A thermal pressurization model for the spontaneous
dynamic rupture propagation on a 3–D fault:

Part I – Methodological approach

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Abstract

We investigate the role of frictional heating and thermal pressurization on earthquake ruptures by modeling the spontaneous propagation of a 3-D crack on a planar fault governed by assigned constitutive laws and allowing the evolution of effective normal stress. We use either slip–weakening or rate– and state–dependent constitutive laws; in this latter case we employ the Linker and Dieterich (1992) evolution law for the state variable and we couple the temporal variations of friction coefficient with those of effective normal stress. In a companion paper we investigate the effects of thermal pressurization on the dynamic traction evolution within the breakdown zone. We solve the 1–D heat conduction equation combined with the Darcy’s law for fluid flow in porous media. We obtain a relation that couples pore fluid pressure to the temperature evolution on the fault plane. We analytically solve the thermal pressurization problem by considering an appropriate heat source for a fault of finite thickness. Our modeling results show that thermal pressurization reduces the temperature increase caused by frictional heating. However, the effect of the slipping zone thickness on temperature changes is stronger than that of thermal pressurization, at least for a constant porosity model. Pore pressure and effective normal stress evolution affects the dynamic propagation of the earthquake rupture producing a shorter breakdown time, larger breakdown stress drop and rupture velocity. The evolution of the state variable in the framework of rate– and state–dependent friction laws is very different when thermal pressurization is of relevance. In this case the evolution of the friction coefficient differs substantially from that inferred from a slip–weakening law. This implies that the traction evolution and the dynamic parameters can be strongly affected by thermal pressurization.

Key words: Earthquake dynamics, pore pressure evolution, frictional heating, thermal
1. Introduction

The role of fluids and pore pressure relaxation on the mechanics of earthquakes and faulting has been largely debated in the literature. Numerous evidences have been discussed to demonstrate that fluid flow and/or pore pressure evolution can affect earthquake ruptures. Hubbert and Rubey (1959) emphasized the importance of fluids on the effective normal stress, in particular when the fault mechanical conditions are moved from a hydrostatic regime ($S_0 \approx 0.4$) to a lythostatic one ($S_0 \approx 1$, where $S_0$ is the dimensionless Sommerfeld number - Sommerfeld, 1950 - that is the ratio between the fluid pressure, $p_{\text{fluid}}$, and the normal stress $\sigma_n$).

Fluids can affect the earthquake nucleation process: high fluid pressure can allow fault reactivation with the inversion of slip direction (such as low angle thrust fault reactivated by normal faulting, see Sibson, 1986; Collettini et al., 2005 and references therein) as well as they can trigger aftershocks controlling the patterns of seismicity both in space and time (Nur and Booker, 1972; Miller et al., 1996, 2004; Yamashita, 1998; Shapiro et al., 2003; Antonioli et al., 2005, among many others). Fluid flow and pore pressure evolution can also affect the dynamic propagation of earthquake ruptures: frictional heating caused by earthquake dislocation can modify the pore pressure and, therefore, the effective normal stress acting on the fault surface. This process, called thermal pressurization, has been proposed in the literature long time ago (Sibson, 1973; Lachenbruch, 1980; Raleigh and Everden, 1981; Mase and Smith, 1985, 1987) and more recently by Andrews (2002). Other authors (Irwin and Barnes, 1975; Rudnicki and Chen, 1988; Byerlee, 1990; Rice, 1992; Lockner and Byerlee, 1995, among several others)
proposed various physical mechanisms that are able to maintain high fluid pressure in fault zones. Despite the numerous papers in the literature, the presence and the role of fluids on dynamic fault weakening is still a matter of debate within the scientific community. More recently Kanamori and Heaton (2000) emphasized again that during large earthquakes the thermal pressurization phenomenon is important. In this work we aim to discuss the role of thermal pressurization on earthquake ruptures by modeling the spontaneous propagation of a 3–D rupture governed by assigned constitutive laws allowing the evolution of effective normal stress ($\sigma_{n}^{eff} = \sigma_{n} - p_{fluid}$).

The increase of temperature caused by dynamic slip episodes during earthquake ruptures is usually named frictional heating (see for instance Fialko, 2004 and references therein). The temperature changes in a poro–elastic medium modify pore pressure, leading to temporal variations of the effective normal stress acting on the fault surface. This is the basic idea we follow in constructing our numerical experiments that we will discuss in this study. Although we limit our study to coseismic processes, it is important to remind here that pore pressure can also change during the interseismic period due to compaction and sealing of fault zones (Blanpied et al., 1992; Sleep and Blanpied, 1992). Sibson (2003) presents an exhaustive discussion on the structure and the thickness of the slipping zone, pointing out the role of frictional heating to induce partial melting of fault gouge. He emphasizes that one meter of slip on a slipping zone few millimeters thick would produce temperature changes larger than 1000 °C under adiabatic conditions (i. e. with no exchange of heat), and therefore cause melting of most crustal rocks (see also Fialko, 2004). Sibson (2003) raises an interesting question related to the apparent scarcity of pseudotachylyte and the common evidence of slip localization in fault zones, suggesting that this might be explained either if friction–melting is a
rare phenomenon or if pseudotachylyte, created at depth, is not preserved in exhumed fault zones. Melting has been also observed in laboratory experiments (Friedman et al., 1974; Teufel and Logan, 1979; Tsutsumi and Shimamoto, 1997). If melts have a very low viscosity they may lubricate faults (Spray, 1993; Fialko, 2004) and cause rapid changes in friction. Several mechanisms have been proposed in the literature to explain this sudden drop of the friction coefficient and the associated dynamic fault weakening; they include melting (Jeffreys, 1942; McKenzie and Brune, 1972; Richards, 1976; Cardwell et al., 1978; Sibson, 1977; Allen, 1979), acoustic fluidization (Melosh, 1979, 1996) and mechanical lubrication (Spray, 1993; Brodsky and Kanamori, 2001; Kanamori and Brodsky, 2001). Melting of gouge materials can also arrest the dynamic rupture by viscous braking, as also observed in laboratory experiments by Tsutsumi and Shimamoto (1997). This may happen when an uninterrupted layer of melt with a relatively high viscosity is formed along the fault surface; in such conditions viscous deformation controls the resistance to slip and not the usual Terzaghi or Mohr–Coulomb law (Fialko, 2004).

