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**PlatEMotion: a Matlab[®] Tool for
Geodetic Reference Frame Definition**

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PLATEMOTION: A MATLAB® TOOL FOR GEODETTIC REFERENCE FRAME DEFINITION

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Introduction

An important concept related to plate tectonics theory is the internal rigidity of tectonic plates together with Euler's theorem, which allows modelling the relative motion of plates quantitatively. Roughly speaking, plates can be viewed as rigid caps on the surface of a sphere. The motion of a plate on a sphere can be described by a rotation about a virtual axis that passes through the centre of the sphere (Euler's theorem). This implies that an angular velocity vector originating at the centre of the Earth can describe motions of plates. The most widespread parameterization of such a vector is using latitude and longitude to describe the location where the rotation axis cuts the surface of the Earth, and a rotation rate that corresponds to the magnitude of the angular velocity. Formally, the latitude and longitude of the angular velocity vector constitute the so called 'Euler pole'.

Here, we describe the main features of the developed software tool, called PlatE-Motion (PEM), which allows calculating both the expected velocity value for any point located on the Earth providing a Euler pole (direct problem) and inferring the Euler pole parameters by inverting the observed velocities at a set of sites located on a rigid block (inverse problem). PEM is developed in Matlab[®] framework and is designed for easy-to-use file exchange with the GAMIT/GLOBK software package [Herring *et al.*, 2006], and has a user-friendly graphical user interface (GUI). The file formats of the data used by PEM are not only restricted to the GAMIT/GLOBK file formats, but they can also be simple ASCII files generated from other kind of software. The tool is available for the scientific community. The interested user can freely obtain it by simply contacting the authors.

1. Mathematical background

In a mathematical formulation, let a point P on the Earth's surface be defined by its Cartesian coordinates X, Y and Z (in meters). We have chosen to work in Earth-Centred-Earth-Fixed (ECEF) Cartesian coordinates because the plate motion calculations are much easier in this frame using rotation vectors. In an ECEF frame, at a given location P, for a plate rotation defined by the rotation vector $\Omega(\omega_x, \omega_y, \omega_z)$ (in rad/yr), the velocity vector in geocentric coordinates $V(v_x, v_y, v_z)$ (in m/yr) is given by the cross-product:

$$\bar{V} = (\bar{\Omega} \times \bar{P}) = \begin{pmatrix} Z\omega_y - Y\omega_z \\ Z\omega_z - X\omega_x \\ Y\omega_x - X\omega_y \end{pmatrix} \quad (1)$$

For the direct problem, the resulting site velocities are then in ECEF frame so they need to be converted to local NEU frame. A valid velocity conversion at the corresponding geodetic location, defined with geodetic latitude, longitude and height (above ellipsoid) respectively (λ, φ, h) , is a combination of the 3 rotations needed to align the ECEF frame with the NEU frame:

$$\bar{L} = \bar{R}\bar{V} \quad (2)$$

where R is the rotation matrix:

$$\bar{R} = \begin{bmatrix} -\sin \lambda \cos \varphi & -\sin \lambda \sin \varphi & \cos \lambda \\ -\sin \varphi & \cos \varphi & 0 \\ \cos \lambda \cos \varphi & \cos \lambda \sin \varphi & \sin \lambda \end{bmatrix} \quad (3)$$

and L (N,E,U) are the components of the velocity in the local frame.

Regarding the inverse problem, in order to calculate the pole parameters by knowing the velocities of some reference sites, the direct problem shown in equation (1) can be written in matrix form. In particular for a given site, and taking into account the measurement errors, the equation becomes:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = A\bar{\Omega} + \varepsilon = \begin{bmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \varepsilon \quad (4)$$

where A is the design matrix and ε is the vector of measurement errors. Having 3 unknowns and 3 equations per site, if there are at least 2 sites with known velocities, the problem is over-determined and can be solved by minimizing the residual model errors through the use of the least squares approach [Tarantola and Vallette, 1982]:

$$\bar{\Omega} = (A'WA)^{-1} A'W\bar{V} \quad (5)$$

where W is the data weight matrix that is typically chosen as the inverse of the measurement error matrix. The model covariance matrix is then:

$$C_{\Omega} = (A'WA)^{-1} \quad (6)$$

2. Goodness-of-fit

In the Euler pole estimation, assessing the quality of a model solution becomes a fundamental step to evaluate the quality of the whole process of plate-frame identification. Different tests can be carried out for this goal; the most important ones are implemented in PEM for an easy model evaluation.

