ASSESSMENT AND MODELING OF LAVA FLOW HAZARD ON ETNA VOLCANO
Annalisa Cappello\textsuperscript{1,2}, Annamaria Vicari\textsuperscript{1}, Ciro Del Negro\textsuperscript{1}
(1) Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Catania, Italy
(2) Dipartimento di Matematica e Informatica, Università di Catania, Italy

Abstract
A methodology for constructing a probability map of lava inundation by considering the past eruptive behavior of the Mt Etna volcano is described. The basic a priori assumption is that new vents will not form far from existing ones and that such a distribution can be performed using a Gaussian kernel. The methodology is based on several steps: computation of susceptibility map that provides the spatial probability of vent opening; evaluation of the temporal probability for the occurrence of the hazard during the considered time interval; characterization of the expected eruptions; numerical simulations of lava flow paths, and elaboration of the hazard map. The application of MAGFLOW code, a physical-mathematical model, for simulating the lava flow paths represents the central part of this methodology for the hazard assessment at Etna. The simulation approach, to assess lava flow hazard, provides a more robust and locally accurate analysis than a simple probabilistic approach and accounts for the influence of the actual topography on the path of future lava flows.

Introduction
Mt Etna in Sicily (Italy) is one of the most active volcanoes in the world, and during the past 400 years it erupted over sixty times from vents on its flanks, while eruptive activity at its summit was nearly continuous. Its eruptions are often characterized by lava flows that spread along its flanks. Such eruptions can potentially reach the villages located to medium-low elevations. Even the area where city of Catania is settled was reached in the past by the flows outpoured from eruptive fractures opened at lower elevations. In the last century, the village of Mascali was destroyed by lava flows in 1928, while the villages of Fornazzo and Randazzo 1981 were threatened by lava flows. More recently, several tourist facilities have been repeatedly destroyed, with serious damage to the local economy specifically in the 2001 and 2002-03 eruptions.

The analysis of lava flows as a volcanic hazard at the Mount Etna is of special importance for the authorities and Civil Defense to take decisions in case of an eruptive crisis. In order to estimate the amount of damage that can be caused by a lava flow, it is useful to be able to predict the size and extent of such flows. A map showing areas that would be likely affected by future volcanic activity is extremely useful for long-term land use planning. Several hazard maps have already been proposed to help in risk management at Mt Etna. Some of these maps (e.g., Andronico and Lodato, 2005; Behncke et al., 2005) are mainly based on detailed databases and past eruptions, where the morphology of the volcano is only qualitatively considered despite its fundamental effect in determining lava paths. An alternative approach is to estimate the probabilities of lava-flow inundation from the combined estimate of the probability of an eruption occurring anywhere on the volcano, the probability of the eruption location, the probability of a
lava flow being generated, and the probability of specific lava-flow parameters. Wadge et al. (1994) have already used this approach for Mount Etna, where 380 lava flows were simulated by using both a stochastically chosen vent site and a set of parameters from a library of such parameters for lava flows erupted between 1763 and 1989. Favalli et al. (2009) applied the same approach and obtained hazard maps by simulating the inundation areas for a large number of possible future eruptions using an empirical relationship for the maximum length of lava flows.

On the base of our experience in modeling of the lava flow dynamics during past Etna eruptions, we present a new methodology for the identification of the zones that have the highest probability of being affected by lava flows, taking into account the location, extent and eruptive history of the source areas. The application of physical-mathematical models for simulating the lava flow paths represents the central part of this methodology for the hazard assessment at Etna. Recently, we have made significant progress in the hazard assessment at Etna through the development of accurate and robust physical-mathematical models able to forecast the spatial and temporal evolution of lava flows. With such simulations, one can explore a large number of eruption scenarios and these can specifically be used to estimate the extent of the inundation area. The probability of vent opening, the temporal probability, and the results obtained from the numerical simulations, are processed in order to obtain a final map showing for a given area at Mount Etna the probability of being affected by lava-flow inundation during the considered time interval.

**Methodology**

Lava-flow hazard can be defined as the probability for a given area being inundated by a lava-flow during a considered time interval. Many different approaches have been used for the generation of lava-flow hazard maps. However, we consider that any methodology for the elaboration of a volcanic hazard map, for a specific volcanic area and a specific time interval, should necessarily compute the following steps: computation of susceptibility map that provides the spatial probability of vent opening; evaluation of the temporal probability for the occurrence of the hazard during the considered time interval; characterization of the expected eruptions; numerical simulations of lava flow paths, and construction of the hazard map.

