How to promote earthquake ruptures:

Different nucleation strategies

in a dynamic model with slip–weakening friction

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Abstract

The introduction of the linear slip–weakening friction law permits the solution of the elasto–dynamic equation for a rupture which develops on a fault, by removing the singularity in the components of stress tensor, thereby ensuring a finite energy flux at the crack tip. With this governing model, largely used by seismologists, it is possible to simulate a single earthquake event but, in absence of remote tectonic loading, it requires the introduction of an artificial procedure to initiate the rupture, i.e, to reach the failure stress point. In this paper, by studying the dynamic rupture propagation and the solutions on the fault and on the free surface, we systematically compare three conceptually and algorithmically different nucleation strategies widely adopted in the literature: the imposition of an initially constant rupture speed, the introduction of a shear stress asperity, and the perturbation to the initial particle velocity field. Our results show that, contrarily to supershear ruptures which tend to “forget” their origins, subshear ruptures are quite sensitive to the adopted nucleation procedure, which can bias the runaway rupture. We confirm that that the most gradual transition from imposed nucleation and spontaneous propagation is obtained by initially forcing the rupture to expand at a properly chosen, constant speed (0.75 times the Rayleigh speed). We also numerically demonstrate that a valid alternative to this strategy is an appropriately smoothed, elliptical shear stress asperity. Moreover, we evaluate the optimal size of the nucleation patch where the procedure is applied; our simulations indicate that its size has to equal the critical distance of Day (1982) in case of supershear ruptures and to exceed it in case of subshear ruptures.

Key words: Earthquake dynamics; Computational seismology; Nucleation process; Earthquake ground motions; Rheology and friction of fault zones.
1. Introduction

1.1. Overview

A large quantity of information about physical processes occurring during an earthquake event can be inferred from the results of dynamic models of seismic sources. In these mixed boundary condition problems the slip is assigned outside the region experiencing the rupture (typically assumed at rest or in a stable sliding regime) and the traction components are assigned inside this region. The rupture occurrence at a point on the surface (or in the volumetric region) of discontinuity of the medium (the “fault”) is determined by a fracture criterion, expressed in terms of maximum frictional resistance or in terms of energy. The singularities (in components of stress tensor and energy) at the tip of the rupture are removed by the introduction of a governing law which relates the magnitude of traction on the fault surface to some physical observables, such as the slip, the slip velocity, etc.. This makes it possible to obtain a non–singular solution of the elasto–dynamic equation in a discontinuous medium.

In the recent literature there is a lively debate about the most reasonable and realistic (from a physical point of view) analytical formulation of a fault governing law (Bizzarri and Cocco, 2006; Rice and Cocco, 2007) and the issue is still open (Bizzarri, 2009c). The most widely adopted (see for instance Harris et al., 2009) constitutive model is the slip–weakening (SW thereinafter) law, which prescribes that the magnitude \( \tau \) of fault traction decreases for increasing cumulative fault slip (Ida, 1972). SW law, motivated by the cohesive zone models developed for tensile fractures by Barenblatt (1962), Dugdale (1960) and Bilby et al. (1963), is conceptually simple and its incorporation within the numerical codes is straightforward compared to other more elaborated friction laws, such as rate– and state–dependent friction
laws (e.g., Dieterich, 1979). Moreover, it contains perhaps the most physically reasonable feature of a constitutive model, that the stress on the fault decreases (due to abrasion of surface asperities) as the rupture propagates and the fault slip accumulates. This attribute has been clearly recognized (Cocco and Bizzarri, 2002) also in the laboratory–derived rate– and state–dependent friction laws.

Contrary to rate– and state–dependent friction laws, by assuming the linear SW law it is impossible to simulate repeated ruptures on the same fault (i.e., to model the whole seismic cycle) and, more interestingly for the present matter, it is impossible to numerically reproduce the spontaneous rupture nucleation, unless external, time variable loading is inserted in the model (e.g., the tectonic load). As a matter of fact, the linear SW law does not contain any hardening effect (i.e., the strength increase for increasing slip; see for instance Matsu’ura et al., 1992) and it prescribes that the fault remains locked into its initial equilibrium state until the static level of friction is reached. Therefore the fault friction has to be increased, in some way, from the initial value ($\tau_0$, the stress distribution prior to the rupture) up to the static level. Evidently, in the specific case of a single dynamic rupture controlled by a linear SW law, the artificial increase of fault friction described above is not a physical, but rather a purely numerical procedure necessary to produce the desired rupture which expands on the fault in a dynamic fashion. Obviously, the dynamic models resulting from the application of this procedure have to satisfied some criteria, that will be described in section 5.

1.2. Critical lengths for nucleation

Two of the crucial aspects of the introduction of the artificial nucleation are the size and the shape of the fault patch where the nucleation procedure is applied. In the remainder of the
paper we will denote with $I_{\text{nucl}}$ this region, named the initialization (or nucleation) area, having border $\partial I_{\text{nucl}}$ (Figure 1). In the literature several critical lengths have been introduced to quantify the size of $I_{\text{nucl}}$; in the following of the paper we will quantify the dimensions of $I_{\text{nucl}}$ by referring to these quantities and to their mutual relationships.

Starting from energy balance considerations, Andrews (1976b) analytically derived an expression for the half–length that a 2–D, purely in–plane (i.e., mode II), bilateral crack has to reach in order to be able to spontaneously propagate farther:

$$L_{c}^{(\text{II})} = \frac{2}{\pi} G \frac{\lambda + G}{\lambda + 2G} \frac{\tau_u - \tau_f}{(\tau_0 - \tau_f)^2} d_0$$  \hspace{1cm} (1)$$

Equation (1) has its counterpart in the case of a 2–D, purely anti–plane (i.e., mode III), bilateral crack (Andrews, 1976a):

$$L_{c}^{(\text{III})} = \frac{G}{\pi} \frac{\tau_u - \tau_f}{(\tau_0 - \tau_f)^2} d_0$$  \hspace{1cm} (2)$$

In equations (1) and (2) $\lambda$ and $G$ are the Lamé constants, $\tau_u (= \mu_u \sigma_n^{\text{eff}})$ is the static stress in the SW model ($\sigma_n^{\text{eff}}$ is the effective normal stress), $\tau_f (= \mu_f \sigma_n^{\text{eff}})$ is the kinetic level of traction and $d_0$ is the characteristic SW distance (defining the breakdown, or cohesive, zone, where the stress drop is realized).

By considering an initially circular, uniformly expanding 3–D crack and by balancing the strain energy release rate and the energy dissipation rate at the crack edge, Day (1982) estimated the critical fundamental length scale for the dynamic solution as:
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\[ r_c^{(D)} = \frac{7\pi}{24} G \frac{\tau_u - \tau_f}{(\tau_0 - \tau_f)^2} d_0 \]  \hspace{1cm} (3)

which, for a Poissonian medium (i.e., when \( \lambda = G \)), is systematically greater (for the same parameters) than \( L_c^{(II)} \). Due to its theoretical derivation, the length scale defined in (3) appear to be appropriate to our fault model (see section 3). Even if the dynamics of some large strike–slip earthquakes can be understood by considering them as mode II rupture (since they are dominated by the in–plane sliding), in general, it is well known that 3–D problems are not simply a combination of modes II and III (e.g., Bizzarri and Cocco, 2005). In the following of the paper we will consider also the critical length scales defined by equations (1) and (2) since there are some attempts to quantify the extension of the initialization zone in 3–D geometries in terms of \( L_c^{(II)} \) and \( L_c^{(III)} \).

By generalizing to a 3–D geometry the 2–D analysis made by Uenishi and Rice (2003) in the case of an infinite, homogeneous, elastic space, Uenishi and Rice (2004) analytically found universal nucleation lengths for fault instability, which are the major and minor semi–axis of an elliptical initialization zone. In the case of Poisson’ s ratio \( \nu = 0.25 \) they are expressed as

\[ a_c^{(UR)} \approx 1.299G \frac{d_0}{\tau_u - \tau_f} \]  \hspace{1cm} (4)

and

\[ b_c^{(UR)} \approx 0.9755G \frac{d_0}{\tau_u - \tau_f} \]  \hspace{1cm} (5)
respectively. Interestingly, the ratio $a_{c}^{(UR)}/b_{c}^{(UR)} = 1.33$ equals the ratio $L_{c}^{(II)}/L_{c}^{(III)}$ for a Poissonian medium. When these critical nucleation lengths are reached, the quasi–static regime no longer exists and the instability is then dynamically controlled by the friction law and the rupture grows spontaneously. We notice that, contrarily to the other critical lengths, $a_{c}^{(UR)}$ and $b_{c}^{(UR)}$ are independent on the pre–stress $\tau_{0}$. The Uenishi and Rice’s model additionally include a condition specifying the shape of slip and traction; in the following of the paper we will not directly compare our results with the Uenishi and Rice’s model, but we will simply consider the critical lengths defined in equations (4) and (5) when we will evaluate the size of $I_{nuc}^{r}$.