2.1 Error estimation

In order to take into account all the possible uncertainty sources, a Monte Carlo resampling step, named Jackknife, has been implemented in PEM for propagating the uncertainties through the whole inversion process and obtaining robust solutions with valid associated covariance matrix.

2.2 χ^2 test

The quality of the modelled Euler pole can be evaluated statistically by using the chi-square test. The chi-square test (χ^2 test) considers as null hypothesis the chi-square distribution of the asymptotic (by increasing the samples) sampling distribution of residues. The chi-square statistic is defined by this sum:

$$\chi^2 = E' C_E^{-1} E \quad (7)$$

where E is the vector of residuals $E=(V_x-v_x, V_y-v_y, V_z-v_z)$ and C the associated covariance matrix. The method of least-squares is built on the hypothesis that the optimum description of a set of data is one which minimizes the weighted sum of squares of deviations, between the data, V , and the fitting model v [Tarantola and Vallette, 1982]. Hence the smaller the chi-square value, the better the model.

The ratio between the estimated variance in the residues and the theoretical variance of the hypothetical distribution can be estimated by χ^2/ν , where, for an Euler pole estimation, $\nu = 3N - 3$ is the degree of freedom, N is the number of sites. χ^2/ν is called the reduced chi-square statistic and for a good model it assumes values around 1.

In PEM, the reduced chi-square of the residues can easily be calculated, allowing to compare different models with different degrees of freedom.

2.3 F-test

This test allows comparing the results from two least-square estimations in order to detect which model best fits the data, according to the degree of freedom (*i.e.* the number of observations minus number of unknowns) [see Nocquet et al. 2001 for more details]. As described in the previous section, the least-squares estimation of an Euler pole minimizes the term in equation (7).

For a given set of observations, adding unknown parameters in least-squares inversion makes χ^2 decrease. The F-ratio evaluates if the decrease of χ^2 from a model with n_2 versus n_1 ($n_1 > n_2$) degrees of freedom is significant. Since the statistic (7) is χ^2 distributed with n degrees of freedom, the ratio

$$F = \frac{[\chi^2(n_1) - \chi^2(n_2)] / (n_1 - n_2)}{\chi^2(n_2) / n_2} \quad (8)$$

follows a Fisher-Snedecor distribution with n_1 versus n_2 degrees of freedom. This experimental ratio is compared to the expected value of a $F(n_1 - n_2, n_2)$ distribution, for a given risk level α ($\alpha = 1\%$ or 5% , corresponding to a 95% or 99% confidence level).

For our objectives, the null hypothesis, following the rigid plate tectonics assumption, is that all the site velocities can be modelled with the same Euler pole. We therefore test

$$F \leq f_{\alpha}^{n_1 - n_2, n_2} \quad (9)$$

where $f_{1-\alpha}^{n_1, n_2}$ is the $\alpha\%$ fractile for a Fischer-Snedecor law with (n_1, n_2) degree of freedom. We use this test to verify if the velocity of an individual site is consistent with a set of sites. In particular, we compare the two estimations of the Euler pole with and without an individual site. The degree of freedom for a Euler pole estimation is $n_1 = 3 \cdot N - 3$ for N site velocities, and it becomes $n_2 = 3 \cdot N - 3 - 3$ by subtracting a site. If (9) is not verified, we can assert, with a risk of $\alpha\%$ of being wrong (equivalent to a $100 - \alpha\%$ confidence level), that this site velocity is not consistent with the velocity of the other sites.

PEM allows to estimate the F-test, also showing the sites that are not consistent with the other velocities.

2.4 Correlation coefficient

In order to estimate the consistency of the results, a correlation coefficient r has also been implemented on PEM to evaluate the statistical significance of the correlation:

$$r = \frac{3N \sum V_i v_i - \sum V_i \sum v_i}{\sqrt{3N \sum V_i^2 - (\sum V_i)^2} \cdot \sqrt{3N \sum v_i^2 - (\sum v_i)^2}} \quad (10)$$

where V_i and v_i represent the observed and modelled values respectively, and N represents the number of sites. The r absolute values can take on any value between 0 and 1, with a value closer to 1 indicating a better fit. The meaning of r can be explained by the coefficient of determination, r^2 , which indicates the percentage of data P_i that in turn can be explained by data O_i . According to statistical decision theory, $r^2 > 70\%$ ($r = 0.837$) is an acceptable test for prediction [Mulargia and Gasperini, 1995].