**Vent opening spatial distribution**

The spatial probability of vent opening, named here as volcanic susceptibility, can be a critical step for the evaluation of the lava flow hazard (Cappello et al., 2009). The estimation of the probability of volcanic eruptions is necessary to estimate the recurrence rate of volcanic events up until the time of investigation. Such estimates are based mainly on the past eruptive behavior of the volcano obtained from geological field observations, chronological and geophysical data. During recent years, new insights on the behavior of Mt Etna have been gained regarding the understanding of past eruptive activity, the dynamics of the volcano, the magma transfer processes, and the geophysical and geochemical monitoring. We use two different kinds of datasets. The first dataset collects the main volcanological parameters of all eruptions at Mt Etna since 1607: beginning and end of the eruption, the extent of the invasion area, the lava volume
erupted and the geographic coordinates of main vents (Coltelli et al., 2009). The second dataset comprises the geographic coordinates of faults, dikes and eruptive fractures (Neri et al., 2009).

In order to estimate the spatial recurrence rate \( \lambda_{xy} \) (eruptive vent density) we use the most common and largely used spatial point process model, which is based on the kernel technique. A kernel function is a probability density function (PDF) that is symmetric about the origin and spreads probability away from the event (Diggle, 1985). It is used to obtain the intensity of volcanic events at a sampling point \( P(x,y) \), calculated as a function of the distance to nearby structures and a smoothing constant \( h \). Different kernel functions can be used including the Cauchy kernel (Martin et al., 2004), the Epanechnikov kernel (Lutz and Gutmann, 1994) and the Gaussian kernel (Connor and Hill, 1995). It is widely agreed that the shape of kernel function chosen in this type of analysis generally has a trivial impact on probability calculations compared to other parameters (Connor and Hill, 1995; Lutz and Gutmann, 1994). We used the Gaussian function for the Mt Etna because vents are treated as discrete events in time and space, and the Gaussian model responds well to the patterns generally recognized in volcano distributions, such as clustering of vents. The formula for bivariate Gaussian kernel is given by:

\[
\lambda_{xy} = \frac{1}{2\pi h^2 N} \sum_{i=1}^{N} e^{-\frac{d_i^2}{2h^2}}
\]  

where \( d_i \) is the distance from the point \( P(x,y) \) to the \( i \)-th vent location, \( h \) is a smoothing parameter that controls the size of the zone to which each data point contributes an increased intensity, and \( N \) is the number of volcanic events considered in the calculation. Due to the fact that \( N \) occurs in the denominator, the integral \( \lambda_{xy} \) across the map will be unity. Therefore the spatial density is a bivariate probability density function.

Probability estimate made using Eq.1 depends on the value chosen for \( h \). The choice of the kernel function with appropriate values of \( h \) has some consequences for the parameter estimation, because it controls how \( \lambda_{xy} \) varies with distance from existing structures. Using a bivariate Gaussian kernel, events will have a high estimated probability in proximity to existing vents if the value chosen for \( h \) is small, but low estimated probability away from the vent. On the other hand, a large value of \( h \) will yield a more uniform estimate of probability distribution across the region. In the Gaussian kernel, the smoothing factor is equivalent to the standard deviation of a symmetric, bivariate Gaussian distribution. Therefore, the choice of the smoothing coefficient depends on the combination of several factors including size of the volcanic fields and degree of clustering. An optimum value of smoothing coefficient varies proportionally with the volcanic field size and vent density. A possible approach consists in comparing the observed nearest-neighbor distance between the vents with the expected distribution of nearest-neighbor distances (Weller at al., 2006). Based on the bivariate Gaussian kernel, the cumulative distribution function for fraction of vents located within distance \( D \) of their nearest neighbor is:

\[
f(D, h) = \text{erf} \left[ \frac{D}{h\sqrt{2}} \right]
\]
As shown in Fig. 1, plots for \( h = 750 \) m and \( h = 2200 \) m give the upper and lower bounds to curves generated by plotting cumulative nearest-neighbor distances of volcanic events at Mt Etna. A medium value (1500 m) between the lower and upper bounds has been chosen.

In order to calculate the final spatial PDF, we use the following datasets: location of faults, location of fractures, and location of dikes. We calculated the spatial density \( \lambda_{xy} \) for each dataset and a relevance value for each PDF has been given, which measures its importance and the quality of the dataset with respect to the evaluation of the susceptibility. We used a procedure of back analysis for determining the better weights for the different datasets: 5\% to faults, 5\% to dikes and 90\% fractures. Finally, the PDFs and their relative values are combined through a weighted summation to obtain the single spatial density containing all information.