Finally, we notice that, for a Poissonian medium, if $(1 + S)^{2} > 3.06$ (where $S = \frac{\tau_{u} - \tau_{0}}{\tau_{f} - \tau_{0}}$ is the strength parameter; Andrews, 1976b), it results that $a_{c}^{(UR)} < L_{c}^{(II)} < r_{c}^{(D)}$ and $b_{c}^{(UR)} < L_{c}^{(III)} < r_{c}^{(D)}$.

2. Goals of the present study

The study of the nucleation process is an extremely challenging problem from a numerical point of view (see Lapusta et al., 2000 for a discussion) and the efforts spent are motivated by several reasons. First of all, nucleation has a fundamental importance in the physics of earthquakes per se. Secondly, it has immediate practical implications (e.g., Iio, 1995; Lapusta and Rice, 2003). Finally, because the relation between strength of its initiation (i.e., the nucleation size) and the ultimate size of the ensuing earthquake event is still the matter of an animated debate (Ellsworth and Beroza, 1995; Kilb and Gomberg, 1999).

A detailed study of the nucleation process, accounting for the underlying physics
(describing the evolution of slip on a pre-existing main frictional surface or in an increasing coalescence of distributed micro-cracks in a rock volume, solicited by progressive loading), is beyond the purposes of the present paper. For some connections between nucleation phase and properties of a dynamic rupture we recall here, among others, Festa and Villotte (2006) and Shi and Ben-Zion (2006). On the contrary, the aim of the present study is to provide a methodological tutorial on algorithmic issues associated with the problem of the initiation of a synthetic rupture. We emphasize that without a systematic comparison of the different nucleation strategies it is impossible to establish a priori how much the resulting rupture propagation is biased by the nucleation procedure, and what are the optimal parameters to be used (size and shape of initialization patch, inherent parameters of each nucleation algorithm).

As we will discuss in section 4, different nucleation strategies have been used in the various implementation of SW law presented in the literature (Day, 1982; Andrews, 1985; Ionescu and Campillo, 1999; Bizzarri and Cocco, 2005; Dunham and Bhat, 2008 among many others), but they have not been rigorously and systematically compared. This paper aims to fill this gap.

The scientific objectives of the present paper can be summarized as follows: i) to explore and quantify, through numerical experiments representative of typical crustal earthquakes, the effects of the different nucleation procedures on the further rupture propagation and on the synthetic signals on the free surface; and ii) to establish the parameters that have to be used in the various strategies to obtain the “desired” solution of the dynamic problem. In the comparison of the results of the numerical experiments we will rely on the quantitative criteria described in section 5.
3. Fault model and numerical method

In this paper we consider an isolated, planar, strike–slip fault embedded in a perfectly elastic, isotropic half–space, initially at rest and subjected only to stress perturbations excited by the earthquake source. The considered fault geometry is reported in Figure 1. The elasto–dynamic problem is numerically solved by neglecting body forces, by using the conventional grid, finite difference code described in Bizzarri and Cocco (2005), which is 2nd–order accurate in space and in time and OpenMP–parallelized. The rupture developing on the fault is fully dynamic, since we include full account of inertial effects (see also Bizzarri and Belardinelli, 2008) and truly 3–D, since we independently solve the equations of motion for both the two components of physical observables, allowing rake rotation. Each component of the solutions (slip, slip velocity and traction) depends on both the two on–fault spatial coordinates \( x_1 \) and \( x_3 \) and on time \( t \). Since we consider identical materials properties on both sides of the fault plane, in order to reduce computational effort we exploit the existing symmetries, as described in detail in Bizzarri (2009a).

The fault is subjected to the linear SW governing law in the following form (Ida, 1972):

\[
\tau = \begin{cases} 
\tau_u - (\tau_u - \tau_f) \frac{u}{d_0}, & u < d_0 \\
\tau_f, & u \geq d_0
\end{cases}
\]

(6)

where \( u \) is the magnitude of the fault slip. In the interest of simplicity, we neglect here the possible changes in pore fluid pressure \( p_{\text{fluid}} \), moreover, uniform material properties guarantee a constant value of the normal stress of tectonic origin, \( \sigma_n \), and therefore also \( \sigma_n^{\text{eff}} (= \sigma_n - p_{\text{fluid}}) \) is constant through time.
4. Different nucleation strategies

As previously pointed out, in the case of linear SW law and in absence of tectonic load or stress perturbations coming from other neighboring faults, the nucleation procedure is a numerical artifact needed to induce the rupture to spontaneously propagate, i.e., to enlarge without prior assigned rupture velocity ($v_r$ is itself a solution of the problem and, depending on fault rheology, it can potentially assume a very complicated distribution; see for instance Bizzarri and Spudich, 2008; their Figure 11e). In all the numerical experiments presented and discussed in the remainder of the paper the earthquake hypocenter H (see Figure 1) is imposed (it is located in $(x_1^H, x_2^H, x_3^H)$) and it is in the centre of the initialization zone $I_{\text{nucl}}$. In the next three sub–sections we will describe the various nucleation strategies considered in this study, schematically illustrated in Figure 2.

4.1. Initially non–spontaneous rupture propagation

We assume that the rupture is initially non–spontaneous, in that it propagates with a constant rupture velocity, $v_r = v_{\text{force}}$, as in Andrews (1985). Namely, the fault friction is specified as follows (see Figure 2a):

$$
\tau = \begin{cases} 
\tau_{\text{nucl}} = \min\{\tau^{(\text{SW})}, \tau^{(\text{TW})}\}, & \forall (x_1, x_3) \in I_{\text{nucl}} \\
\tau^{(\text{SW})}, & \forall (x_1, x_3) \notin I_{\text{nucl}}
\end{cases}
$$

(7)

where $\tau^{(\text{SW})}$ is expressed as in equation (6) and $\tau^{(\text{TW})}$ given by (Bizzarri et al., 2001)

$$
\tau^{(\text{TW})} = \begin{cases} 
\mu_f - (\mu_u - \mu_f) \left(\frac{t - t_{\text{force}}}{t_0}\right) \sigma_n^{\text{eff}}, & t - t_{\text{force}} < t_0 \\
\mu_f \sigma_n^{\text{eff}}, & t - t_{\text{force}} \geq t_0
\end{cases}
$$

(8)
Formally, equation (8) can be regarded as a constitutive model, the linear time–weakening (TW henceforth) law, in which the fault friction explicitly depends on time, instead of on slip, as in the SW model. In (8) \( t_{force} = \frac{\sqrt{(x_1 - x_1^H)^2 + (x_3 - x_3^H)^2}}{v_{force}} \) is the instant of time at which a rupture propagating at the forcing velocity \( v_{force} \) reaches the point \((x_1, x_3)\) and \( t_0 \) is a characteristic time (the temporal counterpart of \( d_0 \)). At a certain time (which depends on the adopted frictional parameters) the SW takes over and then the rupture begins to propagate spontaneously. To briefly illustrate this strategy, let we consider, for sake of simplicity, to be in \( H; \) here, at \( t = 0 \), the fault traction has magnitude \( \tau_0 \) and fault strength is defined by \( \tau^{(TW)} \) and it equals \( \tau_u \) (since \( u = 0 \)). Then the fault strength diminishes linearly through time, accordingly to the \( \tau^{(TW)} \) function (8). When it reaches \( \tau_0 \) the sliding begins and it causes a stress redistribution in the surrounding fault points, which are loaded. This load can be such that the upper yield stress \( \tau_u \) is reached and therefore also these points start to slip. The additional parameters that come into the model as a consequence of the implementation of this nucleation strategy are \( v_{force} \) and \( t_0 \).