3. Functionality

PEM is an application that runs in Matlab 7.1 and above. The current version is 1.0 but the tool is still being developed and future versions are expected. An example of the PEM screenshot is shown in Figure 1 with some of the main features:

- 1: panel for loading input data files (see section 3.1 for details about the format of input files). Once the data file is selected, a box message will appear, asking for time alignment (this message will appear only for `org_file` and `apr_file` formats, because they contain information about the observation epoch). This could be useful to align all site positions to the same observation epoch. Once the data file is loaded, its name is added in the box list. Several files can be loaded at the same time. The formats of the input data will be described in a following section.
- 2: panel for loaded stations. Once an input data file is loaded, all stations contained in it will be listed here and unwanted stations can be selected and deleted from further computations.
- 3: panel for station parameters.

- 4: panel for displaying site locations, velocity vectors and Euler pole location in a simple map projection.
- 5: panel for managing plots on panel 4.
- 6: panel for Euler pole estimation. The “Pole estimation” button performs the pole estimation from the selected loaded data file (inverse problem). The “Calculate Rotation” button runs the expected velocity vector from a given (or computed) Euler pole at sites contained in a selected loaded data file (direct problem). The “Calculate Residue” button computes the residual velocity vector at selected sites from a given (or computed) Euler pole. Once a pole is estimated, it can be plotted on the map projection (“plot pole” button on panel 5).
- 7: “send pole” button. It allows transferring the current Euler pole solution to other loaded data frame in the list 1.
- 8: “F-test” button. It calculates a F-ratio test [Nocquet et al. 2001] in order to test if the velocity of one or more sites are consistent with the other set of velocities.
- 9: “chi2” and “r” buttons. They are useful for estimating the consistency of the results as described in the previous sections.

