Density Distribution Functions and Correlations of Earthquake Parameters

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Abstract. The statistical analysis of the source parameters of 9 earthquake sets of different types (aftershocks, scattered events, swarms) and of different seismic regions shows that the density distribution function (*ddf*) of the linear dimension l of a fault is represented by a negative power law, as well as the *ddf* of the static stress drop σ and of the scalar seismic moment M_o. It is then suggested, and tentatively verified, that also the *ddf* of the root mean square ground acceleration, defined as a function of l and σ, may be represented by a negative power law and that, at least in the cases examined, it scales like the *ddf* of σ. It is seen that the variability of the static stress drop is significant from one region to another, as is well known, but it seems remarkable also in the same seismic region (in particular in California, σ varies by several orders of magnitude) and in the different sets of events of a given region (as observed again for California). It is hypothesized that a correlation, although weak, between the stress drop and the linear dimension of a fault exists and the analyses seem not to contradict that σ may be a decreasing function of l. Finally, it is suggested that the seismicity of a region may be represented two-dimensionally as a function of the *ddf* of the stress drop and of the linear dimension of a fault instead of the classic b and b_o values.

Key words: stress drop, source radius, scalar seismic moment, ground acceleration, density distribution function.

1. Introduction and Purposes

Earthquakes are a strong manifestation of the instabilities that exist in the interior of the Earth. They may be characterized by several physical parameters, for instance, the magnitude M or the more rigorous scalar seismic moment M_o as in our analyses. Moreover, considering that tectonic earthquakes are caused by the slip of two lithospheric blocks along a fault, we may study the linear dimension l of such faults and then analyze the static stress drop σ.

The first step in the quantitative scientific study of seismic events is the production of recorded earthquake catalogues and thus proceed with the statistical analysis of the source parameters. The aim of this article is then to focus on the study of the source parameter range of variation, of their density distribution functions (*ddfs*) and of their possible correlations.

In particular the variability of the static stress drop has recently come to the attention of several seismologists, some of those we consider here (Smith and Priestley, 1993; Abercrombie, 1995; Hardebeck and Hauksson, 1997; Mori et al., 2003; Tusa et al., 2006). On the contrary to standard constant stress drop models (e.g., Aki, 1967; Kanamori and

Anderson, 1975), some support the idea that σ may vary from one region to another, however, they suggest that σ has a limited variation in the same seismic region (e.g., Hanks and McGuire, 1981; Abercrombie and Leary, 1993). Others, instead, do not set limits on the values of σ, except those imposed by the properties of rocks, and argue that it may also vary in the same seismic region and that the variations may be of several orders of magnitude (e.g., Caputo, 1981, 2005). In this note we will try to compare the ranges of the values of the static stress drop in different seismic regions and different types of earthquake sets in order to offer a contribution to previous discussions.

As suggested above, we also expect to improve the knowledge on the *ddfs* of single source parameters other than the magnitude M. These analyses may be of interest in determining the characteristics of different seismic zones and of different types of earthquake sets for a better understanding of the world seismicity. Moreover, the variations of the parameters that define the statistical laws of the *ddfs* could be used as precursors of incumbent earthquakes (Knopoff et al., 1982; Caputo, 1982); in addition, the *ddfs* of l and of σ could be helpful in estimating the elastic energy stored in the Earth's crust (Caputo, 1987), or for the study of a possible equipartition of the seismic moment or of the release of elastic energy, or for the study of the excitation of the Chandler wobble (O'Connell and Dziewonski, 1976). Moreover, the *ddfs* of l and σ may tentatively lead to the forecast of the possible acceleration of the ground during earthquakes (Caputo, 1981); in this direction we will tentatively develop a model (see paragraph 2.3) giving the *ddf* of ground acceleration based on those of l and σ and try to verify it with the available data sets.

We greatly appreciate the many recent articles that deal with ground motion caused by earthquakes (e.g., Goda and Hong, 2008; Rhoades et al., 2008; Mancilla et al., 2008; Ameri et al., 2008; Massa et al., 2008; Mezcua et al., 2008). Since the response of structures to ground motion is non-linear, many prefer to use direct ground motion, however, in some cases peak ground acceleration is of interest as is velocity of the ground or root mean square acceleration. Here we will discuss the problem of the r.m.s. ground acceleration.

Finally, we will also investigate the possible correlation between the source parameters l and σ. The length of the fault can be independently determined from the spectrum of seismic waves (e.g., Brune, 1970, 1971; Boatwright, 1980); whereas the static stress drop, on the other hand, must be calculated through l and M_o, then σ is not independently and directly measured. Thus, through the analysis of the correlation between l and σ we cannot rigorously demonstrate the possible relationship between the two parameters, but only try to verify a physical hypothesis that smaller faults generate large stress drops more frequently, per unit time and stress accumulation that is supposed to be uniform, compared to larger faults (Caputo, 2005) (see Figure 1), although small stress drops are also possible in small faults. As a consequence, for small earthquakes it could be possible to see localized points of high (and low) stress drop, while for the larger fault dimensions, the static stress drop may represent an average over a large area. The estimates of the correlation values for all data sets now considered will thus enable one to see if the hypothesis that l is a decreasing function of σ is plausible, as is suggested in previous works (e.g., Caputo, 1987, 2005). Moreover, we will investigate the possibility of a 2D representation of the earthquake statistics based on l and σ. <Insert Figure 1>.

We have examined 9 data groups in this note that are related to different seismic regions and to different types of earthquakes: aftershocks, swarms and scattered earthquakes. Some of these sets have already been examined in previous works (e.g., Caputo, 2005), but here we will use slightly different methods and perform further analyses and, in a few cases, obtain more accurate results.

2. Assumptions and Models

2.1 Source Parameters Scalar Seismic Moment, Equivalent Radius and Stress Drop

A more rigorous measure of earthquake size than magnitude is the *scalar seismic moment* M_o which is usually defined by the formula (Aki, 1966)

$$M_o = \mu A < s > \tag{1}$$

where μ is the rigidity of rocks in the fault region, A is the fault area and $< s >$ is the average final slip on the fault.

Assuming that the seismic moment is known, one can also estimate the *linear dimension l of the fault*, usually known as the *equivalent radius*, through Brune's (1970, 1971) formula, which, however, would be appropriate only for small circular faults (as is the case of the events of this paper), using seismic wave spectra

$$l = \frac{2.34\,\beta}{2\pi\, f_c} \tag{2}$$

This formula refers to the S-wave spectrum, β is the shear wave velocity and f_c is the *corner frequency* of the spectrum often determined using the conventional method of Brune (1970, 1971) (as is the case of most of the sets herein considered); 2.34 is a factor which includes the average value of the radiation pattern.

One may, however, introduce other methods to calculate the source dimensions in order to avoid the difficulties concerning the determination of the corner frequency values. In fact, as it has been noted (e.g., Hanks, 1982), the seismograph site response and the whole-path attenuation may distort the spectrum masking the value of f_c and causing significant errors (Mori and Frankel, 1990), mainly in the case of earthquakes with small source area. So, another possible way to estimate the source dimension, as followed in some cases (e.g., Smith and Priestley, 1993; Hardebeck and Hauksson, 1997; Mori et al., 2003), is through the *source rupture duration* obtained from the initial body-wave pulse width (e.g., Frankel and Kanamori, 1983; Smith and Priestley, 1993); the formula used is Boatwright's (1980):

$$l = \frac{\tau}{\frac{1}{j\,\beta} - \frac{\sin\theta}{\alpha}} \tag{3}$$

where τ is the *source rupture duration*, α is the P-wave velocity, j is a factor generally assumed to be in the range from 0.75 to almost unity which, with β, defines the rupture velocity $v = j\beta$, and, finally, θ is the angle between the normal to the fault plane and the direction of the outgoing seismic ray.