4.2. Introduction of an initial shear stress asperity

Starting from the hypothesis of Benioff (1951) and Reid (1910) that a fracture occurs when the stress in a volume reaches the rock strength, and from the concept of asperity in the sense of Kanamori (1981), it is physically reasonable to assume that within a region close to the hypocenter the shear stress is higher than in the remaining portions of the fault and that here the rupture is prone to start to propagate. In accordance to its conceptual simplicity, the
numerical implementation of this nucleation strategy is rather trivial; at \( t = 0 \) the fault traction is (see Figure 2b):

\[
\tau = \begin{cases} 
\tau_{nucl} = \tau_u + \Delta \tau_{nucl} , & \forall (x_1, x_3) \in I_{nucl} \\
\tau_0 , & \forall (x_1, x_3) \not\in I_{nucl}
\end{cases}
\]  

(9)

where \( \Delta \tau_{nucl} \), the additional parameter inserted into the model by the introduction of this nucleation strategy, is a (small) perturbation to \( \tau_u \) (namely it is a static overshoot). We will discuss in the remainder of the paper (section 7) a possible refinement in this strategy, consisting in tapering \( \tau_{nucl} \) from \( \tau_u + \Delta \tau_{nucl} \) to \( \tau_0 \) over a finite distance, instead of having an abrupt transition between \( \tau_u + \Delta \tau_{nucl} \) and \( \tau_0 \) at \( \partial I_{nucl} \).

4.3. Perturbation to the initial particle velocity

It is well known that stress redistribution following an earthquake corresponds to a propagation of seismic waves in the medium surrounding the earthquake source (see for instance Bizzarri and Belardinelli, 2008 among many others). This wave excitation causes perturbations of the particle velocity in the medium that can lead to dynamic triggering, which in some situations can even be relatively abrupt. Starting from this physical basis, the third type of artificial nucleation we consider assumes that in a volume surrounding the imposed hypocenter the particle velocity \( V \) is non–null. Formally, following with appropriate modifications Ionescu and Campillo (1999) and Badea et al. (2004), we prescribe that at \( t = 0 \) the components of \( V \) are expressed as:
where $v_0$ is the modulus of the initial fault slip velocity in H, $\phi$ is the rake angle, $l_1$ and $l_2$ parameterize $I_{\text{nuc}}$ in $x_1$– and $x_3$–direction, respectively, $d_{\text{nuc}}$ is a sensitivity factor, controlling how rapidly the perturbation to $V_i$ decreases to the reference value of 0, moving in the direction perpendicular to the fault plane $x_2 = x_2^f$ (see Figure 2d; we recall that the medium is initially at rest and therefore $V = 0$ is the reference state of the elastic medium) and $\Theta(.)$ the Heaviside function. In the framework of the traction–at–split–nodes numerical technique (see Bizzarri and Cocco, 2005 and references therein for further details) the components of the initial fault slip velocity are expressed as: $v_i(x_1,x_3,0) = V_i^+(x_1,x_3,0) - V_i^-(x_1,x_3,0)$, where $V_i^+(x_1,x_3,0)$ and $V_i^-(x_1,x_3,0)$ are the components of $V$ in the “positive” and “negative” parts of the medium separated by the fault, respectively (see Figure 1), and $i = 1, 2, 3$. From equation (10) we have that in H: $V_1^+(x_1^H,x_3^H,0) - V_1^-(x_1^H,x_3^H,0) = v_0 \cos \phi$, $V_2^+(x_1^H,x_3^H,0) - V_2^-(x_1^H,x_3^H,0) = 0$ and $V_3^+(x_1^H,x_3^H,0) - V_3^-(x_1^H,x_3^H,0) = v_0 \sin \phi$, in agreement with our formal definition of $v_0$. This nucleation strategy causes fault points within $I_{\text{nuc}}$ to move at $t = 0$ with a velocity which is maximum in H (where it is $v_0$, as previously noticed) and which is exponentially tapered to 0 at the border $\partial I_{\text{nuc}}$ (see Figure 2c). The difference between $V_i^+$ and $V_i^-$ induces the a differential force between the split nodes, which will cause in turn a differential acceleration, ultimately
leading to a readjustment in the fault traction. When a fault point is slipping, the fault friction is then determined by the governing law (equation (6)). The basic difference of this nucleation strategy with respect to those described in sub–sections 4.1 and 4.2 is that the previous strategies change the reference state of the variables only on the fault plane (fault strength and pre–stress, respectively), while the current strategy introduces a modification in $V$ in a volume surrounding the hypocenter. The additional parameters inserted into the model with this nucleation strategy are $d_{nucl}$ and $v_0$.

5. Quantitative criteria for the evaluation of the nucleation strategy

Since the spontaneous, fully dynamic rupture problem does not have a closed–form analytical solutions (even in homogeneous conditions) there is no a theoretical solution of the problem which we can take as reference against whom compare the different solutions obtained numerically by adopting the various nucleation strategies. In the evaluation of the various solutions presented in the following of the paper we will consider the following criteria that have to be satisfied.

I) As previously noticed, the inertia is always considered (i.e., we do not use the quasi–static approximation to solve the elasto–dynamic equation).

II) The transition between the early stages of the rupture, primarily controlled by the nucleation strategy, and the further spontaneous dynamic rupture propagation, controlled by the adopted constitutive law, has to be gradual in both space and time, without abrupt discontinuities in rupture velocity, stress drop, etc.
III) The rupture speed of the ongoing dynamic rupture has to satisfy the rules based on the value of the strength parameter (see for instance Dunham, 2007); e.g., a low strength fault (like configuration A described in the next section) would accelerate up to supershear speeds, while a high strength fault (like configuration B in section 7) would remain subshear.

IV) The extension of the initialization zone has to be as small as possible and, once nucleated, the rupture has to propagate spontaneously and dynamically outside $I_{\text{nuc}}$.

V) At fault nodes located outside $I_{\text{nuc}}$ the rupture has to reproduce the imposed SW law, with its constitutive parameters.

A solution that better satisfies all the above–mentioned criteria characterizes what we select as the “desired” solution.

6. Results from numerical experiments: supershear rupture propagation

In this paper we consider a set of parameters which is representative of a typical crustal earthquake occurring at a depth of 5 km. In particular, we adopt the same parameterization of the medium surrounding the fault adopted in Version 3 of the Southern California Earthquake Center (SCEC) benchmark problem (e.g., Harris et al., 2004); the other frictional parameters are listed in Table 1. We consider the idealized situation of homogeneous rheology, i.e., neglecting frictional heterogeneities on the fault (except for $I_{\text{nuc}}$). Of course, this might not represent a realistic assumption for natural fault (e.g., Rivera and Kanamori, 2002), but here we are interested in the effects of the nucleation on the rupture propagation and therefore we wanted to disregard any complication arising from a potentially complex fault rheology, such
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as local transitions to supershear regime (Liu and Lapusta, 2008; Bizzarri et al., 2009). We consider two different sets of parameters that are representative of two distinct classes of rupture regimes; configuration A is a low strength fault \((S = 0.4)\), which can produce supershear ruptures, while configuration B is a high strength fault \((S = 2)\), where ruptures remain subshear. If not otherwise explicitly mentioned, the parameters of various nucleation strategies are those tabulated in Table 2.

In Figure 3a we report the distribution on the fault plane of the rupture times \((t_r(x_1,x_3))\) for the three nucleation procedures described in section 4. In the case of initial shear asperity we consider three shapes: a circle (as suggested by the meaning of the critical radius \(r_c^{(D)}\) of Day, 1982) an ellipse (as suggested by the two critical lengths \(L_c^{(II)}\) and \(L_c^{(III)}\) of Andrews, 1976a, 1976b and also by \(a_c^{(UR)}\) and \(b_c^{(UR)}\) of Uenishi and Rice, 2004) and a square (as in SCEC benchmarks; Harris et al., 2009). The rupture time \(t_r\) at a generic fault point is defined as the instant at which the slip velocity at that point exceeds \(v_l = 0.01\, \text{m/s}\), a threshold value which appropriately captures the initiation of dynamic slip (see Bizzarri and Spudich, 2008 and references therein). We then calculate the rupture velocity \((v_r(x_1,x_3))\) as the inverse of slowness:

\[
v_r(x_1,x_3) = \frac{1}{\left\| \nabla_{(x_1,x_3)} t_r(x_1,x_3) \right\|}.
\]

We recall that in the case of configuration A the maximum allowable rupture speed (Burridge et al., 1979) is: \(v_{r\text{max}} = v_p\) (where \(v_p\) is the \(P\) wave speed), which is in fact attained in our models, accordingly with previous studies (e.g. Bizzarri and Cocco, 2005; Liu and Lapusta, 2008) and with laboratory experiments (e.g., Xia et al., 2004).