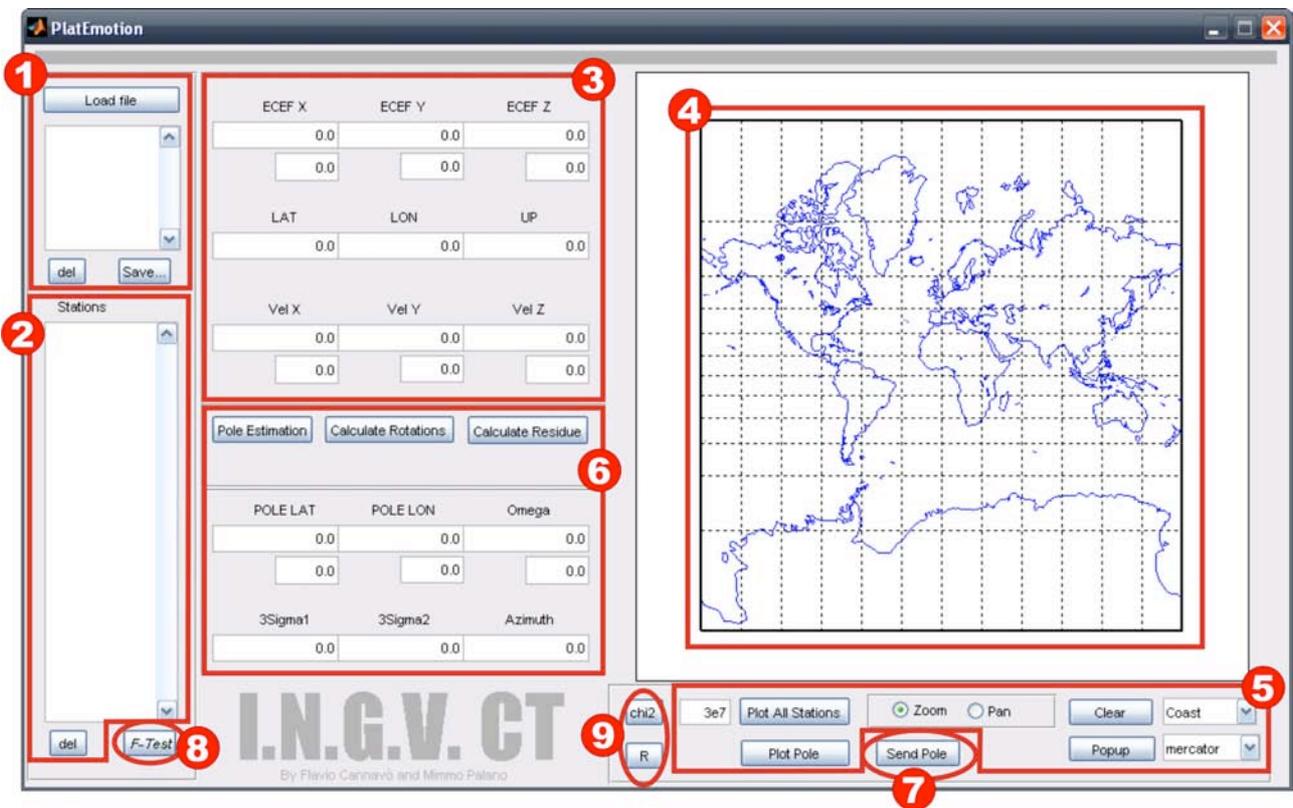


Figure 1. PEM screen view showing main functions: 1): panel for loading input data files; 2) panel for loaded stations; 3) panel for station parameters; 4) panel to display site locations, velocity vectors and Euler pole location in a map projection; 5) panel for managing plots on panel 4; 6) panel for Euler pole estimation; 7) “send pole” button; 8) “F-test” button. 9) “chi2” and “R” buttons. Please see text for details.

3.1 Loading and editing data

The formats of the input data files used by the tool are mainly specific for GAMIT/GLOBK, but they are simple ASCII files that can be generated from other file formats. In the following, we describe the formats of the input files accepted by the software.

org_file: this file represents the output from the run of the *glorg* module of GAMIT/GLOBK. In particular, this file contains a summary of the final solution, including estimates and uncertainties of the parameters and various representations of these parameters (e.g. baseline components). For more details about this file, please refer to the GAMIT/GLOBK reference manual (<http://www-gpsg.mit.edu/~simon/gtgk/docs.htm>).

apr_file: this file contains the a priori coordinates and velocities to be used for a specified set of sites in the *glorg* solution. This file has the form as the Cartesian version of the GAMIT L-file with a blank character in the first column:

```
ALAC_GPS 5009051.16577 -42072.14540 3935057.76804 -0.00968 0.02008 0.01329 2007.239 0.0006 0.0005 0.0005
ALME_GPS 5105220.08201 -219278.46965 3804387.16865 -0.00754 0.01894 0.01326 2007.643 0.0007 0.0005 0.0006
VILL_GPS 4849833.63613 -335048.90209 4116015.00659 -0.01028 0.02006 0.01203 2010.999 0.0003 0.0001 0.0002
MADR_GPS 4849202.36404 -360328.87149 4114913.21558 -0.00904 0.01933 0.01262 2007.190 0.0006 0.0005 0.0006
.....
```

Each release of GAMIT/GLOBK software packages contains several *apr_file*, including the version of last ITRF solutions and additional files rotated to plate-specific frames. However, after an initial *glorg* solution, each user can create specific *apr_file*, containing selected sites by extracting them from the *org_file* output of *glorg*. Specifically, the *org_file* includes a listing of the estimated cartesian coordinates and velocities at the midpoint time of the solution in *apr_file* format but with the character string 'Apr.' in the first four columns; hence the command `grep 'Apr.' [apr file name] > temp.apr` will write these coordinates to the file *temp.apr*; to complete the task, 'Apr.' must be removed from the first columns of the output. Please refer to the GAMIT/GLOBK reference manual for more details.

vel_file: this file must contain the coordinates and velocities of selected sites. It should have an extension *vel* and should have the following format:

```
Site Long. Lat. E N E_σ N_σ
ALAC 359.519 38.339 20.00 16.53 0.07 0.07
ALME 357.541 36.853 18.60 15.62 0.09 0.08
VILL 356.048 40.444 19.30 16.70 0.01 0.01
MADR 355.750 40.429 18.60 16.38 0.07 0.07
...
```

Entries should be separated by a space. The first line of the data file is a header which is in free format. E and N describe the East and North velocities component expressed in mm/yr and E_{σ} and N_{σ} the associated errors.

