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If Space is Material, What Inertia Should Be?

Rediscovering a Dismissed Awareness of Ernst Mach

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Extended Abstract

Ernst Mach (1838-1916) was personally educated by his father, the idealist Johann Mach, until he enroll to the High School on 1853 (Dragoni et al., 1999). It could be a plausible supposition that the father's tuition had a strong and never completely overcomed influence on the intellectual pathways of his son. Ernst Mach became renown (Musil, 1908) with the book *Die Mechanik in ihrer Entwickelung historisch-kritisch dangerstellt* (Mach, 1883) and in particular for the criticism of the Newtonian concept of inertia founded on the experiment of the water in a rotating bucket.

Albeit the criticism to the exagerations of his time in proposing mechanical *ad hoc* imaginative interpretations for all physical phenomena were fully justified, he did not fully understand that the search for the causes of the natural phenomena is one among the main trigger for new progress in the sciences (Musil, 1908). His argument of an alleged simmetricity among the rotation of the bucket with respect to water and of the water with respect to the bucket, with the inertial centrifugal forces produced only by the motion with respect to all the distant masses of the universe, was adopted by scientists and philosophers as an explanation of the presence of 'inertia' in the sensible world, and is universally called "Mach's Principle". It is considered for more than a century as the highest expression of the philosophical rationality of the western world, but I will try to proof that, on the contrary, it is built on a unstable ground and with uncomplete assumptions: it is a masterpieces of captious logic of which the same Ernst Mach had some awareness.

Fig. 1. Ernst Mach (February 18, 1838, Brno, Austrian Empire – February 19, 1916, Munich, German Empire) became Professor of the History and Theory of Inductive Science at the University of Vienna.

The Mach's Principle is very often misinterpreted by scientific community with an illegitimate extrapolation of the Mach words. I can propose some critics arguments to fix my views:

(1) – Mach's aim was only to pose a limit, a boundary, to the possible truthful statements about the experiment of Newton of the rotating bucket filled with water.

(2) – He did not intend to assign particular and still unknown properties to the distant mass of the Universe, but only and simply to assume them as a suitable reference frame. Indeed, Mach (1883) said: *No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick. The one experiment only lies before us, and our business is, to bring it into accord with the other facts known to us, and not with the arbitrary fictions of our imagination.* (pag. 232, English transl., 1919)

(3) – Many followers of Mach, but with a lower philosophical level, had the unfortunate idea to force the argument toward the possible existence of an unclear influence of all the distant masses on the laboratory test mass. This is somewhat equivalent to try to restore the concept of "action at a distance", so lowed by idealism.

(4) – In defining inertia, meditating about the rotating buckle experiment is not sufficient. It is more important to observe that a body can be deviate from its natural rectilinear path only by applying a force F, and that in the time window of the action of F "inertial forces" appears (e.g. Coriolis forces).

(5) – The preceding point (4) is a fundamental clue that cannot be disregarded about the *local* and *symmetric* nature of what we have to search for.

(6) – A process that fulfills the above requested properties of *locality* and *symmetricity* is the motion of bodies in a fluid. Theory of hydrodynamics states that a solid body can move with constant velocity without friction in a perfect fluid. It undergoes resistance only if accelerates – perceiving only now the influence of the otherwise undetectable sourrounding milieu. Symmetrically, a constant velocity wind of a perfect fluid is not perceived by a solid body, while if the wind accelerates, a force is experienced by the body – perfectly analogous to a gravity force and nothing but an *equivalence principle*.

Fig. 2. The Dutch mathematician, astronomer, physicist and horologist, Christiaan Huygens (14 April 1629, The Hague, Netherlands – 8 July 1695, The Hague, Netherlands) was friend of René Descartes. He worked about the explanation of gravity adopting a Cartesian point of view. Many others have followed his way, like Newton, Fatio, Le Sage, Yarkovsky (1888), Hilgenberg (1976) (Aiton, 1969; Van Lunteren, 2002: Edwards, 2002; Scalera & Jacob, 2003) in the past and many others in the present (Blinov, 2011; Cahill, 2009; among many others). The continuity and ineradicability of these ideas in the sciences (Musil, 1908) are demonstration that science-methods cannot be limited by the Mach's philosophy.