In Figure 3b we superimpose the boundary lines separating the fault points experiencing supershear rupture velocities (points on the left of each line) from those remaining subshear
(points on the right of each line). From these two panels we can see that the overall behavior of the rupture is nearly the same; the shapes of the rupture, at a given time level, are similar and all solutions satisfy all the criteria I) to V) in section 5. By looking at the details of each numerical experiment, it emerges that there is a temporal difference in the arrival of rupture front. This delay is also viewable from the time evolutions of the fault slip velocity (Figure 3c) and those of particle velocity components (Figure 4). The peaks are nearly the same, even if particle velocity perturbation (magenta curve) produces differences in $V_1$ (Figure 4a). The numerical oscillations are practically the same in all models; this indicates that the accuracy of the simulated rupture is primarily controlled by the spatio–temporal discretization and not by the choice of the nucleation procedure.

In all previous simulations the size (radius, major semi–axis, side) of $I_{nucl}$ was the same and equal to $r_c^{(D)}$, a conservative choice. On the other hand, by imposing an elliptical asperity having $r_a = L_c^{(II)}$ and $r_b = L_c^{(III)}$, respectively, the rupture initially starts to propagate, but very rapidly dies (therefore criteria IV) and V) are not satisfied). This is not surprising, given the fact that $L_c^{(II)}$ and $L_c^{(III)}$ have been theoretically derived for purely 2–D problems. The same occurs by setting $r_a = 1.36 L_c^{(II)}$ and $r_b = 1.36 L_c^{(III)}$, respectively; this suggest that the multiplicative factor of 1.36 suggested by Galis et al. (2009) for supershear ruptures is not universal, but strongly depends on the adopted frictional parameters and in particular is not correct in the case of configuration A. We have extensively explored the parameters space and we found that the minimum value of this multiplicative factor, guaranteeing spontaneous rupture propagation, is 1.45. However, from the synoptic comparison between elliptical shear asperities reported in Figure 5, we can see that the dynamic propagation is significantly
delayed as we reduce the size of the shear asperity (i.e., as we decrease the multiplicative factor of $L_c^{(II)}$ and $L_c^{(III)}$). Just for an example, at the hypocentral depth the rupture tip arrives at a strike distance from H of roughly 5 km at $t = 1.5$ s for $r_a = 2.2L_c^{(II)}$ ($= r_c^{(D)}$ for our parameters) and at $t = 2.5$ s for $r_a = 1.45L_c^{(II)}$. This indicates that in the latter case the rupture takes more time to be able to propagate on its own outside $I_{nuc}^r$.

By construction, the TW–driven rupture is that which better satisfies the criteria II) and IV) and can be regarded as the “desired” solution. Among the other possibilities presented above, the case which better agrees with it is the rupture forced with an elliptical asperity — with a static overshoot $\Delta \tau_{nuc} = 0.5 \% \tau_u$ in the initial shear stress — with semi–axes $r_a = r_c^{(D)}$ ($= 2.2L_c^{(II)}$ for our parameters) and $r_b = 2.2L_c^{(III)}$.

7. Results for subshear rupture propagation

The differences between the nucleation strategies become significant in the case of configuration B, the subshear rupture, for which the maximum allowable rupture speed is $v_{r,max} = v_R$. The behaviour of the rupture is quite sensitive (definitely more than in the case of the supershear rupture presented in previous section) to the parameters of each nucleation procedure (see Table 2), as deeply discussed in Appendix A. In the present section we compare the best cases for the three nucleation strategies described in section 4.

First of all, we have verified that, for all the considered nucleation procedures, a nucleation patch with a dimension along $x_1$ (radius, major semi–axis or side) equal to $L_c^{(II)}$ is not large enough to produce a dynamic instability which is able to spontaneously propagate over the
whole fault. Since in this case $a_c^{(UR)} < L_c^{(II)}$ (see Table 3) the same is true when $I_{nucl}$ extends $a_c^{(UR)}$ along $x_1$. Again, this is physically reasonable, since $L_c^{(II)}$ and $L_c^{(III)}$ were derived in 2–D.

In Figure 6a there is a comparison between rupture times obtained for a TW–driven rupture ($I_{nucl}$ of equation (7) is now a circle with radius $1.57 r_c^{(D)}$; blue curve), for two smoothed asperities (circular, green line, and elliptical, red line), and for the case of perturbation of particle velocity (magenta line). In the case of smoothed asperities we assume that $\tau_{nucl}$ of equation (9) overcomes (by the overshoot $\Delta \tau_{nucl}$) $\tau_u$ only in an inner portion of $I_{nucl}$ and it is cosine–tapered to $\tau_0$ at $\partial I_{nucl}$ over the length $l_{aper} = 2.6$ km. In the case of the smoothed circular asperity, $\tau_{nucl}$ exceeds $\tau_0$ in a circular region of radius $r = r_c^{(D)}$ (see Figures 6b and 6c); in the case of the smoothed elliptical asperity $\tau_{nucl}$ exceeds $\tau_0$ in an elliptical region with $r_a = r_c^{(D)}$ ($= 2.2 L_c^{(II)}$ for our parameters) and $r_b = 2.2 L_c^{(III)}$. We can clearly see a delay in the rupture times (Figure 6a) and also a significant difference in the peaks of the resulting fault slip velocity (Figure 6d). We recall that in case of configuration A, on the contrary, the peaks in fault slip velocity were substantially the same (see Figure 3c).

The comparison of the free surface velocity histories (Figure 7) shows that the nucleation obtained by imposing a perturbation in the initial particle velocity (magenta lines) causes the solutions to be more oscillating for early times ($t < 3$ s for this receiver); such high frequency oscillations are spurious artifacts which are absent in the solutions obtained by using the other nucleation strategies. These oscillations are present also in the distributions of the rupture velocity (see Figures A3a and A3b) and are in contrast with criterion II).

Among the different solutions, the TW–driven nucleation over a region of radius $r > r_c^{(D)}$
and with \( v_{\text{force}} = 0.75v_R \) and \( t_0 = 0.1 \text{ s} \) is that which better satisfies criteria in section 5 and can be therefore regarded as the “desired” solution. The other solution which better approaches that behavior corresponds to the smoothed elliptical shear stress asperity, with semi–axes \( r_a = r_c^{(D)} \), \( r_b = r_c^{(D)} \frac{L_c^{(III)}}{L_c^{(II)}} \) and \( l_{\text{taper}} = 2.6 \text{ km} \). We notice that the specific values of \( r \) (in the case of TW–driven rupture) and \( l_{\text{taper}} \) (when asperity is imposed) can depend on the adopted constitutive parameters; their optimal values might have to be numerically obtained by a trial–and–error approach.

8. Discussion and concluding remarks

The numerical simulation of rupture dynamics is fundamental in the attempt to understand the earthquake physics and in the strong ground motion prediction. Coherent modeling of earthquake rupture requires the description of the several space and time scales involved in the rupturing process, such as the nucleation (an initial, aseismic slippage, where inertial effects are negligible), the rapid propagation of the rupture (seismically detectable and associated with the emission of seismic waves and with the stress redistribution in the surroundings of the fault) and the rupture arrest. Space and time steps are numerically controlled by the smallest scale; the nucleation therefore requires very small computational grids as compared to the rest of the process and drastically increases the computation time, even if the nucleation zone is very small compared to the surface which fails. Moreover, the time duration of the nucleation is very much longer than that of coseismic processes. As noticed above, our main interest here is not on the physical details of the nucleation process, which can be modeled by the
considering a tectonically–driven fault (see for instance Liu and Lapusta, 2008). Since we want to focus on the dynamic rupture propagation it is computationally convenient to introduce artificial processes that allow the rupture to spontaneously propagate. Nevertheless, the correct modeling of the nucleation is fundamental to properly retrieve the slip and rupture time distribution on the fault plane, as well as to model the energy content of the radiation and its distribution in the frequency domain.