The entries on each line are 4-character site name, site longitude (positive east, decimal degrees), latitude (decimal degrees), east velocity, north velocity, standard deviations for the east and north velocities. All units for the velocities are mm/year. There are 7 columns in the format.

Output data files are *apr_file* which can be used as input directly in the GLOBK runs.

4. Applications

In the following we report two examples of applications.

Example 1. On Mt. Etna, the use of a consistent and stable reference frame to correctly isolate the ground deformation has become critically important since the first GPS measurements were carried out on the volcano edifice and surrounding areas. In order to isolate volcano deformation, researchers working on Mt. Etna have referred GPS measurements to different reference frames over time, making the direct combination and comparison of results very difficult, especially for computing valid velocity estimates. This problem can be overcome by using a consistent and stable reference frame over time by adopting the same rigorous approach used to describe plate motions. Only the establishing of a local reference frame capable of i) isolating the volcanic deformation (or other local deformation) rigorously and ii) directly allowing the comparison of the ground deformation over time, may prove very useful to researchers wishing to analyse ground deformation on Mt. Etna both for research and monitoring activities.

In order to define a local reference frame to which the solutions of the Etna GPS network may be referred, we have analyzed continuous GPS data collected in southern Italy between 1996.00 and 2009.75 through the use of GAMIT/GLOBK. To provide a consistent and stable reference frame over the past years,

we also processed GPS survey-mode data coming from some benchmarks located around Mt. Etna (Figure 2) and used as reference frame in the past [see Palano et al., 2008 and references therein].

Finally, taking into account the ITRF2005 velocities and associated errors of 13 sites located within 350 km around Mt. Etna, we estimated the three Euler vector parameters (latitude and longitude of pole, rotation rate) for a block by minimizing, (using the PEM tool), the adjustments to two horizontal components of GPS velocity at each intra-block site. Observed and predicted velocities, misfit vectors, and best fit Euler pole are shown in Figure 2. Listings of Euler parameters and their uncertainties are included in Table 1. For more details, please see the paper by Palano et al. [2010].

Block	Latitude N° (<i>deg</i>)	Longitude E° (<i>deg</i>)	ω (<i>deg/Myr</i>)	E RMS (<i>mm/yr</i>)	N RMS (<i>mm/yr</i>)
Etn@ref	38.450 ± 3.870	-107.702 ± 4.588	0.263 ± 0.002	0.7	1.1

Table 1. Parameters of the Etn@ref Euler pole. Associated errors are also reported.

Example 2. In this example we estimated the Euler vector parameters by using as input the example.apr file which is included in the tool. This file contains the ITRF2005 velocities and associated errors of 34 sites located on the Eurasia plate (Figure 3). In a first step, after loading the file, we estimated the Euler vector parameters, whose details are reported in Table 2 under the “Strategy 1” section. As testified by $r^2 = 98.0\%$ ($r = 0.99$) and reduced $\chi^2 = 17.27$, the obtained solution is statistically adequate. However, by using the following strategy, we show how to improve the statistically quality of the results. In this way, after loading the file we performed some F-tests until “all the stations have passed the F-test with 97.5% confidence level”. Then we estimated the Euler vector parameters, taking into account all stations passing the F-test. Resulting Euler vector parameters are reported in Table 2 under the “Strategy 2” section. The $r^2 = 100.0\%$ ($r = 1.0$) and a reduced $\chi^2 = 0.86$, clearly indicate a statistically better quality result. Observed velocities and misfit vectors are shown in Figure 3.

	Latitude N° (<i>deg</i>)	Longitude E° (<i>deg</i>)	ω (<i>deg/Myr</i>)	r	χ^2
Strategy 1	54.265 ± 0.494	-101.614 ± 0.844	0.256 ± 0.002	0.99	17.27
Strategy 2	54.236 ± 1.015	-100.251 ± 1.754	0.257 ± 0.003	1.00	0.86

Table 2. Parameters of the Euler vector parameters and associated statistical tests as estimated by using the *example.apr* input file adopting two different strategies (please see text for details).