Ernst Mach showed some awareness of the incompleteness of his reasoning about inertia by writing (1883) with a great intellectual honesty:

It might be, indeed, that the isolated bodies A, B, C . . . . play merely a collateral role in the determination of the motion of

the body K, and that this motion is determined by a medium in which K exists. In such a case we should have to substitute this medium for Newton's absolute space. Newton certainly did not entertain this idea. Moreover, it is easily demonstrable that the atmosphere is not this motion-determinative medium. We should, therefore, have to picture to ourselves some other medium, filling, say, all space, with respect to the constitution of which and its kinetic relations to the bodies placed in it we have at present no adequate knowledge. In itself such a state of things would not belong to the impossibilities. It is known, from recent hydrodynamical investigations, that a rigid body experiences resistance in a frictionless fluid only when its velocity changes. True, this result is derived theoretically from the notion of inertia; but it might, conversely, also be regarded as the primitive fact from which we have to start. Although, practically, and at present, nothing is to be accomplished with this conception, we might still hope to learn more in the future concerning this hypothetical medium; and from the point of view of science it would be in every respect a more valuable acquisition than the forlorn idea of absolute space. (pag. 230-231, English transl., 1919)

And Musil (1908) considered that:

It is difficult to say whether this Newtonian hypotheses non fingo *should be taken to signify no more than a methodological attempt to separate the secure goals of physico-analytic enquiry from the uncertain results of the philosophico-physical considerations erected on the top of these – such a boundary would correspond to the level of knowledge at that time, but could gradually be changed to incorporate the "hypotheses" into what had already been proven.*(pag. 32, English transl., 1982)

1. Appendix

A short synthesis (from text-books of Hydrodynamics) is shown here of the hydrodynamical treatment of the motion of a sphere in a perfect fluid (incompressible, irrotational, without friction, infinite extent, at rest at infinity, no gravity or other body forces acting on it).

The well known equation of continuity

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

with incompressible fluids ($\rho = const$) reduces to: $\nabla \cdot \mathbf{v} = 0$; or, if the fluid is irrotational:

$$\nabla \cdot \nabla \Phi = 0, \tag{1}$$

namely the Laplacian of the scalar field Φ, from which the velocities components can be derived:

$$v_x = \frac{\partial \Phi}{\partial x}, \qquad v_y = \frac{\partial \Phi}{\partial y}, \qquad v_z = \frac{\partial \Phi}{\partial z}.$$

It is straightforward to show that, transforming to polar coordinates, equation (1) can be written:

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Phi}{r \partial \theta}\right) = 0,$$

that should be solved adopting the two boundary conditions:

$$\left(\frac{\partial \Phi}{\partial r}\right)_{r=a} = V\cos\theta, \qquad \text{and} \qquad \left(\frac{\partial \Phi}{\partial r}\right)_{r=\infty} = 0,$$

with V the velocity of the sphere along the X axis, and a the radius of the sphere.

With few algebra it is easy to show that a solution is:

$$\Phi = Ar \cdot \cos\theta + \left(B/r^2\right) \cdot \cos\theta.$$

From the boundary conditions, it is possible to infer the two constants A and B:

$$A = 0, \qquad \text{and} \qquad B = \frac{1}{2}Va^3,$$

and finally

$$\Phi = \frac{Va^3}{2r^2} \cdot \cos\theta.$$

Now we have all the elements to derive the kinetic energy E_k of the fluid:

$$v_r = -\frac{\delta\Phi}{\delta r} = \frac{Va^3}{r^3}\cos\theta,$$

$$v_\theta = -\frac{\delta\Phi}{r\delta\theta} = \frac{Va^3}{2r^3}\sin\theta,$$

$$v^2 = v_r^2 + v_\theta^2 = \left(\frac{Va^3}{r^3}\right)^2 \cdot \left(\cos^2\theta + \frac{1}{4}\sin^2\theta\right),$$

$$E_k = \int_a^\infty \int_0^\pi \frac{1}{2}\rho v^2 \cdot 2\pi r^2 \cdot \sin\theta \cdot d\theta dr =$$

$$\pi\rho V^2 a^6 \int_a^\infty \frac{dr}{r^4} \int_0^\pi (\cos^2\theta + \frac{1}{4}\sin^2\theta)\sin\theta d\theta =$$

$$= \frac{1}{3}\pi a^3 \rho V^2 = \frac{1}{4}M'V^2.$$

The constant quantity M' is the mass of the fluid displaced by the sphere and $\frac{1}{2}M'$ is the apparent mass of a massless sphere. This mass produce an inertial effect, whatsoever was the mass of the sphere, a fact that points towards the uselessness of the concept of mass of the bodies. Both mass and inertia could well be attributes coming from a pervading medium.

The astonishing and wonderful thing in this simple derivation of the inertial effect is that we can imagine the sphere as a simple massless *geometrical* entity. This means that a meditation deserves the role of *geometry* in this conception. Could the distorsions of the space be redefined in a more local manner? Are waiting pure *shapes* and *extents* for a deeper role in fundamental physics? Obviously the agreement with all the relativistic effects sould be searched for.

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