In this paper we have considered 3–D dynamic ruptures spontaneously spreading on a planar fault, which obey the linear slip–weakening (SW) law (equation (6)) and which are embedded in a homogeneous, elastic medium, free from external tectonic loading. We have considered various nucleation strategies, largely employed in the literature, that are conceptually and algorithmically different: the initially non–spontaneous, time–weakening (TW)–driven rupture propagation (section 4.1); the introduction of an asperity in the initial shear stress (section 4.2); and the perturbation to the initial particle velocity (section 4.3). We have systematically compared the resulting solutions by considering the agreement with respect a “desired” solution. The latter is the solution which better satisfies all the criteria described in section 5. We have also tested the effects of the size and of the shape of the initialization zone, $I_{\text{nucl}}$. In the comparison of the solutions we have considered the rupture times ($t_r$), the rupture velocities ($v_r$), the solutions on the fault and the synthetic motions on the free surface of the resulting dynamic ruptures.

One interesting conclusion is that in the case of supershear ruptures the above–mentioned nucleation strategies produce results not dramatically dissimilar one from the others (see section 6). Basically, different strategies lightly change in time the occurrence of the transition to supershear rupture speeds (see Figure 3b), in agreement with the results of Festa and Villotte.
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(2006) and Liu and Lapusta (2008). Interestingly, our results indicate that supershear ruptures tend to “forget” their origins more than subshear ruptures do. In fact, in the case of ruptures that develop with a speed equal to a fraction of the shear wave velocity — which seem to represent the majority of real–world earthquake events (e.g., Heaton, 1990) — the modeler has to carefully tune the parameters of each individual nucleation strategy (see section 7 and Appendix A).

Our results also demonstrate that, among the different critical nucleation lengths introduced in the literature (see section 1.2), the key parameter to be used to quantify the extension of $I_{nuc}$ is $r_{c}^{(D)}$ (equation (3)). For $I_{nuc}$ having a length along the in–plane direction less than $r_{c}^{(D)}$ the rupture does not spontaneously propagate outside the nucleation patch or quickly dies a few fault nodes outside $I_{nuc}$ (contradicting criterion IV). We are not aware of the fact that there is a conceptual problem of the applicability of the critical length theoretically derived for 2–D geometries to 3–D problems. We have performed these tests to numerically verify this and since some authors (e.g., Galis et al., 2009) claim that even in 3–D the size of $I_{nuc}$ has to be quantified trough multiples of $L_{c}^{(II)}$ and $L_{c}^{(III)}$ for both sub– and supershear ruptures.

The numerical simulations presented in this paper confirm that the TW–driven nucleation strategy, with $v_{force} = 0.75v_{R}$ and $t_{0} = 0.1$ s, produces the “desired” solution when the size of $I_{nuc}$ along the in–plane direction (namely where the condition of equation (7) is evaluated) is $r_{c}^{(D)}$ for supershear ruptures and greater than $r_{c}^{(D)}$ in the case of subshear ones.

Moreover, we found that the “desired” solution is approached when a smoothed elliptical asperity in the initial shear stress is adopted. While in the case of supershear ruptures the
smoothing of the asperity (i.e., the portion of $I_{\text{nucl}}$ where $\tau_{\text{nucl}}$ is gradually tapered from $\tau_u + \Delta\tau_{\text{nucl}}$ to $\tau_0$) is of secondary importance, we have shown that in the case of subshear ruptures the smoothing distance $l_{\text{aper}}$ is important (see Appendix A). We found that the optimal parameters are a static overshoot $\Delta\tau_{\text{nucl}}$ equal to 0.5\% of $\tau_u$ (in agreement with Liu and Lapusta, 2008 who use 1\% of $\tau_u$) and major and minor semi–axes of $I_{\text{nucl}}$ given by $r_a = r_c^{(D)}$ and $r_b = r_c^{(D)} \frac{L_c^{(III)}}{L_c^{(III)}}$, respectively. Numerical results indicate that the optimal value of $l_{\text{aper}}$ is 2.6 km, but its specific value might change by varying the models parameters and therefore it has to be found by a trial–and–error procedure. We want to emphasize that, within $I_{\text{nucl}}$, these two nucleation strategies (TW–driven and asperity) have a different Kostrov energies (Kostrov and Das, 1988): in the first case, when the rupture propagates at the fixed velocity $v_{\text{force}}$, the Kostrov term is $(\tau_0 - \tau_u)$, while in the second case, since the initial traction is raised up to $\tau_u$, it is $(\tau_u - \tau_f)u$.

The third type of the nucleation strategy is conceptually interesting because it consists of the introduction of a perturbation of the initial reference state of the fault system (the static equilibrium) within a volume surrounding the fault. This is particularly appealing since we have various evidence of the complexity of a fault structure and we are aware that a plane is only a mathematical approximation of the volume where non–elastic processes take place (see Bizzarri, 2009b and references therein for a comprehensive discussion). In spite of this, the adoption of this nucleation strategy produces results close to the “desired” solution for supershear ruptures, but not for subshear ones. In the latter configuration the results are in conflict with criterion II) since they are affected by large, high frequency, spurious oscillations.
(see Figure 7) that can not be removed even with a careful exploration of the parameter space.

Different nucleation procedures potentially have different stress drops within the initialization zone. A quantitative estimate of the significant differences caused by the various strategies and, for the same numerical procedure, by the adoption of different values of the nucleation parameters, is represented by the temporal evolution of the (dynamic) seismic moment, $M(t)$, which accounts for the cracked area and for the developed cumulative fault slip during the considered time window. It is expressed as $M(t) = \sqrt{M_{21}(t)^2 + M_{23}(t)^2}$, being

$$M_{21}(t) = \int_{\Sigma} G u_i(x_1, x_3, t) d x_1 d x_3,$$

where $i = 1$ and $3$; $\Sigma$ is the fault and $u_1$ and $u_3$ are two components of fault slip (see Bizzarri and Belardinelli, 2008 for further details). A synoptic comparison between $M(t)$ pertaining to the whole ensemble of the numerical experiments presented in this paper is reported in Figure 8. In panels (a) and (b) of that figure the thick lines refer to the optimal cases for each nucleation strategy, i.e., the configurations that, for that nucleation strategy, better approaches the “desired” solution (marked as “DS” in the legend and plotted with the thickest blue line). In the case of configuration B (subshear ruptures) we can see that the tuning of the nucleation parameters reduces the large differences between the results obtained with the various nucleation strategies and the “desired” solution. While in the case of configuration A (supershear ruptures) the maximum of the absolute value of the difference between the seismic moment obtained with TW–driven nucleation and elliptical shear asperity is of the order of 10 %, in the case of configuration B it increase up to 80 % (Figure 8c). After about 1 s the solutions are very similar in the supershear case, while they differ by nearly 30 % after about 2 s in the subshear case.

Finally, we want to highlight that the results of a dynamic model of a synthetic earthquake
can be potentially affected and biased from the adoption of a nucleation strategy for linear slip–weakening governing law. This can be due, for instance, to an improper size or shape of the initialization patch, to an excessively high static overshoot, to an incorrect forcing rupture velocity. This is true especially in the case of high strength faults; in some configurations, we have found that a huge initial shear stress asperity can lead to the crack front bifurcation at depth and to the transition to supershear rupture speeds, contradicting criterion III).

As an overall conclusion, we point out that the modeler has to carefully check the obtained numerical solution, compare it against other results and painstakingly tune the nucleation parameters. In this paper we have presented some practical recipes with the aim to serve as guidance in performing these efforts.

**Acknowledgements.** I would like to thank Paul Spudich for an insightful review of the paper; his comments greatly improved the presentation. I also acknowledge Yehuda Ben–Zion for fruitful comments on a preliminary version of this paper and Sara Bruni for the assistance during the preparation of some figures. I’ m grateful to the Associate Editor (Michel Bouchon) and to two anonymous referees for their useful and constructive comments.
Appendix A. Sensitivity to the nucleation parameters

While in the case of supershear ruptures the differences between ruptures forced to develop using dissimilar nucleation strategies are relatively small (see section 6), in the case of subshear ruptures (see section 7) they become more significant. Moreover, the nucleation parameters (listed in Table 2) have important effects on the further dynamic rupture propagation and therefore a thorough tuning of them is required in order to cause a dynamic propagation consistent with all the criteria listed in section 5, as discussed in the remainder of the present appendix.