So, concerning formula (3), Mori et al. (2003), for instance, used the empirical Green's function deconvolutions to remove path and site effects from the seismic waveforms and extract source time functions; using the latter, in turn, they estimated the source rupture durations τ which, thereby, have some correction for these effects.

Others, such as Smith and Priestley (1993), refer to the method of Frankel and Kanamori (1983). As these authors state, the initial pulse width measured on the seismogram is a function of the rupture duration, of the instrument response and of the broadening caused by the apparent attenuation along the path, including both intrinsic attenuation, scattering and site effects. Frankel and Kanamori (1983) noted that for events in Southern California below a certain magnitude, the pulse width is no longer decreasing and remains about constant; hence, these small events could be assumed as point sources and their waveforms as the impulse response of the combination of the path and instrument factors. Then, the rupture duration τ of larger events may be estimated approximating the deconvolution by subtracting the initial pulse width of small co-located aftershocks or foreshocks from that of the correspondent larger main shock. Thus, the pulse broadening due to factors common to both events, including whole-path attenuation and site response, is eliminated and the source information of the larger event is isolated obtaining estimates of the main shock rupture duration corrected for these effects (as Mori at al.(2003)).

Considering the uncertainties in θ, those in the wave velocities and in j, one may reasonably associate an error factor to l of about 2 (or 0.3 in the log scale), comparable to experimental errors that are sometimes accepted in M and M_o (Caputo, 2005).

Using the estimates of M_o and l, then, one can calculate the values of the *static stress drop* σ. In fact, from Neuber's (1937) theory, the expression for the average final displacement on a fault $< s >$ is derived, assuming $\lambda = \mu$ for the elastic parameters

$$< s > = \frac{16\,\sigma\, l}{7\,\pi\,\mu} \tag{4}$$

Thus, substituting in equation (1) formula (4) and $A = \pi\, l^2$ for a circular fault with equivalent radius l, one obtains σ as function of M_o and l

$$\sigma = \frac{7\, M_o}{16\, l^3} \tag{5}$$

Adding the uncertainty of l to those possibly arising from the various steps to calculate σ, it is clear that an error factor larger than 2 (or 0.3 in the log scale) may affect the stress drop (Caputo, 2005).

2.2 Density Distribution Functions of M_o, l and σ

After introducing the source parameters, we focus on their *ddfs*. We also do this to tentatively infer the *ddf* of the ground acceleration as proposed in paragraph 2.3.

The statistical analyses of earthquakes have for a long time been characterized by the b or the b_o value related respectively to the *ddf* of the magnitude M and to that of the scalar seismic moment M_o. In particular, in analogy to Gutenberg and Richter's (1954) relation for M, the *ddf* of M_o, $n_0(M_o)$, may be represented by a power law (as theoretically suggested for instance by Caputo (1987) and experimentally verified by many authors, e.g. Kagan (2002)), which, in logarithmic form, is

$$\log n_0(M_o) = a_o - b_o \log M_o \qquad (6)$$

In preceding studies (e.g., Caputo, 2005), moreover, it is verified that the *ddfs* of l, $n_l(l)$, and of σ, $n_\sigma(\sigma)$, too may be expressed by power laws with characteristic exponents, respectively ν and $(-1+\alpha)$

$$n_l(l) = L\, l^{-\nu} \qquad (7)$$

$$n_\sigma(\sigma) = \Sigma\, \sigma^{-1+\alpha} \qquad (8)$$

where L and Σ are normalizing factors.

In addition, in the two-dimensional model of the statistics of earthquakes developed by Caputo (1987) it is shown that formulae (7) and (8) tentatively imply the validity of equation (6) and that the two following relations exist respectively between b_o and ν or b_o and $(-1+\alpha)$, depending on the ranges of l and σ:

$$b_o = (\nu+2)/3 \qquad \text{when} \qquad l_2^3\, \sigma_1 > l_1^3\, \sigma_2 \qquad (9)$$

$$b_o = -(-1+\alpha) \qquad \text{when} \qquad l_1^3\, \sigma_2 > l_2^3\, \sigma_1 \qquad (10)$$

where l_1, l_2, σ_1 and σ_2 are the lower and upper limits of the ranges of l and σ (see next paragraph).

Formulae (7) and (8) have been tested (e.g., Caputo, 1981, 2005) with several data sets and the obtained estimates of $(-1+\alpha)$ generally result in the range [-2, -1]. Moreover, the analysis of the world data of M_o (Caputo, 1987), collected and processed at Harvard University (Dziewonski et al., 1987), gives $b_o= 1.61 \pm 0.05$ from which $\nu = 2.83 \pm 0.15$ is inferred, which is not very far from $\nu = 2.5$ obtained (Caputo, 1982) through the statistics of the data on the California surface fracture pattern gathered by Wallace (1976). These estimates have recently been confirmed by Kagan (2002) which, using the data of the Harvard CMT catalogue from 1977 to 1999, gives a worldwide b_o-value in the range [1.60, 1.65].

In our work, using 9 sets of earthquakes, we want to reconfirm the validity of equations (6), (7) and (8) and also to verify formulae (9) and (10) comparing the values of ν and $(-1+\alpha)$ obtained from direct analysis of l and σ with those obtained from the *ddf* of M_o. The latter study will allow one to see if the analysis of the values of the two independent parameters M_o and l may lead to the same *ddf* of the equivalent source radii for a seismic region and, also, if the hypothesis of representing the statistics of earthquakes two-dimensionally is plausible.

Moreover, the *ddfs* of l and σ are also the point of departure in investigating ground acceleration as discussed in the following paragraph.

2.3 Density Distribution Function of the Root Mean Square Ground Acceleration a_{rms}

Firstly, we consider McGuire and Hanks's (1980) relation for the *root mean square ground acceleration* a_{rms}

$$a_{rms} = 0.317\, \frac{\sigma}{\rho R}\left(\frac{f_{max}}{f_c}\right)^{1/2} \qquad (11)$$

where ρ is the density of rocks reasonably assumed in the crust 2.8 g/cm^3, R is the hypocentral distance and f_{max} is

$$f_{max} = \frac{Q\,\beta}{\pi\,R} \tag{12}$$

where Q is the quality factor often assumed around 200/300 in the crust and for the seismic waves of interest. An expanded discussion on the Q factor is, for example, to be found in Knopoff (1964); concerning the Q values, Hanks (1982), for instance, for California assumes a reasonable whole-path Q greater than or about equal to 250, while Mitchell (1973, 1975) for North America suggests values of Q_S roughly in the range [200, 300], whereas Castro and Munguia (1993) for the Oxaca, Mexico, subduction zone give, in a common case, $Q_P = 22\,f^{0.97}$ and $Q_S = 56\,f^{1.01}$, with f frequency. When f_{max}, mainly because of the site effect and of the attenuation along the seismic ray path, is near or smaller than f_c, as is sometimes observed for small events, the seismic wave spectrum is affected by the so called f_{max} effect (Hanks, 1982) and this implies the difficulties cited in paragraph 2.1 in determining the real value of the corner frequency.

For our purposes, we may substitute in equation (11) the equation (12) and the expression for f_c obtained from formula (2) and find

$$a_{rms} = \frac{0.29}{\rho}\,(Q)^{1/2}\,\frac{\sigma}{R}\left(\frac{l}{R}\right)^{1/2} = k\,\sigma\,l^{1/2} \tag{13}$$

Since ρ and Q are fixed (although somewhat arbitrarily) parameters and R is chosen depending on the point where one wants to estimate a_{rms}, the acceleration of the ground is thereby only a function of l and σ, for this reason, if the *ddfs* of these two parameters are known, then the *ddf* of a_{rms}, $n(a_{rms})$, can be defined.

The model we developed to determine the *ddf* of a_{rms} given those of l and σ follows the seminal work of Caputo (1981).