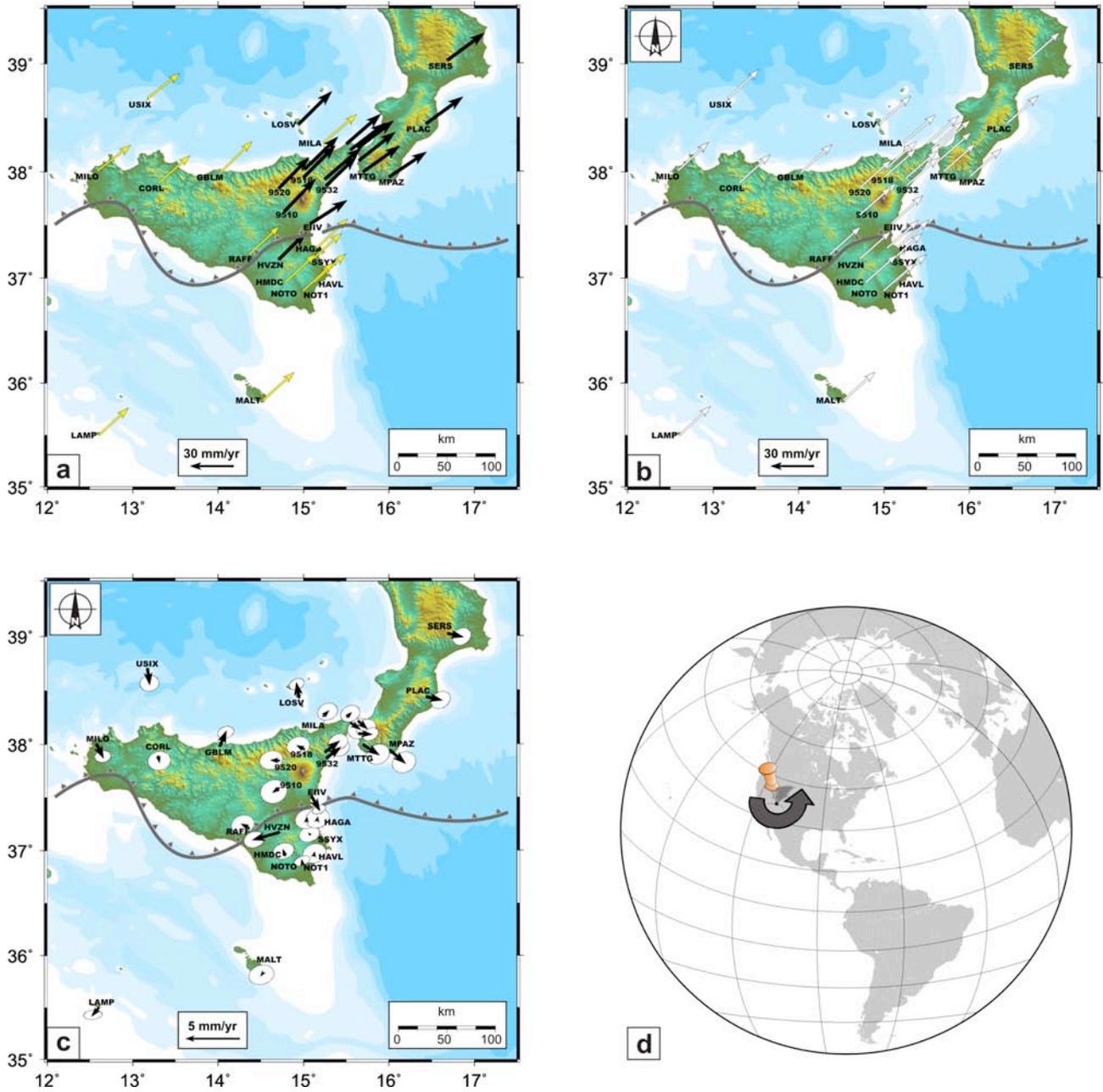


Figure 2. a) GPS velocities in ITRF2005 reference frame. The uncertainties have been excluded from the figure because their values are small with respect to the scale of displacements. Sites used to estimate Euler pole are reported in yellow. b) predicted velocities; c) misfit vectors and 95 percent confidence ellipses. The estimated errors also take into account the Euler pole error, computed through a Monte Carlo method; d) Euler pole derived in this study.

