ON THE UNCERTAINTIES OF SEISMIC PARAMETERS: A BAYESIAN FRAMEWORK FOR THEIR ESTIMATION USING BRUNE’S MODEL

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ABSTRACT

The estimation of seismic parameters from ground motion records is subject to many uncertainties, such as: (i) parameterization, modeling procedures and underlying hypotheses, (ii) approximated input parameters, (iii) instrumental errors on records and their impact in data processing, (iv) procedures to estimate model parameters. However these uncertainties are rarely treated and propagated to the final results. For example, on one side, density of rocks, velocity model, geometrical spreading, radiation pattern are just some of the common parameters needed to estimate the main seismic parameters of an earthquake and are generally used as average values. On the other side, uncertainties derived from the acquisition system and processing of the data are often neglected. Nevertheless, in many cases these uncertainties may be particularly important, as for example in the analysis of historical earthquakes, where both instrumental response and treatment of analog records intrinsically imply non negligible sources of uncertainty.

Here, we present a new Bayesian procedure to estimate seismic parameters that allows: (i) to obtain a robust estimation of the Brune’s model parameters (Brune 1970, 1971) and relative uncertainties, (ii) to account for the uncertainty related to the Earth model parameters used, and (iii) to propagate such uncertainties on the estimation of seismic parameters (seismic moment, moment magnitude, radius of the circular source zone and static stress drop).

It is important to highlight that this study does not intend to discuss the validity or the physical significance of the Brune’s model, but it is focused on the details of how to fit it on a dataset in order to evaluate the seismic parameters, accounting and properly propagating a rather large range of uncertainties. These capabilities of the proposed procedure are finally demonstrated through an illustrative application analysing seismic records from historical events.

CASE STUDY AND DATA: the 3rd April 1909 Benavente (Portugal) earthquake and its historical seismograms.

The reassessment of the seismic parameters analysing seismograms of an historic earthquake is a complex work. The treatment of analogic records of seismic waves poses different technical challenges; The waveforms obtained are the result of different operations performed to overcome the numerous problems that may occur during the analog to digital conversion of a historical seismogram (Rafail et al, 2008).

Furthermore, the instrumental characteristics of the seismometers, Information necessary for proper correction of the waveform, are often approximate or lost (see table 1, also).

As case study to test our methodology, we analyzed the Benavente earthquake (Portugal) occurred in April 3, 1909 (Fig. 1). This earthquake has already been studied by various authors, and estimates of the seismic parameters of this event based on historical records are available in literature (Teves-Costa et al., 1999).

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MODEL PARAMETER ESTIMATION: A Bayesian approach

We start assuming that our data d = (ds, dz, ... , de) and the model parameters are linked by a specific model (the “forward operator” often used in inverse theory, e.g. Menke 1989, Tarantola 2005). Our task is to infer the parameter values of a given model function g(·) that we sample in presence of noise: d = g(µ) + e

where e represents the “error” component in the observed data. Let be the vector containing the model parameters, f(·) the density of the prior distribution for d, f(·|d) the likelihood for the data. Then, Bayes’ theorem states:

\[ f(µ|d) = \frac{f(d|µ) f(µ)}{\int f(d|µ) f(µ) dµ} \]

Bayes’ theorem provides a tool for converting a initial set of “beliefs” about d, as represented by the prior distribution f(µ), into a posterior distribution f(d|µ), that includes the additional information provided by the data d.

Integrating out the data domain, the marginal probability density in the model parameter space provides the posterior distribution in the space of the model parameters. To obtain samples of the posterior distribution in the space of the model parameters we explore the model domain using a Markov-chain Monte Carlo (MCMC) approach based on the Metropolis (Metropolis and Ulam 1949, Metropolis et al. 1953; Hastings 1970). From the MCMC output (after eliminating a burn-in period and thinning procedure), we get information about model parameters: an empirical probability distribution (CDF or PDF) for each model parameter, and (where possible) compute percentiles, moments, etc. to produce both best gross values and associated uncertainties (e.g., see Fig 2).

ACKNOWLEDGEMENTS

We are extremely grateful to Josep Batlló, Ramon Macia, Jose Morales, Daniel Sich and Paula Teves-Costa for the great availability demonstrated in sharing with us their dataset on the earthquake studied. Thank you very much!