We consider a Cartesian system with axes l and σ and define l_1, l_2, σ_1 and σ_2 respectively the minimum and the maximum values of l and σ for the considered set of earthquakes; these extreme values delimit the rectangle in Figure 2 with area Γ.

Then, we assume that each point of the plane (l, σ) represents an earthquake whose r.m.s. ground acceleration a_{rms} is obtained from formula (13) and we define $a_{rmsm} = k\,\sigma_1\,l_1^{1/2}$, $a_{rmsM} = k\,\sigma_2\,l_2^{1/2}$, $a_{rms1} = k\,\sigma_2\,l_1^{1/2}$ and $a_{rms2} = k\,\sigma_1\,l_2^{1/2}$, reported in Figure 2.

Moreover, let us assume that the *ddfs* of l and σ are represented respectively by the power laws (7) and (8), then in the elementary rectangle defined by the points (l, σ) $(l+dl, \sigma)$ $(l, \sigma+d\sigma)$ $(l+dl, \sigma+d\sigma)$ the number of earthquakes is

$$L\,\Sigma\,l^{-\nu}\,\sigma^{-1+\alpha}\,dl\,d\sigma \tag{14}$$

We firstly calculate the cumulative distribution of a_{rms} in order to obtain the *ddf* later on; thus, to determine the number $\bar{n}\,(a_{rms})$ of earthquakes with r.m.s. ground acceleration less than a_{rms} we have to integrate relation (14) in the portion of the area Γ where l and σ, according to (13), give an r.m.s. ground acceleration less than a_{rms}, that is in the portion of Γ obtained by intersecting area Γ with curve (13) where a_{rms} is constant. The equation of this curve, the obtained area of integration and the final formulae depend on the values of l_1, l_2, σ_1, σ_2 and a_{rms}. <Insert Figure 2>.

As the first case we consider that in Figure 3 (a) for which the ranges of l and σ satisfy the relation

$$l_2^{1/2}\,\sigma_1 > l_1^{1/2}\,\sigma_2 \tag{15}$$

and a_{rms} is in the interval

$$k\sigma_2 l_1^{1/2} = a_{rms1} < a_{rms} < a_{rms2} = k\sigma_1 l_2^{1/2} \tag{16}$$

By integrating (14) over the area in Figure 3 (a) that is delimited by the curve a_{rms} = *cost* for this case and by $l = l_1$, $\sigma = \sigma_1$ and $\sigma = \sigma_2$ we derive the $\overline{n}\,(a_{rms})$, then, differentiating the latter, we finally obtain the *ddf* of a_{rms}

$$n(a_{rms}) = \frac{L\Sigma\left(\sigma_2^{\alpha+2\nu-2} - \sigma_1^{\alpha+2\nu-2}\right)}{(1-\nu)(\alpha+2\nu-2)\,k^{-2\nu+2}}\,(-2\nu+2)\,a_{rms}^{-2\nu+1} \tag{17}$$

whose logarithm is

$$\log n(a_{rms}) = \log C - (2\nu\text{-}1)\log a_{rms} \tag{18}$$

where C includes the two factors which multiply a_{rms} in (17). As one may note, in this case, log $n(a_{rms})$ is a linear function of log a_{rms} with a slope related to ν, the slope of the *ddf* of l.

The second case is shown in Figure 3 (b): the condition satisfied and the interval of a_{rms} are respectively

$$l_1^{1/2}\,\sigma_2 > l_2^{1/2}\,\sigma_1 \tag{19}$$

$$k\ \sigma_1 l_2^{1/2} = a_{rms2} < a_{rms} < a_{rms1} = k\sigma_2 l_1^{1/2} \tag{20}$$

By integrating (14), this time over the area in Figure 3 (b) that is delimited by a different curve a_{rms} = *cost* and by $l = l_1$, $l = l_2$ and $\sigma = \sigma_1$, we derive $\overline{n}\,(a_{rms})$, and, differentiating it, the following $n(a_{rms})$ is obtained

$$n\,(a_{rms}) = \frac{L\Sigma\left(l_2^{-\nu-\frac{\alpha}{2}+1} - l_1^{-\nu-\frac{\alpha}{2}+1}\right)}{\left(-\nu-\frac{\alpha}{2}+1\right)k^{\alpha}}\,a_{rms}^{\alpha-1} \tag{21}$$

Labelling the factor which multiplies a_{rms} as C_1, the logarithm of (21) is

$$\log n\,(a_{rms}) = \log C_1 - (1\text{-}\alpha)\log a_{rms} \tag{22}$$

again a linear function of log a_{rms}, but this time with a slope equal to that of $n_\sigma(\sigma)$, (-1+α).

We will calculate $n(a_{rms})$ for all the nine considered data sets in order to verify the validity of the suggested model and, so, see if it is possible to develop a statistical study of ground acceleration during earthquakes by only using $n_l(l)$ and $n_\sigma(\sigma)$.

On the basis of the developed model, it is also worth noting that, if the range of the l values is limited with respect to that of the σ values, thus clearly the *ddf* of a_{rms} scales like that of σ, our data analysis will confirm this, as discussed in section 4.3. <Insert Figures 3 (a), (b)>.

3. The Data

In this note we examine 9 sets of earthquakes related to different seismic regions and to different types of sets of events; these are at present the only ones available and suitable for our analyses. Five of these nine groups, Smith and Priestley (1993), Abercrombie (1995), Hardebeck and Hauksson (1997), Jin et al. (2000) and Mori et al. (2003), have been partially studied earlier (Caputo, 2005). These 5 sets are generally the most recent, then more accurate, available, they are relatively numerous and have previously led to promising results; now, in the light of new aims, we consider these groups again and more thoroughly than before, comparing them to the other four groups to support the consistency of the results. In addition, there are 3 sets, Thatcher and Hanks (1973), Tucker and Brune (1973) and Hartzell and Brune (1977), whose events were recorded in the '70s; these groups were preliminarly analyzed in some previous works in order to verify the model which is suggested for the *ddfs*, and these provided one of the first tentative checks of it. So, although their data have an accuracy and a reliability which are less than those of the more recent ones, they are reanalyzed in this note also for the new objectives of this article (e.g. concerning the a_{rms}) that are presented in the preceding sections. Finally, we use the most recent data set of Tusa et al. (2006).

Seven of the 9 sets of events have been recorded in different areas of California; instead, the data of Jin et al. (2000) are related to Central Japan and those of Tusa et al. (2006) to Southeastern Sicily. Moreover, 4 groups are made up of aftershocks of a main event, another 4 groups concern earthquakes that are scattered in a given region and in a certain time interval and, finally, Hartzell and Brune's (1977) set is a swarm of earthquakes. This may allow one to tentatively see if there are some differences in the statistical results depending on the seismic region and the type of set considered, even though the unbalanced number of sets per region or per type leads us to suppose that a final conclusion will not be possible with our data.

In six of the groups examined, the values of l, and then those of σ, have been calculated using the conventional corner frequency f_c (Brune, 1970, 1971), and introducing corrections for the attenuation and site effects, perhaps less accurate in the sets of the '70s. Moreover, in the remaining sets that are three of the most recent, the authors used Boatwright's (1980) formula to calculate l, introducing an implicit correction for the whole-path attenuation and the local site effect (calculating τ as discussed in paragraph 2.1). This will also allow a comparison of the statistical properties of l and of σ that are determined with different methods.

The results of the analyses performed in this note are presented in Table 1, 2, 3 and 4 and discussed in detail in Section 4. In what follows we briefly outline the main characteristics of each set studied.

The set of **Smith and Priestley (1993)** involves 85 aftershocks of the 1984 Round Valley, California, earthquake. The local magnitude of the main event is 5.8, those of the aftershocks in the range [2.7, 4.4]. The authors determined the values of l using an adaptation of the initial P-wave pulse width time-domain deconvolution technique of Frankel and Kanamori (1983) and then calculated σ from l and M_o.