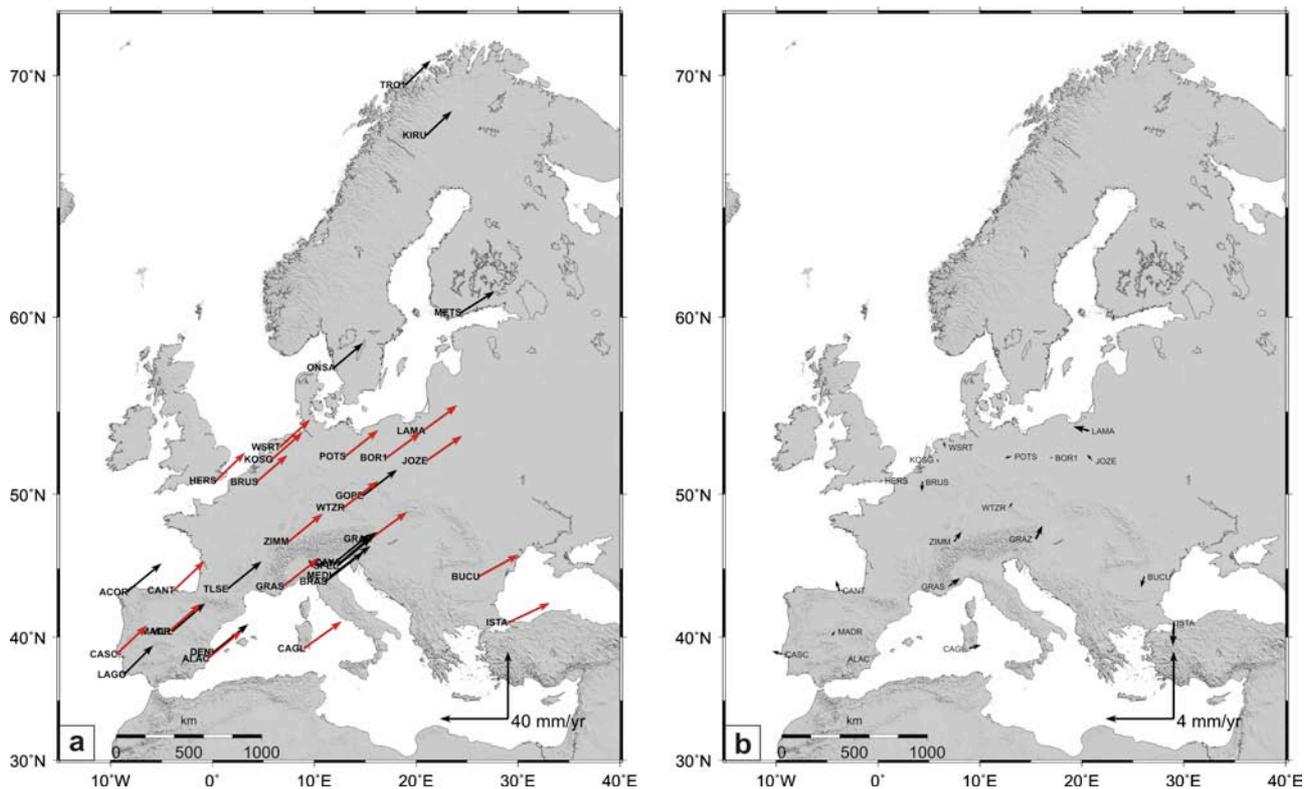


Figure 3. a) GPS velocities in ITRF2005 reference frame. The uncertainties have been excluded from the figure because their values are small with respect to the scale of displacements. Sites which have passed the F-test are reported in red. b) misfit vectors as obtained from “Strategy 2” (see text for details).

5. Disclaimer

PEM tool is provided free of charge for academic, non-profit use. Considerable time, effort and expense have gone into the development and documentation of PEM. The program has been thoroughly tested and used. In using the tool, however, the user accepts and understands that no warranty is expressed or implied by the developers on the accuracy or reliability of the program. The user must explicitly understand the assumptions of the program and must independently verify the results. Further information and documentation on PEM may be obtained from the developers.

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