A.1. The case of TW–driven ruptures

In Figure A1 we compare the solutions obtained by varying the parameters of the TW–driven nucleation. The behavior of the dynamic rupture at a radial distance greater than $1.5r_c^{(D)}$ from H is practically identical in all cases. The most important differences appear at lower hypocentral distances, where the rupture velocity is forced to equal $v_{\text{force}}$. In cases plotted in Figures A1a to A1d $v_r$ has large fluctuations near $\partial I_{\text{nucel}}$ which are in contrast with criterion II) (see section 5) and are very difficult to justify physically. When the rupture becomes spontaneous, $v_r$ increases and soon it decreases a lot (blue annular region in Figures A1a to A1d) and it finally increases again up to its limiting velocity. This indicates that even if the SW takes over, the solution is still affected by the imposed nucleation. This behavior becomes more evident as $t_0$ increases (compare Figures A1a and A1c); from the rupture times reported in Figure A1f we can see that for $t_0 > 0.1$ s the rupture is affected by a significant delay. The resulting $v_r$ for the “desired” solution, which better satisfies all criteria in section 5, is reported
in Figure A1d (corresponding to the blue curve in Figures 6a, 6d and 7), in which the radius of \( I_{\text{nucl}} \) where the condition of equation (6) is evaluated is greater than \( r_c^{(D)} \). The specific value of \( r = 1.57r_c^{(D)} \) might depend on the adopted governing parameters and therefore it might have to be found numerically by a trial–and–error procedure.

**A.2. The case of initial stress asperity**

Figure A2 summarizes the comparison between different types of asperities introduced in the initial shear stress field. We first emphasize that by forcing the rupture by assuming a circular asperity with radius \( r = r_c^{(D)} \) the rupture hits the free surface with so much energy to cause the birth of a sustained supershear pulse, which is in contrast with criterion III). Moreover, in all cases presented in the previous section the supershear patch was noticeably smaller and the supershear pulse died very soon (see Figure A1 and also next Figure A3), while in the present case it continues to propagate up to the boundary of the computational domain. These results clearly indicate that, in the case of configuration B, the initial shear stress asperity as defined in equation (9) has to be modified in order to obtain the “desired” solution. We therefore consider a smoothed asperity as described in section 7 (see also Figures 6b and 6c); we set \( I_{\text{nucl}} \) with \( r = r_c^{(D)} \) and we progressively increase \( l_{\text{taper}} \) (as a consequence of this variation we also decrease the size of the inner region (having radius \( r_c^{(D)} - l_{\text{taper}} \)) where \( \tau_{\text{nucl}} \geq \tau_u \)). Results for two cases are reported in Figures A2b and A2c. We can see that by progressively increasing \( l_{\text{taper}} \), the supershear patch is reduced. The result that better agrees with the “desired” solution (that reported in Figure A1d) corresponds to the smoothed elliptical
asperity (Figure A3d), having $r_a = r_c^{(D)}$, $r_b = r_c^{(D)}$, $l_{\text{taper}} = 2.6$ km; a further increase of $l_{\text{taper}}$ will cause the rupture to die and to not propagate dynamically outside the initialization zone (and this is barely in contrast with criterion IV)). A similar results is obtained with a smoothed circular asperity (Figure A2c). From the comparison of the rupture times (Figure A3e) we can see that the best agreement is between the configurations of panels (c) and (d), as expected.

Finally, we note a small, semicircular patch at the hypocentral depth which is slightly supershear. This small numerical artifact (which tends to be against criterion II)) becomes larger and more pronounced as the static overshoot $\Delta \tau_{\text{nucl}}$ increases.

A.3. The case of the perturbation to the initial particle velocity

Our numerical simulations indicate that the prominent parameter in this nucleation procedure is $d_{\text{nucl}}$, which controls the extension of the volume, in the direction perpendicular to the fault, where $V$ is perturbed at $t = 0$. If this region is too wide ($d_{\text{nucl}} > 50$ m) the rupture is not able to dynamically propagate outside $I_{\text{nucl}}$, even for large values of $v_0$; this contradicts criterion IV). In Figure A3 we compare the resulting normalized rupture velocities for two of the numerical simulations we performed. We can see that just outside $I_{\text{nucl}}$ the rupture decelerates. This behavior, which is in contrast with criterion II), is more evident as $d_{\text{nucl}}$ increases. Within the annular blue region enclosing $I_{\text{nucl}}$ the rupture velocity is highly oscillatory; this region is modulated by the value of $d_{\text{nucl}}$ and in extreme cases ($d_{\text{nucl}} > 50$ m) inhibits the dynamic rupture propagation.
References


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Figure Captions

Figure 1. Geometry of the considered seismogenic model. The black star denotes the imposed hypocenter H and $I_{\text{nucl}}$ is the initialization zone, with border $\partial I_{\text{nucl}}$. The light grey plane indicates the fault $x_2 = x_2^f$, oriented through its normal unit vector $\hat{n}$ and having aspect ratio $L^f/W^f$. The dotted gray box marks the portion of the computational domain where calculation are performed, due to the exploitation of the symmetry about H and about the fault plane.

Figure 2. Schematic representation of the three nucleation strategies compared in this paper. (a) Initially TW–driven rupture (section 4.1). (b) Initial shear stress asperity (section 4.2). (c) and (d) Perturbation to the initial particle velocity (section 4.3); (c) $V_1/v_0$ as a function of on–fault coordinates; (d) $V_1/v_0$ as a function of $x_1$ and $x_2$. In all panels the imposed hypocenter and the initialization zone are indicated.

Figure 3. Comparison between solutions for ruptures developing on a low strength fault ($S = 0.4$) obtained by using different nucleation strategies. (a) Contours of rupture times plotted every 0.5 s, with line corresponding to 2 s marked with thick lines for better clarity. (b) Boundary between super– and subshear regimes: fault points located on the right of each line experience subshear rupture velocities. (c) Slip velocity time histories and (d) Phase portrait in a fault point located at hypocentral depth and at a distance of 7 km from H.
Figure 4. Time histories of the resulting particle velocity for the five models of Figure 3 on a free surface receiver located at a strike distance of 7 km from the epicenter and at a distance of 1 km from the fault trace. (a) Fault–parallel component of $V$ (namely $V_1$). (b) Fault–normal component of $V$ (namely $V_2$). (c) Vertical component of $V$ (namely $V_3$).

Figure 5. (a) Comparison between rupture times (contour lines plotted every 0.5 s) for low strength ruptures nucleating with the imposition of an initial shear asperity of elliptical shape and having different values of the major and minor semi–axes: $r_a = \alpha L_c^{(II)}$ and $r_b = \alpha L_c^{(III)}$. The values of the multiplicative factor of $\alpha$ are indicated in the legends. We remark that the case $\alpha = 1.5$ roughly corresponds to have semi–axes equal to critical values of Uenishi and Rice (2004; see previous equations (4) and (5). Namely $r_a = a_c^{(UR)}$ when $\alpha = 1.56$). To emphasize the different locations of the rupture tip in the various numerical experiments the curves corresponding to 2 s are displayed with thick lines. (b) Time evolution of the slip velocity in the same fault point of Figures 3c and 3d. The result of the initially TW–driven rupture (blue line) is reported in both panels for comparison.

Figure 6. (a) Comparison between rupture times (lines plotted every second) obtained in case of ruptures developing on a high strength fault ($S = 2$). Blue line refers to an initially TW–driven event ($I_{nucl}$ of equation (7) is a circle with $r = 1.57r_c^{(D)}$, $t_0 = 0.1$ s and $v_{force} = 0.75v_R$ (= 2.4 km/s for our parameters)). Green line refers to a solution where a smoothed circular asperity is applied, as reported in panels (b) and (c): $\tau_{nucl} > \tau_0$ in a
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circular region with \( r = r_c^{(D)} \) and \( \tau_{nucl} = \tau_u + \Delta \tau_{nucl} \) only in an inner portion of \( I_{nucl} \) and it is cosine–tapered to \( \tau_0 \) at \( \partial I_{nucl} \) over the length \( l_{taper} = 2.6 \) km. Red line refers to the case of a smoothed elliptical asperity (analogous to previous case, but now with major semi–axis \( r_a = r_c^{(D)} \) and minor semi–axis \( r_b = r_c^{(D)} \frac{L_c^{(III)}}{L_c^{(II)}} \)). (d) Comparison between fault slip velocity histories at the a fault point located at hypocentral depth and at a distance of 8 km from H.