Abercrombie (1995) analyzed 111 tectonic earthquakes, some with very small magnitude, recorded at 2.5 km depth, in granite, in the Cajon Pass scientific drill hole, Southern California, about 4 km from the San Andreas fault. She used 4 different models for the spectra of both P and S waves, all based on Brune's (1970, 1971) f^{-n} with n=2 model, in order to compute M_o and f_c and then to infer the values of l; we consider her model 1 as basic in which n=2 is fixed, Q of P waves varies in the range [581, 1433] and Q_S in [879, 132]. We only use the data from the S waves, because of their greater accuracy, and from the values of l and M_o we estimate σ values by relation (5). Three values of the stress drop calculated for this set are much larger than 1000 bar and seem too high to be reliable; thus, we consider these values as outliers and we neglect them in all the analyses concerning σ, also implying that the range of the stress drop decreases to [0.87, 1118] bar and the log of the ratio of the extremes of the range to 3.11 (in Table 1 there is also the range and the log of the ratio considering all 111 data, for completeness). In addition, we note that about 87% of the values of l are in the limited range [10, 100] meters.

Hardebeck and Hauksson (1997) analyzed a large number, 279, of aftershocks of the 1994 Northridge, California, earthquake, whose local magnitudes are between 2.5 and 3.9. The authors calculated the l values using the estimates of the source rupture duration τ. The values of M_o have been determined from the local magnitude M_L of each event using an empirical relationship for Southern California earthquakes (Thatcher and Hanks, 1973): $\log M_o = 1.5\ M_L + 16.0$, with M_o in dyne*cm. Finally, the authors estimated the stress drops by using equation (5), averaging the values obtained for the same event at each of the four stations used. The estimates of σ for this set cover a range of about 3 orders of magnitude, as one notes in Table 1, and the authors suggested that this variability is caused not only by the measurement uncertainties but there is also a significant heterogeneity in the stress drops of the aftershocks, which is not surprising considering the heterogeneous state of stress in the examined region.

At nine local stations **Jin et al. (2000)** recorded 102 small earthquakes in the Atotsugawa fault zone (including the Ushikubi, Mozumi and Atotsugawa faults) in Central Japan; these occurred between March 1995 and October 1997. The magnitudes of the events are in the range [0.5, 3.6], but mostly (about 70%) equal to or less than 2.0. From the S-wave spectra the authors determined the values of f_c, M_o and σ (from f_c and M_o). We calculate the values of l using those of f_c (formula (2)).

Mori et al. (2003) selected 55 aftershocks of the 1994 Northridge, California, earthquake with clean P-wave arrivals on the seismogram and local magnitude greater than or equal to 4.0. The events were recorded at 3 different stations and the authors used their estimates of the source rupture duration and M_o to calculate the σ values; then, we determine l through relation (5). The stress drops of this set belong to a wide range of about 2.5 orders of magnitude (Table 1) and also for this group, as for that of Hardebeck and Hauksson (1997), the variability of σ seems (Mori et al., 2003) to be caused not solely by the errors attributed to this parameter, although they are significant.

Thatcher and Hanks (1973) analyzed 138 earthquakes scattered throughout Southern California from March 1930 until February 1971 and with local magnitude in the range [2.0, 6.8]. The authors estimated the values of l from those of f_c and the values of M_o from the spectra; they then determined the stress drops using relation (5). The earthquake catalogue of this set is incomplete for small magnitudes (only four values of M_L are small than 3.0), but it may however be considered a statistically significant sample.

Tucker and Brune (1973) examined 165 aftershocks of the 1971 San Fernando, California, earthquake. The magnitude of each event is not specified in their note which reports only that M_L varies between about 0.5 to 4.5. The authors calculated M_o from the S-wave spectra, l from f_c and σ from l and M_o using equation (5). Moreover, they suggested that the ranges attributed to M_o, l and σ of this set are too wide to be caused only by the errors that were estimated for these parameters and, thus, may represent their real variability.

Hartzell and Brune (1977) analyzed 62 events of the January 1975 Brawley–Imperial Valley earthquake swarm, whose local magnitudes are in the range [1.0, 4.7]; M_o was calculated from the spectra, l from f_c and σ with relation (5). To correct the S-wave spectra for the attenuation effect, the authors, due to the presence of a 6 km thick accumulation of Cenozoic sediments that covers the region, used Q=150 and not the values usually assumed for the Earth's crust ($Q \approx 200/300$, e.g. Hanks (1982), Mitchell (1973, 1975), Castro and Munguia (1993)). The value of Q that was somewhat arbitrarily chosen by the authors implies an error factor of 1.5 in f_c and of 3 to 4 in σ (Hartzell and Brune, 1977).

Finally, **Tusa et al. (2006)** studied 135 small local earthquakes that occurred in Southeastern Sicily from June 1994 until May 2001 and whose magnitudes ranged from 0.6 to 2.5. From the P-wave spectra corrected for attenuation and site effects, the authors calculated the values of M_o and f_c; from the latter, in turn, they estimated l using Hanks and Wyss's (1972) formula (Brune's formula modified for P-waves); finally, σ was calculated from relation (5). In order to determine the source parameters, Tusa et al. (2006) averaged the estimates from the 9 stations of the SE Sicily Seismic Network and used the method of the 'holed spectra' (spectra that were lacking corrected spectral amplitudes within a certain frequency range) with success, because P-wave Q and site factor to correct spectra are known only in two limited frequency ranges for SE Sicily (Giampiccolo et al., 2003). The values of the stress drop estimated for this set are very low (between 0.02 and 7.75 bar), thus implying probably low stress accumulation in SE Sicily. Moreover, the values of l are generally in a limited range (Table 1) and in particular 76% within the narrow range [200, 300] meters. <Insert Table 1, 2, 3 and 4>.

4. Discussion of the Results

4.1 The Ranges of the Source Parameters and their Correlations

In Table 1 we report the estimated ranges of the parameters for all the 9 sets considered and also the log of the ratio of the extremes of the ranges, which gives the orders of magnitude covered and also a comparison of the ranges of physically different parameters.

Analyzing this table, one can see that the ranges of M_o and σ in each of the 9 sets are always larger than two orders of magnitude, whereas the ranges of l are smaller. However, the ranges of the source parameters examined here seem in general to be sufficiently wide to obtain acceptable statistics of the correspondent data sets.

Comparing Table 1 in this note and Table 1 in Caputo (2005) there are some differences in the ranges of the parameters of the 5 sets considered by both works, in some cases negligible, in others significant to justify a revision, as for example the range of log M_o of Hardebeck and Hauksson (1997), that of σ of Abercrombie (1995) and that of l of Mori et al. (2003). They are due to some misprints in Caputo (2005).

In particular, we are interested in the range of variation of the static stress drop, and from our analysis we may infer that, contrary to some previous studies, σ varies from one region to another but also significantly in the same seismic region and also in the different sets of a given region. In fact, one notes in Table 1 that the ranges covered by σ and their extensions are different depending on the different sets considered and each set has a variability of σ larger than two orders of magnitude. Moreover, the sets of Hardebeck and Hauksson (1997) (that with the maximum range of σ) and that of Mori et al. (2003) concern the aftershocks of the same earthquake but with not overlapping magnitudes, and thus, putting them together, one finds that σ covers 4.82 orders of magnitude which is the maximum range of σ for a set of aftershocks. Therefore, one may suggest that the characteristics of a seismic region may influence the values of σ causing the differences between one region to another; however, the same seismic region too may be affected by significant internal heterogeneity which is a possible cause of the scatter in the values of σ of the earthquakes that occurred in it.

The above conclusions regarding the variability of the stress drop are shown in this article for the Californian region only (7 out of 9 datasets are from California), whereas they are merely suggested for other two regions, Sicily and Japan, and to be confirmed they need further future analyses with a larger number of sets out of California.