**Rate of volcanic activity and probability estimates**

We calculated the temporal recurrence rate \( \lambda_t \) using an approach based on the repose-time method (Ho et al., 1991). In this method the duration of an eruption is ignored; only the onset date is considered as the most physically meaningful parameter and repose times from one onset date to the next are measured. Based on the definition of repose times, a volcanic recurrence rate \( \lambda_t \) is defined using a maximum likelihood estimator that averages events over a specific period of volcanic activity:

\[
\lambda_t = \frac{\sum_{i=1}^{N} e^{-\frac{t_i}{\tau}}}{\tau e^{-\frac{t_i}{\tau}} - \tau}
\]  

(3)
where $E$ is the total number of eruptions, $T_o$ is the age of the oldest eruption, and $T_y$ is the age of the youngest eruption. Mulargia et al. (1985) demonstrated that the time series of occurrence of flank eruptions at Mt Etna follow a stationary Poisson process. So the frequency of eruptions is represented by a Poisson distribution, a discrete distribution that describes the number of random events on an interval in space or time. Therefore the cumulative probability of having at least one eruption in a time interval $\Delta t$ is given by:

$$P(\Delta t) = 1 - e^{-\lambda \Delta t} \quad (4)$$

Once the spatial density (events per kilometer) and the temporal recurrence rate (events per year) are defined, we calculate the probability of an event occurring at each grid point $P(x,y)$ by using a Poisson distribution. If $\lambda_{xy}$ represents the intensity function normalized to unity across the entire area and $\lambda_t$ represents the regional recurrence rate, then:

$$P_{xy}\{N(t) \geq 1\} - \left( 1 - e^{-\lambda_t \Delta t} \right) \left( 1 - e^{-\lambda_{xy} \Delta x \Delta y} \right) \quad (5)$$

where $N(t)$ represents the number of future volcanic events that occur within time $\Delta t$ and area $\Delta x \Delta y$. We selected $\Delta x \Delta y$ as 1 km$^2$, and a time period of interest equal to 50 years (Fig. 2).

Fig. 2: Spatio-temporal vent opening probability map
Characterization of expected eruption

For the characterization of volcanic events, we used the knowledge of the main volcanological parameters of all eruptions at Mt Etna since 1607 to fix three different values of emitted lava volume (Tab. 1): 30, 100 and more than 100 million of cubic meters (i.e. 200). Then we established short-medium and large times of eruptions, setting respectively to 30 and more than 30 days of simulation (i.e. 90). Combining these values with random distribution, we obtained six possible functions representing the variation of flux rate in relation to the time of eruption. The shape of the curves has been considered as a kind of bell, in which the eruption starts from a low value of flux rate, reaching its maximum value after a 1/4 of the entire time of simulation. After 2/3 of the maximum time of simulation reaches 1/3 of the maximum value, finally, gradually decreases until the end of the eruption is reached (Fig. 3).

<table>
<thead>
<tr>
<th>Duration (days)</th>
<th>[0 - 30]</th>
<th>[30 - 100]</th>
<th>[ &gt; 100]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0 - 30]</td>
<td>34%</td>
<td>14%</td>
<td>0%</td>
</tr>
<tr>
<td>[ &gt; 30]</td>
<td>7%</td>
<td>23%</td>
<td>22%</td>
</tr>
</tbody>
</table>

Tab. 1: Statistics obtained analyzing the past eruptions of Mt. Etna

![Figure 3: Values of flux rate-days of eruption used for the simulations](image-url)
After calculation of the spatial-temporal vent opening probability map, which assigns a probability of activation to each vent in the grid, we need to estimate maps that assign a probability to each type of considered effusion rate and duration (event probability), devised on the basis of the emission behavior analysis of the study area.

For every type \(i\) of eruption, the location of volcanoes are been used to estimate the probability \(p_i\) for each point \(P(x,y)\) of the grid. When \(i\) is a type of category, \(n_i\) is the total number of events in a category, then:

\[
p_i(x,y) = \sum_{i=1}^{n_i} \frac{d_j^{-1}}{d_j \leq h}
\]

where \(d_j\) is the distance between the point \(P(x,y)\) and the \(j\)-th volcano if \(d_j\) is less or equal to \(h\).

So the value of \(p_i(x,y)\) at a given point depends on the number of vents found within a distance \(h\) of the point. If no vents are located within \(h\) of the point for a type of eruption, a virtual point is inserted at a distance \(2h\) in order to obtain a probability greater than zero. The estimates \(p_i(x,y)\) are then rescaled such that:

\[
\sum_{i=1}^{n_i} p_i (x,y) = 1
\]

For each category of event types we calculated a related smoothing factor using Eq. 2.