**Figure 7.** The same as in Figure 4, but now for configuration B.

**Figure 8.** (a) Temporal evolution of the (dynamic) seismic moment, \( M(t) \), for different nucleation strategies and for various values of the nucleation parameters in case of configuration A. (b) The same as panel (a), but in case of configuration B. Blue lines refer to TW–driven ruptures, green curves to nucleation with circular asperity, red curve to nucleation with elliptical asperity and magenta ones to nucleation with perturbation to \( \mathbf{V} \). The optimal cases for each type of nucleation strategy are plotted with thick lines. The “desired” solution (marked as “DS” in the legend) is plotted with the thickest blue line. (c) Temporal evolution of the percent differences of seismic moments obtained by adopting the TW–driven nucleation and the (smoothed) elliptical asperity (namely, in ordinate axis we plot: \( 100 \frac{M^{(ASP)}(t) - M^{(TW)}(t)}{M^{(TW)}(t)} \)). In the cases of shear asperity, all the fault point where \( \tau_{nucl} \geq \tau_u \) start to move immediately at \( t = 0 \); this causes the big differences in the first time levels. On the top of the horizontal axis we have also indicated
the time levels corresponding to times reported on the bottom horizontal axis.

**Figure A1.** Effects of the nucleation parameters in the case of TW–driven rupture in case of high strength ruptures. Normalized rupture velocity distributions (namely, \(v_r/v_S\)) for \(t_0\) of equation (8) equal to 0.05 s (panel (a)); for \(t_0 = 0.1\) s (panel (b)); for \(t_0 = 0.5\) s (panel (c)); for \(t_0 = 0.1\) s and \(v_{\text{force}} = 1.2\) km/s (panel (d)). In all these numerical experiments \(r = r_c^{(D)}\).

Panel (e) refers to a case with \(t_0 = 0.1\) s and \(r = 1.57 r_c^{(D)}\). (f) Resulting rupture times for all tests reported in previous panels, where the curves corresponding to 3.5 s are displayed with thick lines for a better comparison. In panels (a) to (e) fault patches where \(v_r\) is locally supershear are indicated, as well as the extension of \(I_{\text{nuc}}\).

**Figure A2.** Effects of the nucleation parameters when an asperity in initial shear stress is imposed in case of high strength fault. Ratios \(v_r/v_S\) when at \(t = 0\) is applied a circular asperity of radius \(r = r_c^{(D)}\) (panel (a)), a smoothed circular asperity with \(r = r_c^{(D)}\) and \(l_{\text{taper}} = 1.5\) km (panel (b)), a smoothed circular asperity with \(r = r_c^{(D)}\) and \(l_{\text{taper}} = 2.6\) km (panel (c)) and a smoothed elliptical asperity with \(r_a = r_c^{(D)}\) and \(r_b = r_c^{(D)} \frac{L_e^{(III)}}{L_e^{(II)}}\) (panel (d)).

Fault patches where \(v_r\) is locally supershear and the extension of \(I_{\text{nuc}}\) are indicated. (e) Comparison between resulting rupture times; the curves corresponding to 3.5 s are displayed with thick lines.
Figure A3. Effects of the nucleation parameters when a perturbation in particle velocity is assumed in the case of high strength fault. Ratios \( \nu / v_s \) when a perturbation to the particle velocity is applied at \( t = 0 \) within a fault patch parametrized by \( l_1 = l_2 = r_c^{(D)} \) in the case of \( d_{\text{nucl}} = 20 \text{ m} \) (panel (a)) and \( d_{\text{nucl}} = 40 \text{ m} \). (panel (e)). Comparison between the resulting rupture times; the curves corresponding to 3.5 s are displayed with thick lines.

Tables

Table 1. Model discretization and constitutive parameters adopted in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamé constants, ( \lambda = G )</td>
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<tr>
<td>Rayleigh velocity, ( v_R )</td>
<td>3.184 km/s</td>
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<tr>
<td>( S ) wave velocity, ( v_S )</td>
<td>3.464 km/s</td>
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<tr>
<td>Eshelby velocity, ( v_E = \sqrt{2} v_S )</td>
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<tr>
<td>( P ) wave velocity, ( v_P )</td>
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</tr>
<tr>
<td>Maximum allowed rupture velocity, ( v_{r_{\text{max}}} )</td>
<td>( v_P )</td>
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<td>Cubic mass density, ( \rho )</td>
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</tr>
<tr>
<td>Fault length, ( L )</td>
<td>30 km(^{(a)})</td>
</tr>
<tr>
<td>Fault width, ( W )</td>
<td>10 km(^{(a)})</td>
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<td>Spatial grid sampling, ( \Delta x_1 = \Delta x_2 = \Delta x_3 \equiv \Delta x )</td>
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</tr>
<tr>
<td>Time step, ( \Delta t )</td>
<td>( 1.2 \times 10^{-3} ) s</td>
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<tr>
<td>Courant–Friedrichs–Lewy ratio, ( \omega_{\text{CFL}} = ) ( v_s \Delta t / \Delta x )</td>
<td>0.166</td>
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</table>
Critical frequency for spatial grid dispersion, \( f_{acc}^{(0)} = \nu_s/(6\Delta x) \)  

<table>
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<tr>
<th>Location of the fault, ( x_2 )</th>
<th>4.975 km</th>
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</thead>
<tbody>
<tr>
<td>Coordinates of the imposed hypocenter, ( H = (x_1^H, x_2^f, x_3^H) )</td>
<td>(15,4.975,5) km</td>
</tr>
</tbody>
</table>
| Domain boundary conditions | \( x_1 = 0: ABC^{(b)}; = x_1^H: symmetry^{(c)} \)  
\( x_2 = 0: ABC^{(b)}; = x_2^f: symmetry^{(d)} \)  
\( x_3 = 0: \text{free surface}; = x_3^\text{end}: ABC^{(a)} \) |

### Fault Constitutive Parameters

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Configuration A</th>
<th>Configuration B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial rake angle, ( \phi_0 )</td>
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<tr>
<td>Magnitude of the initial shear stress, ( \tau_0 )</td>
<td>73.8 MPa</td>
<td>63.88 MPa</td>
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<tr>
<td>Magnitude of the effective normal stress, ( \sigma_{\text{eff}} )</td>
<td>120 MPa</td>
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<tr>
<td>Static level of friction coefficient, ( \mu_u )</td>
<td>0.677 ((\leftrightarrow) ( \tau_u = 81.24) MPa)</td>
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</tr>
<tr>
<td>Kinetic level of friction coefficient, ( \mu_f )</td>
<td>0.46 ((\leftrightarrow) ( \tau_f = 55.2) MPa)</td>
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<td>Dynamic stress drop, ( \Delta\tau_{df} = \tau_0 - \tau_f )</td>
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<td>8.68 MPa</td>
</tr>
<tr>
<td>Strength parameter, ( S )</td>
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<td>2</td>
</tr>
<tr>
<td>Characteristic slip–weakening distance, ( d_0 )</td>
<td>0.4 m ((f))</td>
<td></td>
</tr>
</tbody>
</table>

\((a)\) Dimensions of the fault guarantees the transition up to supershear speeds in the case of low strength fault; extrapolating results from Dunham (2007; his Figures 5 and 10) we have that \( L' > L_c = \frac{1}{4}Gd_0/\alpha(S)\left(\tau_u - \tau_f\right) = 3.072\) km and \( W' > W_c = 0.4L_c = 1.229\) km.

\((b)\) The absorbing boundary conditions are described in details Bizzarri and Spudich (2008; their Appendix A).

\((c)\) The symmetry about the strike location of the hypocenter (\(x_1 = x_1^H\)) is exploited as described in Bizzarri (2009a): denoting with \((i,j,k)\) the triplet identifying a node in the \(Ox_1x_2x_3\) Cartesian coordinate system (see also Figure 1), the components of the particle velocity will satisfy the following rules: \(V_1(+i,j,k) = V_1(-i,j,k)\); \(V_2(+i,j,k) = -V_2(-i,j,k)\); \(V_3(+i,j,k) = -V_3(-i,j,k)\), where minus and plus signs in\(\ldots\)
Nucleation strategies with slip–weakening friction

front of the $i$–index denotes a point with $x_1$ coordinates lower and greater than $x_1^H$, respectively.