The correlation coefficients between log σ and log l for each set are found in Table 2, where the slopes of the linear regressions with their associated standard deviations *std* are also reported.

The values of the correlation coefficients for the 5 sets already examined by Caputo (2005) are almost the same as previously determined and, together with those of the other 4 sets considered here, all have modulus equal to or less than 0.53 (Table 2). So, the correlation is weak, in spite of the fact that σ is computed using the values of l; this implies that l and σ, mainly because of the physical and tectonic significance of σ, seem to be possible candidates for the development of a 2D model of seismicity for which it would rigorously be necessary to use two independently determined parameters such as the couple M_o-l.

Despite the low correlations, however, we may note that the slopes of the linear regressions of log σ vs. log l are negative for all the 9 sets, although different depending on different regions and types of earthquake sets (Table 2). It is therefore the consistency of the results that, now for 9 sets, seems to support the validity of the hypothesis that log σ is a linearly decreasing, although weakly, function of log l. This in turn implies that σ and l would be related by a power law of the form $\sigma \propto l^{\gamma}$ with negative γ, although not rigorously proved since σ depends on l.

The trend of log σ vs. log l for the set of Hardebeck and Hauksson (1997), the most numerous, is in Figure 4. <Insert Figure 4>.

Dealing with the reliability of the parameters used for the statistics, in order to conclude this paragraph, we can see that in our paper the linear size of the fault (then the stress drop) has been estimated by different methods. The sets of Smith and Priestley (1993), Hardebeck and Hauksson (1997) and Mori et al. (2003) used Boatwright's (1980) formula (expression (3)); the values of τ in this formula are, as explained in paragraph 2.1, determined by removing the path and site effects from the observed pulse width measures, so the correspondent values of l seem not to be significantly affected by the f_{max} effect. On the contrary, the remaining sets used formula (2) with the conventional corner frequency. Given that Brune's (1970, 1971) values of f_c in relation (2) (then the l values), mainly for events with small source area, may be affected by the f_{max} effect unless the attenuation and site effects are explicitly taken into account in the source parameter estimates, all considered sets which calculate l from Brune's f_c have made a further correction for these effects.

In order to establish that the use of Brune's corner frequency estimates, with an attenuation correction, does not significantly influence the results of our statistics, we use the set of Shi et al. (1998) that is made up of 49 earthquakes in the Northeastern United States. Eight of these events each have an aftershock small enough to be treated as its empirical Green's function; thus, the authors estimated a corner frequency f_{ce} accurately for these events using the empirical Green's function deconvolution method of Shi et al. (1996). Moreover, for all events, they also applied a frequency-dependent Q correction to the observed wave displacement spectra, and from the latter, they estimated corner frequencies by two other methods: firstly, from the intersection of the asymptotic low-frequency spectral level and a line with slope about f^{-2} indicating the source spectral decay above the comer frequency, that was Brune's (1970, 1971) conventional method; the other, by best fitting the corrected spectra with the f^{-2} source model (Aki, 1967). Considering the eight events for which all the three above described methods can be used, Shi et al. (1998) compared the corner frequency estimates and concluded that the corner frequencies from the simulation process, f_{cs}, are more consistent with those from the empirical Green' s function, f_{ce}, than the corner frequencies from the conventional method, f_c; then, the f_{cs} may be used to analyze the earthquakes that do not have an available smaller event that can be used as their empirical Green's function.

It is interesting that Shi et al. (1998) provide corner frequency estimates (and then l estimates) for the same events from different methods, one of which is Brune's (1970, 1971).

Plotting l obtained from f_c, named l_c, as a function of the l obtained from f_{cs}, named l_{cs}, we find the line shown in Figure 5 expressed by the relation $l_c = (1.35 \pm 0.04)\ l_{cs}$. Although the spectra were corrected for attenuation, the f_c are in general smaller than the f_{cs}, then the l_c are in general somewhat larger than the corresponding l_{cs}. However, the difference between the two estimates in log scale is 0.13 which is smaller than the error of 0.3 that is usually assumed for log l. Thus, the values of l obtained from Brune's method seem to be in acceptable agreement with those obtained from the simulation process, which in turn are largely consistent with the accurate estimates from the empirical Green's function method which was considered by Shi et al. (1998) as being the best way for estimating source corner frequency.

Moreover, using the set of Shi et al. (1998) and considering σ_c and σ_{cs} obtained from l_c and l_{cs} respectively, we calculate the correlation and the slope of the linear regression of log σ_c vs. log l_c obtaining 0.38 and 0.85 ± 0.30 respectively, and of log σ_{cs} vs. log l_{cs} obtaining 0.33 and 0.88 ± 0.37 respectively. Since the slopes are within the associated standard deviations, the two results may be tentatively considered as being in agreement.

In general, we have shown that, although the use of Brune's (1970, 1971) method, even when it has attenuation correction, may often cause relatively smaller values of f_c and larger values of l than in reality (implying smaller values of σ), the differences are not large and the relationship between l and σ may not be significantly affected, then, the resulting discrepancies are not too relevant for our statistics. From this result we may infer that the estimates of l

from Brune's corner frequency with attenuation correction appear as valid for our analyses as do those obtained from τ and thus the reliability of the parameters used in this note seems to be confirmed. <Insert Figure 5>.

4.2 The Density Distribution Functions of M_o, l and σ

The slopes of the *ddfs* of l, σ and M_o are listed in Table 3, together with their standard deviations *std*. Depending on the ranges of l and σ, we also calculate and present the values of ν and of $(-1+\alpha)$ obtained from b_o, again with their standard deviations, in this table.

To study the *ddfs* one would need to have complete catalogues of the examined parameters, whereas for the analysis of the correlations a less numerous data set may be sufficient. However, the catalogues are generally incomplete, in particular for the smaller M_o and then for the smaller values of σ and l. Moreover, as was generally done earlier (e.g., Caputo, 2005), in calculating the *ddf* for a generic parameter x, $n(x)$, we neglect the smaller and larger values of x in order to tentatively select the interval of linearity of log $n(x)$ as a function of log x.

Concerning the 5 sets already studied by Caputo (2005) our work highlights some discrepancies from the latter, such as the slope of $n_\sigma(\sigma)$ and $n_0(M_o)$ of Hardebeck and Hauksson (1997), that of $n_l(l)$ of Abercrombie (1995) and Jin et al. (2000), and that of $n_l(l)$ and $n_0(M_o)$ of Smith and Priestley (1993). These differences justify our reanalysis of the 5 sets and may be due to a different selection of the interval of linearity of the *ddfs* and to a different width of the cells used in the histograms.

However, for the cited 5 groups, as in Caputo (2005), and for the other 4 sets herein considered, the slopes of $n_l(l)$, $n_\sigma(\sigma)$ and $n_0(M_o)$ are all negative. Thereby, although the ranges of the source parameters are sometimes narrow, and the slopes scattered probably because of the differences between the tectonics of the various regions and types of earthquake sets, the examined *ddfs* are decreasing functions of their parameters represented by negative power laws: the suggested model summarized in relations (6), (7) and (8) is then validated.

We note that the values of $(-1+\alpha)$ obtained in this note for the 9 sets are all in the range [-2, -1] that was previously determined, regardless of the method used to calculate the values of σ.

The estimates we obtained for ν and b_o are sometimes different from the world averages, respectively 2.83 ± 0.15 and 1.61 ± 0.05 (Caputo, 1987) or b_o= [1.60, 1.65] (Kagan, 2002), and from the experimental estimate of ν, around 2.5 (Caputo, 1982; Wallace, 1976). These differences may be due to many causes, sometimes also coupled; among others, the lack of small events (probably the case of b_o and ν for Thatcher and Hanks (1973)) or the limited range of the parameters (probably the case of ν for Abercrombie (1995) and Tusa et al.(2006)). Additionally, the different characteristics of each region are averaged in the world estimates of Caputo (1987) and Kagan (2002); moreover, the linearity of the *ddfs* of M_o versus log M_o has not been proved over the whole range of the seismic moment. Finally, the small overlap with the M_o ranges of the considered sets gives values of b_o and ν different from the world averages. Because of all the above issues, the reliability of the comparison is still limited and future checks, when more data are available, are needed.

