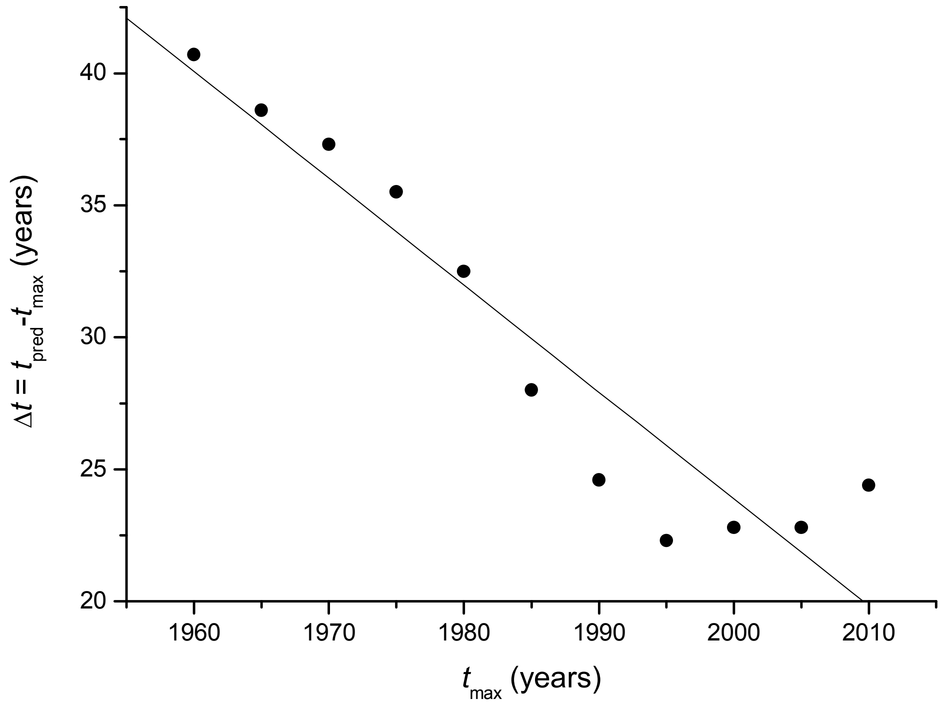

Figure 10. Behavior in time of the difference Δt between prediction (t_{pred}) and epoch (t_{max}) at which the prediction is made for the last 50 years. A linear fit predicts Δt=0 at around 2060, confirming that the most recent prediction (made at t_{max}=2010) of $t_c \approx 2034\pm3$ yrs is probably underestimated, or, alternatively, it is the error of ±3 yrs that is underestimated. However, in the last 15 years Δt almost stopped to decrease, so also this linear prediction must be taken with some caution.