The symmetry about the fault ($x_2 = x_2^f$) is exploited as described in Bizzarri (2009a): the components of the particle velocity will satisfy the following rules: $V_1(i,+,j,k) = -V_1(i,−j,k)$; $V_2(i,+,j,k) = V_2(i,−j,k)$; $V_3(i,+,j,k) = -V_3(i,−j,k)$, where minus and plus signs in front of the $j$–index denotes a point with $x_2$ coordinates lower and greater than $x_2^f$, respectively.

Initial shear traction is aligned along $x_1$ and defines a left–lateral strike–slip fault.

With this value we obtain a sufficiently good resolution of the cohesive zone (see Bizzarri and Cocco, 2005). A larger value would imply that rupture will take several kilometers to get started on its own.

Table 2. Optimal parameters for the different nucleation strategies considered in this paper.

Numerical values inside round brackets are calculated in the case of the adopted models parameters (listed in Table 1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Configuration A ($S = 0.4$)</th>
<th>Configuration B ($S = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initially non–spontaneous rupture propagation (section 4.1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initially constant rupture velocity, $v_{force}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weakening time, $t_0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nucleation patch, $I_{nucl}$</td>
<td>Circle</td>
<td>Circle</td>
<td></td>
</tr>
<tr>
<td>Dimension of the nucleation patch</td>
<td>$r = r_c^{(D)}$</td>
<td>$r = 1.57r_c^{(D)}$</td>
<td></td>
</tr>
<tr>
<td>Introduction of an initial stress asperity (section 4.2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Static overshoot, $\Delta \tau_{nucl}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5 % $\tau_u$ ($= 0.4062$ MPa)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Nucleation patch, $I_{\text{nuc1}}$

Dimensions of the nucleation patch along $x_1$ and $x_3$ axes

- Dimensions of the nucleation patch along $x_1$ and $x_3$ axes:
  - $l_1 = l_2 = r_c^{(D)}$
  - $l_1 = l_2 = r_c^{(D)}$

**Table 3.** Values of the critical nucleation lengths for the model parameters listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation in the main text</th>
<th>Value</th>
<th>Configuration A $(S = 0.4)$</th>
<th>Configuration B $(S = 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_c^{(II)}$</td>
<td>Critical nucleation in the in-plane ($x_1$) direction, following Andrews (1976b)</td>
<td>(1)</td>
<td>409 m</td>
<td>1880 m</td>
</tr>
<tr>
<td>$L_c^{(III)}$</td>
<td>Critical nucleation in the anti-plane ($x_3$) direction, following Andrews (1976a)</td>
<td>(2)</td>
<td>307 m</td>
<td>1410 m</td>
</tr>
<tr>
<td>$r_c^{(D)}$</td>
<td>Critical nucleation radius, following Day (1982)</td>
<td>(3)</td>
<td>884 m</td>
<td>4058 m</td>
</tr>
<tr>
<td>$a_c^{(UR)}$</td>
<td>Critical nucleation major semi-axis, following Uenishi and Rice (2004)</td>
<td>(4)</td>
<td>639 m</td>
<td>639 m</td>
</tr>
<tr>
<td>$b_c^{(UR)}$</td>
<td>Critical nucleation minor semi-axis, following Uenishi and Rice (2004)</td>
<td>(5)</td>
<td>480 m</td>
<td>480 m</td>
</tr>
<tr>
<td>$\min{L_c^{(II)}, r_c^{(D)}, a_c^{(UR)}}$</td>
<td></td>
<td></td>
<td>$L_c^{(II)}$</td>
<td>$a_c^{(UR)}$</td>
</tr>
</tbody>
</table>
Figure 1

\[ W^f \]

\[ L^f \]
Figure 2

(a) $\tau_{nucl} = \min \{ \tau^{(SW)}, \tau^{(TW)} \}$

(b) $I_{nucl}$

(c) Along depth distance (m)

(d) Along strike distance (m)

Off-fault distance (m)
Figure 3

(a) Distance down dip (km) vs. Distance along strike (km)

(b) Distance down dip (km) vs. Distance along strike (km) with contour lines indicating different regions based on particle velocity ($v_r < v_S$ and $v_r > v_S$)

(c) Slip velocity (m/s) vs. Time (s)

(d) Traction (Pa) vs. Slip velocity (m/s)

Test | Nucleation type
--- | ---
B001 | TW–driven
B004 | Circular asperity; $r = r_c^{(D)}$
B005 | Elliptical asperity; $r_a = 2.2L_c^{(II)} = r_c^{(D)}$; $r_b = 2.2L_c^{(III)}$
B010 | Square asperity; $l = r_c^{(D)}$
B008 | Perturbation to particle velocity

Notes:
- $r_c^{(D)}$, $r_c^{(D)}$, $L_c^{(II)}$ and $L_c^{(III)}$ are specific dimensions for each asperity type.
- $l$ is the length of the square asperity.
- $v_r$, $v_S$ are the particle and slip velocities, respectively.
- $T$, $τ_u$, $τ_f$, $τ_0$ are stress and traction parameters.
Figure 4

(a) $V_1$

(b) $V_2$

(c) $V_3$
Figure 6

(a) Long depth distance (m) vs. Along strike distance (km)

<table>
<thead>
<tr>
<th>Test</th>
<th>Nucleation type</th>
</tr>
</thead>
<tbody>
<tr>
<td>B023</td>
<td>TW–driven</td>
</tr>
<tr>
<td>B016</td>
<td>Smoothed circular asperity</td>
</tr>
<tr>
<td>B018</td>
<td>Smoothed elliptical asperity</td>
</tr>
<tr>
<td>B022</td>
<td>Perturbation to particle velocity</td>
</tr>
</tbody>
</table>

(b) Shear component of the initial shear stress (Pa)

(c) Slip velocity (m/s) vs. Along strike distance (m)

\[ \tau_u + \Delta \tau_{\text{nucl}} \]

\[ \tau_u > \tau_{\text{nucl}} > \tau_0 \]
Figure 7

(a) $V_1$

(b) $V_2$

(c) $V_3$
Figure 8

(a) TW-driven elliptical asperity

(b) Seismic moment (Nm) vs. Time (s)

(c) Normalized difference (%) vs. Time (s)

Legend:
- TW - $r = r_c(D)$; $v_{force} = 0.75v_R$; $t_0 = 0.1$ s
- ASP - circle; $r = r_c(D)$; untapered
- ASP - ellipse; $r_a = 1.45L_c(II)$, $r_b = 2.2L_c(II)$
- ASP - square; $l = r_c(D)$

Twist V - $l_1 = l_2 = r_c(D)$; $v_0 = 20$ microm/s; $d_{nucl} = 20$ m

PERT V - $l_1 = l_2 = r_c(D)$; $v_0 = 20$ microm/s; $d_{nucl} = 40$ m

$S = 0.4$

$S = 2$
Figure A1

(a) \( r = r_c^{(0)} \); \( v_{\text{force}} = 0.75v_R \); \( t_0 = 0.05 \) s

(b) \( r = r_c^{(0)} \); \( v_{\text{force}} = 0.75v_R \); \( t_0 = 0.1 \) s

(c) \( r = r_c^{(0)} \); \( v_{\text{force}} = 0.75v_R \); \( t_0 = 0.5 \) s

(d) \( r = r_c^{(0)} \); \( v_{\text{force}} = 0.38v_R \); \( t_0 = 0.1 \) s

(e) \( r = 1.57r_c^{(0)} \); \( v_{\text{force}} = 0.75v_R \); \( t_0 = 0.1 \) s

(f) Graphical representation of the experimental setup with markers indicating different times and forces.
Figure A2

(a) circle; \( r = r_c^{(u)} \); untapered

(b) circle; \( r = r_c^{(u)} \); \( l_{taper} = 1.5 \text{ km} \)

(c) circle; \( r = r_c^{(u)} \); \( l_{taper} = 2.6 \text{ km} \)

(d) ell.; \( r_a = r_c^{(u)} \); \( r_b = 2.2L_c^{(m)} \); \( l_{taper} = 2.6 \text{ km} \)

(e) [Graph showing depth distance vs. along strike distance with various panels marked]
$l_1 = l_2 = r_c^{(b)}; v_0 = 20 \mu m/s; d_{nucl} = 20 m$

$\frac{v_r}{v_S} > 1$

Along depth distance (m)

Along strike distance (m)

0.00 0.50 1.00 1.50

(a) (b)

$\frac{\partial I_{nucl}}{\partial v}$

$0 = 20 \mu m/s; d_{nucl} = 40 m$

d

d

d

(c)