On the base of the model, using relations (9) and (10), we have also calculated ν or $(-1+\alpha)$ indirectly from b_o, depending on the condition satisfied by the ranges of l and σ. Looking at Table 3, then, a comparison can be made between the values of ν and $(-1+\alpha)$ determined directly from the values of l and σ (respectively column 3 changed in sign and column 5) and those indirectly estimated (columns 9 and 11): the indirect values of the exponents of $n_l(l)$ and $n_\sigma(\sigma)$ are in agreement with or only slightly different (Thatcher and Hanks, 1973; and Jin et al., 2000) from the direct values; the only two exceptions are the set of Abercrombie (1995), whose values of l are, however, in a too limited range, and the set of Hartzell and Brune (1977), whose not recent data do not ensure the same accuracy of the recent sets. These results imply that the study of $n_0(M_o)$ confirms the laws suggested for $n_l(l)$ and $n_\sigma(\sigma)$ and the possibility of a 2D representation of the earthquake statistics based on the *ddfs* of l and σ gains a further support. Obviously, it is of particular interest that l and M_o, although they are two independently measured parameters, both provide almost the same *ddf* of the equivalent source radius in a region.

Since the attenuation along the ray path and the site effects may cause, through the f_{max} effect, larger values of l and smaller values of σ than in reality, one could expect that they also influence the exponents of the *ddfs* of these two parameters when an explicit correction for these effects is not made. However, all the considered sets take into account the attenuation and site response, and the statistical results we obtained concerning the *ddfs* are anyhow consistent, although the sets of the '70s used corrections for these effects less reliable and accurate than the more recent ones, as the same authors state.

Examples of the linear regression of $n_\sigma(\sigma)$ and $n_l(l)$ are respectively in Figures 6 (a) and (b) for the set of Hardebeck and Hauksson (1997). <Insert Figures 6 (a), (b), (c)>.

4.3 The Density Distribution Function of a_{rms}

We can now discuss the results concerning the *ddf* of a_{rms}, $n(a_{rms})$, observing Table 4 in which are reported the slopes of $n(a_{rms})$ for all the 9 considered sets, together with their standard deviations *std*.

In order to focus our attention on the trend of the *ddf* of the ground acceleration, in the analyses we neglect the k factor in relation (13), only keeping the dependence of a_{rms} on l and σ by defining a parameter $a = \sigma l^{1/2}$, so that $a_{rms} = ka$ (from (13)) and the slope of $n(a)$ is exactly that of $n(a_{rms})$. Moreover, in calculating this *ddf* we select the interval of linearity of log $n(a)$ as a function of log a, as was made for the *ddfs* of the other parameters (paragraph 4.2).

An example of the linear regression of the *ddf* of the ground acceleration is in Figure 6 (c) for the set of Hardebeck and Hauksson (1997).

Looking at Table 4, first one notes that, as expected from the model presented in paragraph 2.3, the slopes of the *ddf* of a_{rms} for the 9 sets are all negative, thus, as the other *ddfs* above analyzed, $n(a_{rms})$ is a decreasing function of its parameter and may be given by a negative power law.

Moreover, comparing the ranges of the equivalent radius and the stress drop in Table 1, it results that the range of l of each set is smaller than that of σ; therefore, the condition satisfied by the ranges of l and σ is always, for all the sets, that represented by equation (19). Then, as pointed out in paragraph 2.3 in consequence of the proposed model and as it is also expected since a_{rms} depends on σ and $l^{1/2}$ (relation (13)), we can preliminarily suggest that the *ddf* of a_{rms} will scale like that of σ. After analyzing the data, it turns out that, in fact, the slopes of $n(a_{rms})$ are always in the range [-2, -1] and in good agreement with those of $n_o(\sigma)$, with the only slightly relevant exception of the set of Thatcher and Hanks (1973). So, another result of our analysis is that $n(a_{rms})$, at least for the data sets herein considered, seems to have the same form of $n_o(\sigma)$.

Altogether, our study of the *ddf* of the ground acceleration appears promising since the consistency of the results tentatively bears with the proposed model and its expectations. However, the examined data are still too limited and, moreover, for the considered sets, we do not have availability of directly observed values of the r.m.s. ground acceleration to compare with the estimates that we obtained from the source parameters.

5. Conclusions

The variability of the static stress drop from one seismic region to another is well known; in this article, however, we show that it also varies significantly in the different sets of a same region (see the values of σ in Table 1) and, consequently, within the whole given region as, in particular, California, where it ranges from a minimum of 0.02 bar (Hardebeck and Hauksson, 1997) to a maximum of 1307 bar (Mori et al., 2003) (see Table 1). It is worth noting, moreover, that this range is exactly that of the variability of σ for the aftershocks of the same Northridge event (Hardebeck and Hauksson, 1997; Mori et al., 2003). Furthermore, outside California, for instance in Sicily σ varies from 0.02 to 7.75 bar (Tusa et al., 2006) and in Japan from 0.17 to 122 bar (Jin et al., 2000).

Regarding the *ddfs* of l, σ and M_o, although the slopes for the nine considered sets are scattered, they are all negative; this thus confirms, at least in limited ranges, the assumed negative power law models for the *ddfs* of these parameters. Moreover, from the study of the *ddf* of M_o one may indirectly infer a *ddf* of the equivalent radius of faults of a seismic region that is in fair agreement with that obtained from the direct analysis of l. This result also implies that a tentative 2D representation of the statistics of earthquakes based on the *ddfs* of l and σ could possibly supply an alternative model of seismicity, l and σ appearing to be a good couple of candidates mainly because of the tectonic and physical implications of σ and also supported by the generally low values of the correlation between log σ and log l as obtained for the 9 considered sets.

The study of the correlation between log σ and log l has also led to a tentative validation of the physical hypothesis that larger stress drops are more frequent, per unit time and supposing almost uniform stress accumulation, in smaller faults as opposed to large ones. All the slopes of the correlation are, in fact, negative, so log σ appears as a linearly decreasing function of log l, although weakly, and we may tentatively suggest a negative power law relating σ and l. However, the dependence of σ on l and M_o prevents rigorous demonstration.

We have tentatively proven the model that was proposed for the *ddf* of a_{rms}, obtaining that the slope of the latter is negative for all the considered data sets, thereby $n(a_{rms})$ may be given by a negative power law, as is theoretically implied by the *ddfs* of l and σ. Moreover, as expected from McGuire and Hanks's (1980) formula and also due to the more limited range of l in each set than that of σ (see paragraph 2.3 and 4.3), the $n(a_{rms})$ seems to have the same exponent of $n_\sigma(\sigma)$. However, our results, although promising, are preliminary and must be verified by future analyses on both a greater number of data and on directly observed values of a_{rms}.

Although the considered sets are mainly related to California and only one group is a swarm of earthquakes, the discrepancies in the values obtained as results, preliminarily evidenced by this work, lead us to tentatively suggest that the differences between distinct seismic regions and types of earthquake sets may influence the slopes of the *ddfs*, the correlation between parameters and their slopes and the parameter ranges. However, the number of sets considered here is still too limited and not heterogeneous enough to infer specific characteristics of each region or set type; so we defer this interesting study to a time when more suitable data are available.