**Numerical simulations of lava-flow hazards**

To predict lava flow inundation areas we employ the MAGFLOW code (Del Negro et al., 2006), which has been extensively used in lava flow hazard applications at Mount Etna. MAGFLOW is based on cellular automata (CA) in which the states of the cells are the thickness of lava and the quantity of heat. The states of the cells are synchronously updated according to local rules that depend on values of the cell and the values of neighbors within certain proximity. In this way, the CA can produce extremely complex structures from the evolution of rather simple and local rules. The evolution function of MAGFLOW is a steady state solution of Navier-Stokes equations for the motion of a Bingham fluid on an inclined plane subject to pressure force, in which the conservation of mass is guaranteed both locally and globally (Vicari et al., 2007). However, this kind of evolution function induces a strong dependence on the cell geometry and position of the flux, with respect to the symmetry axis of the cell: flows on a horizontal plane spread preferentially in the direction of the mesh (the calculated length of lava flows depends on the relative directions of flow and the mesh). We solved this problem using a Monte Carlo approach, which allows obtaining cell geometry free results as well as calculating large-scale lava flows with no artificial anisotropy (Herault et al., 2008).

Once the activation and the event probabilities are developed, numerical simulations were computed using the MAGFLOW model. The simulations were performed using the typical parameters of Etnean lava flows. A 1 km grid spacing of vents is defined in the study area, and a prefixed number of simulations are executed for each of them, each one characterized by its own effusion rate and duration.
Elaboration of the hazard map and eruption scenarios

Finally, the resulting hazard map is thus compiled by taking into account the probability of vent opening, information on lava flows overlapping and their occurrence probability. This map is obtained by evaluating the hazard at each point in the study area as follows:

- for each simulation, the hazard related to a generic point in the study area is computed as the product of the defined probabilities of occurrence (conditioned probability) if it is affected by the simulated lava flow, zero otherwise;
- for each point, the conditioned probabilities are added over all the performed simulations.

The value assigned to each point of the grid represents the probability of being affected by a lava flow during the considered time interval. The spatial probability of vent opening can be a critical step for the evaluation of volcanic hazard. The most important point in the evaluation of the susceptibility map is how to convert any dataset into a PDF because the opening of a new vent can depend on the characteristic of each dataset. By assigning the same probability of occurrence to all performed simulations, a more trivial criterion of hazard mapping is obtained, simply based on the number of simulated events that affect a given zone. In Fig. 4 a preliminary hazard map of Mount Etna is shown. It is based on 5000 simulations of lava flow paths starting from more than 900 different potential vents. Most of the study area falls in class 1. Medium values of hazard are to be found close to the villages of Bronte and Linguaglossa. Only “Valle del Bove”, a huge depression in the eastern side of the volcano, is affected by the highest values of hazard. This map should help local authorities in making the necessary decisions to deal with ongoing eruptions and to plan long-term land use.

Fig. 4: Preliminary hazard map of Mt Etna
**Conclusions**

Hazard maps provide the probability of the future course of lava flows to enable quantitative hazard assessments and operational guidelines for, potentially, mitigatory actions to be undertaken. We have defined a methodology based on the numerical computer simulation of the flow paths over the surface of the volcano; these simulations are constrained by knowledge from former eruptions of Mt Etna. The simulation approach, to assess lava flow hazard, provides a more robust and locally accurate analysis than a simple probabilistic approach and accounts for the influence of the actual topography on the path of future lava flows. Generating multiple simulations it is possible to evaluate the probability of lava inundation anywhere on the surface of the volcano. This probability is captured as a hazard map, showing the relative frequency of lava flows that could potentially inundate specific areas. Such probability maps indicate the likely areas that could be affected but not which area will be covered by a specific eruption.

Detailed quantitative hazard maps have been produced for Mt Etna using MAGFLOW, a physical-mathematical model based on cellular automata to calculate lava flow paths. This new methodology of applying computer simulation techniques to the assessment of hazard from lava results in a map showing, at each location, the combined spatial and temporal probability to inundate areas. Obviously the accuracy of the results strictly depends on the reliability of the simulation model, on the quality of input data and on the hypotheses on assigning the different probabilities of occurrence. In particular, the activation and event probabilities (Eq. 5 and Eq. 6) constitute a critical step in the chain of this methodology. A key point is how to convert any dataset into a PDF because the opening of a new vent can depend on the characteristic of each dataset. Moreover, by assigning an equal probability of occurrence to all performed simulations, a more trivial criterion of hazard mapping is obtained, simply based on the number of simulated events that affect a given zone.

One of the most interesting possible extensions would be to have near real-time access to different datasets, for example volcanic monitoring data, for the evaluation of short-time susceptibility maps and continuous update of the expected scenarios in the case of unrest.

**Acknowledgment**

This study was performed with the financial support from the V3-LAVA project (INGV-DPC 2007-2009 contract).

**References**