Finally, using the data of Shi et al. (1998), we have statistically tested that the different methods used to calculate the parameters for the analyses do not significantly affect our statistical results, although Brune's method may produce some discrepancies in the parameter estimates even when an attenuation correction is made. The agreement between the outcomes for the 9 used sets, regardless of how l and σ were calculated, contributes to validate this statement. Moreover, even when the attenuation and site corrections are not very accurate and reliable, as in the case of the sets of the '70s, the consistency of the statistics obtained in this paper leads us to suggest that a non-rigorously precise estimate of the f_{max} effect does not significantly influence the correlation between σ and l and the trend of their *ddfs*, although the single estimates of the parameters may be affected by some errors.

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Figure Captions

Figure 1. Schematic representation of two different faults: the dashed lines are the linear dimensions of faults labelled by L or l, the black triangles are the barriers, that is the asperities in our case of equal size, that oppose the slip of the two lithospheric blocks and are subject to increasing regional shear stress labelled by σ; at a fixed time, σ is the same in both cases as is indicated by the arrows in figure. At a fixed time, the large fault L is more prone to apply a larger force ($L^2\sigma$) on the barrier than the smaller fault l ($l^2\sigma$); so, since the barriers are assumed to be the same and supposing that the stress increases with the same rate in both faults, the time needed to the smaller fault to overcome the asperity is greater than that for the larger fault, thereby the smaller fault, in order to rupture the barrier, accumulates more stress and more frequently generates larger stress drop.

Figure 2. Cartesian plane (l, σ). l_1, l_2, σ_1 and σ_2 are respectively the minimum and the maximum values of l and σ for the set of considered earthquakes and they define the area Γ; a_{rmsm} a_{rmsM} a_{rms1} a_{rms2} are defined in the text.

Figures 3. The area of integration where the r.m.s. ground acceleration is less than a_{rms} for the case $l_2^{1/2}\sigma_1 > l_1^{1/2}\sigma_2$ is represented in figure **(a)** and is delimited by the curve $a_{rms} = cost$ and by $l = l_1$, $\sigma = \sigma_1$ and $\sigma = \sigma_2$; instead, the area of integration in the case $l_1^{1/2}\sigma_2 > l_2^{1/2}\sigma_1$ is in figure **(b)** and is delimited by a different curve $a_{rms} = cost$ and by $l = l_1$, $\sigma = \sigma_1$ and $l = l_2$.

Figure 4. Correlation between log σ and log l for the data of Hardebeck and Hauksson (1997). In spite of the scatter, log σ seems to decrease linearly with increasing log l. σ is in bar, l is in meters.

Figure 5. Relation between the l calculated from f_c, l_c, and the l calculated from f_{cs}, l_{cs}, of the set of Shi et al. (1998); l_c and l_{cs} are in meters.

Figures 6 (a) Density distribution function of σ for 279 aftershocks of the 1994 Northridge, California, earthquake (Hardebeck and Hauksson, 1997), σ is in bar. **(b)** *ddf* of l for 279 aftershocks of the 1994 Northridge, California, earthquake (Hardebeck and Hauksson, 1997), l is in meters. **(c)** *ddf* of the ground acceleration for 279 aftershocks of the 1994 Northridge, California, earthquake (Hardebeck and Hauksson, 1997); for sake of simplicity, in order to study this distribution, we used the parameter $a = \sigma l^{1/2}$, so, from formula (13), $a_{rms} = ka$ and the *ddf* in figure is proportional to that of a_{rms}; a is in bar*m$^{1/2}$.

Figure 1.

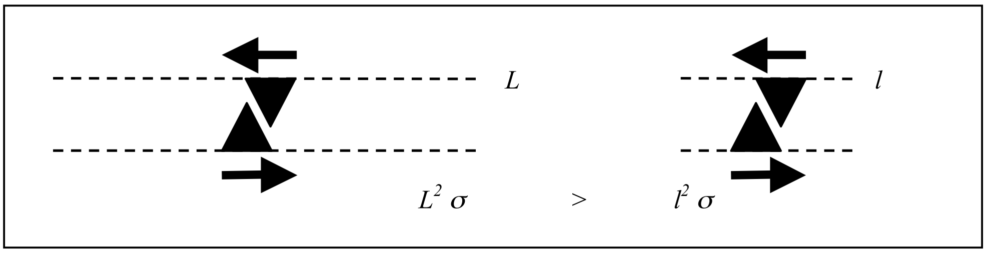
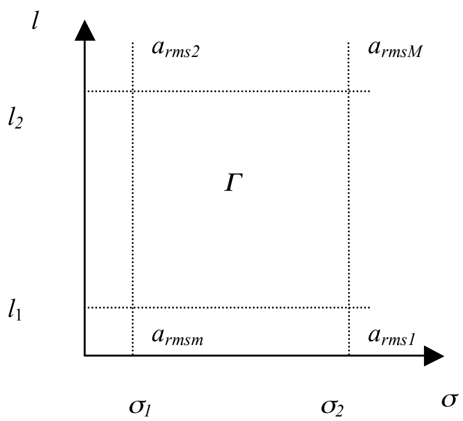
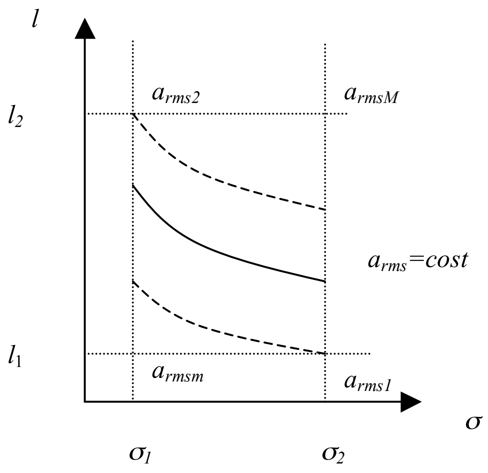


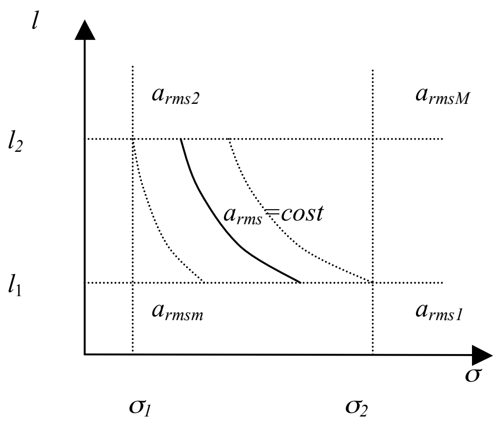
Figure 2.



Figures 3.



(a)



(b)

Figure 4.

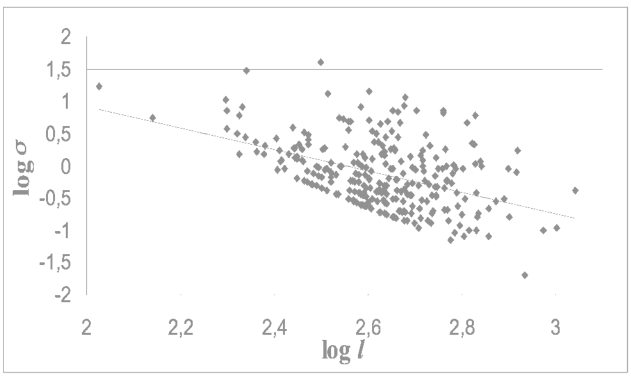
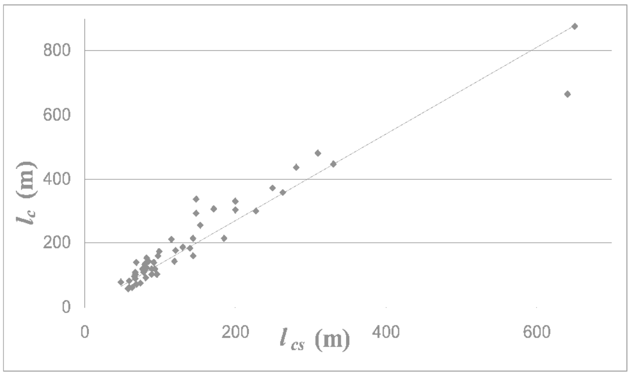
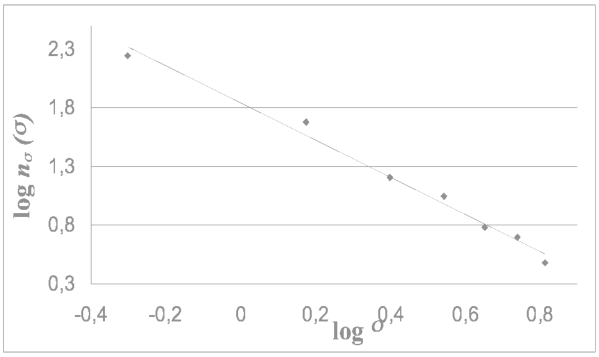


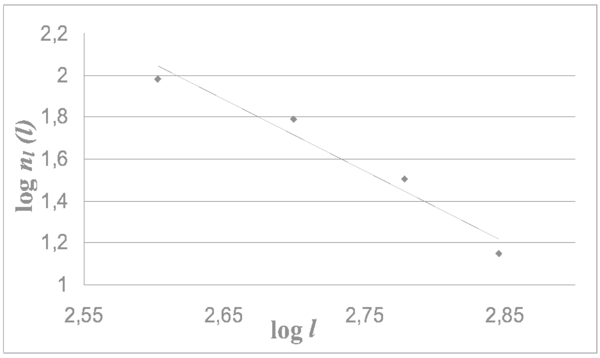
Figure 5.



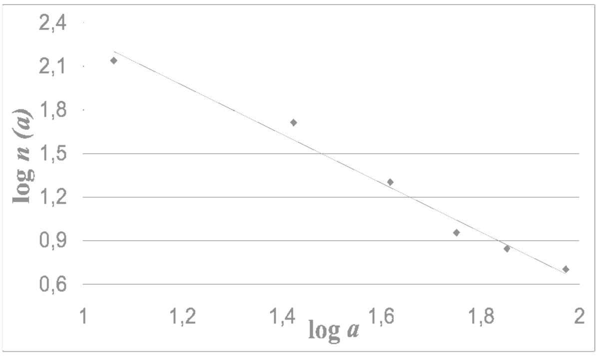
Figures 6.



(a)



(b)



(c)

Table 1. Ranges of the parameters and log of the ratio of the extremes of the ranges for the 9 sets of earthquakes, of different seismic regions and different types, studied in this note; l is in meters, σ in bar and M_o in dyne*cm; AF = aftershocks, SC = scattered events, SW = swarm.

DATA SETS	N° events	Range of l (m)	log ratio	Range of σ (bar)	log ratio	Range of log M_o	difference log
AF Northridge (Calif) *17-1-1994* ***Hardebeck and Hauksson (1997)***	279	106-1098	1.01	0.02-41	3.31	19.75-21.85	2.10
AF S.Fernando (Calif) *9-2-1971* ***Tucker and Brune (1973)***	165	47-448	0.98	0.52-344	2.82	18.36-21.92	3.56
SC South California *3-1930/2-1971* ***Thatcher and Hanks (1973)***	138	400-11900	1.47	0.2-218	3.04	19.71-26.16	6.45
SC South-East Sicily *6-1994/5-2001* ***Tusa et al. (2006)***	135	148-516	0.54	0.02-7.75	2.59	17.22-20.25	3.03
SC Cajon Pass scientific drill hole (South California) ***Abercrombie (1995)***	111	5.13-215	1.62	0.87-991306 0.87-1118	6.06 3.11	15.91-22.31	6.40
SC Atotsugawa fault zone (Central Japan) 3-95/10-97 ***Jin et al. (2000)***	102	54-385	0.86	0.17-122	2.86	17.61-21.47	3.86
AF Round Valley (Calif) *23-11-1984* ***Smith and Priestley (1993)***	85	88-789	0.95	2-282	2.15	19.37-22.23	2.86
SW Brawley-Imperial Valley *January 1975* ***Hartzell and Brune (1977)***	62	72-444	0.79	0.98-340	2.54	18.20-21.90	3.70
AF Northridge (Calif) *17-1-1994* ***Mori et al. (2003)***	55	133-2439	1.26	4-1307	2.51	21.32-24.08	2.76

Table 2. Correlation coefficients and slopes (with their standard deviations *std*) of the trend of log σ as a function of log l for the 9 data sets analyzed in this note; l is in meters and σ is in bar.

DATA SETS	N° events	Correlation (log σ, log l)	Slope (log σ, log l)	*std*
Hardebeck and Hauksson (1997)	279	-0.45	-1.65	0.20
Tucker and Brune (1973)	165	-0.08	-0.27	0.27
Thatcher and Hanks (1973)	138	-0.09	-0.16	0.15
Tusa et al. (2006)	135	-0.22	-1.74	0.67
Abercrombie (1995)	111	-0.11	-0.21	0.19
Jin et al. (2000)	102	-0.14	-0.54	0.39
Smith and Priestley (1993)	85	-0.34	-0.86	0.26
Hartzell and Brune (1977)	62	-0.15	-0.53	0.44
Mori et al. (2003)	55	-0.53	-1.33	0.29

Table 3. Slopes (with their standard deviations *std*) of the density distribution functions (*ddfs*) of l, σ and M_o of the 9 data sets studied in this note. In the last four columns there are the values of ν (with their *std*) or of (-1+α) (with their *std*) determined from b_o depending on the ranges of l and σ of each set. l is in meters, σ in bar and M_o in dyne*cm.

DATA SETS	N° events	Slope log $n_l(l)$	*std*	Slope log $n_o(\sigma)$	*std*	Slope log $n_0(M_o)$	*std*	ν from b_o	*std*	(-1+α) from b_o	*std*
Hardebeck and Hauksson (1997)	279	-3.40	0.53	-1.58	0.08	-1.57	0.05			-1.57	0.05
Tucker and Brune (1973)	165	-1.50	0.59	-1.52	0.07	-1.31	0.10	1.94	0.29		
Thatcher and Hanks (1973)	138	-1.39	0.16	-1.67	0.07	-1.24	0.04	1.73	0.13		
Tusa et al. (2006)	135	-4.81	0.94	-1.84	0.13	-1.56	0.26			-1.56	0.26
Abercrombie (1995)	111	-1.47	0.13	-1.59	0.08	-1.30	0.03	1.91	0.08		
Jin et al. (2000)	102	-2.43	0.55	-1.20	0.14	-1.44	0.02			-1.44	0.02
Smith and Priestley (1993)	85	-1.62	0.62	-1.60	0.15	-1.33	0.01	1.99	0.02		
Hartzell and Brune (1977)	62	-1.43	0.33	-1.61	0.02	-1.33	0.05			-1.33	0.05
Mori et al. (2003)	55	-2.08	0.61	-1.33	0.12	-1.35	0.05	2.06	0.16		
Reference world average or range		**-2.83**	**0.15**	**[-2, -1]**		**-1.61**	**0.05**	**2.83**	**0.15**	**[-2, -1]**	

Table 4. Slopes (with their standard deviations *std*) of the *ddf* of a_{rms} for the nine sets analyzed in this note.

DATA SETS	N° events	Slope log $n(a_{rms})$	*std*
Hardebeck and Hauksson (1997)	279	-1.67	0.12
Tucker and Brune (1973)	165	-1.45	0.06
Thatcher and Hanks (1973)	138	-1.16	0.05
Tusa et al. (2006)	135	-1.70	0.16
Abercrombie (1995)	111	-1.65	0.14
Jin et al. (2000)	102	-1.21	0.15
Smith and Priestley (1993)	85	-1.70	0.22
Hartzell and Brune (1977)	62	-1.60	0.30
Mori et al. (2003)	55	-1.42	0.09