Because in our study we use different constitutive laws to model the spontaneous propagation of an earthquake rupture, we have to account for the temporal changes of effective normal stress caused by thermal pressurization in the analytical formulation of the friction law. To this goal we consider the temporal changes of effective normal stress in the analytical expression of fault friction and, in the case of rate– and state–dependent laws (Dieterich, 1978, 1986, 1994; Ruina, 1980, 1983 among many others), also in the evolution of the state variable (Linker and Dieterich, 1992) as we will discuss in the next Sections. In the literature there are many papers describing laboratory experiments involving normal stress variations and related phenomena. Linker and Dieterich (1992) and Dieterich and Linker (1992) show the effects of
imposed step variations in normal stress on fault friction: a sudden change in normal stress produces a direct response on friction and an evolving relaxation to a new steady state, analogously to a sudden change in load point velocity. More recently Prakash (1998) examined the frictional response due to temporally varying imposed normal stress. Richardson and Marone (1999) presented experimental results about effects on frictional healing of normal stress vibrations.

A thermal pressurization model requires to assign the permeability and the porosity of the medium (see Andrews, 2002 and references therein). In most of these applications, porosity is considered to be constant. However, several authors have investigated the porosity evolution: Segall and Rice (1995) propose different analytical relations to describe the porosity change due to frictional dilatancy and ductile compaction, although they did not considered frictional heating. Sleep (1995a, 1995b, 1997, 1999) and Sleep et al. (2000) propose different analytical relations to account for ductile creep, compaction and other conditions that can modify the porosity. A recent work of Andrews (2002) is focused on the effects of fluid pressure changes due to frictional heating in a 3-D fault model governed by a time–weakening friction law, in which the hydraulic diffusivity is assumed to be constant and the porosity obeys to Biot’s theory of saturated porous media.

In spite of the profusion of experimental and theoretical works, there are no numerical simulations that consider a realistic fault model and that include all these physical phenomena. Miller (2002), using the model of Segall and Rice (1995), and therefore neglecting thermal effects, modeled the influence of fluids on earthquake and faulting, assuming the Radiation Damping Approximation (RDA; see for instance Rice, 1993) and a simple constitutive law with two levels of friction. Perfettini et al. (2003), still adopting the RDA, modeled the effects
of externally imposed analytical variations in normal stress and their effects on a fault obeying to the Linker and Dieterich (1992) constitutive equation.

The aim of the present work is to discuss the influence of thermal pressurization on dynamic earthquake ruptures by performing several numerical experiments using different constitutive laws. We will present in a companion paper (Bizzarri and Cocco, 2005b) the effect of thermal pressurization on the dynamic traction evolution. In the present study we focus on the temperature changes caused by thermal pressurization as well as on the effective normal stress changes affecting the evolution of the friction coefficient.

2. Simulation strategy

In this paper we model the dynamic spontaneous propagation of an earthquake rupture on a vertical strike–slip fault in a homogeneous half–space by solving the elasto–dynamic fundamental equation using a 3–D Finite Difference code (second–order in space and time, see Bizzarri and Cocco, 2005a for further details). The solution of this problem requires the adoption of a governing law to model fault friction, which is necessary to have a finite fracture energy absorbed at the crack tip. The implementation of such a Fault Boundary Condition (FBC) is done by using the Traction–at–Split–Nodes (TSN) technique, proposed in 3–D by Day (1977), Archuleta and Day (1980), Day (1982a, 1982b) and Andrews (1999), which allows the calculation of all components of traction and slip at the same fault position.

The shear traction degradation for increasing slip during the dynamic rupture propagation is controlled by the adopted constitutive law. This allows us to define the spatial extension of the cohesive zone as well as the duration of the breakdown process (i. e. the duration of the decrease of the fault friction from the upper yield value down to the kinetic residual level). In
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the following of this study we refer to the breakdown processes as the physical mechanisms controlling the shear stress degradation within the cohesive zone at the crack tip. The FBC on the fault surface is represented by either slip–weakening or rate– and state–dependent friction laws, which will be briefly described in the next Section.

Because in this study we are mostly interested in modeling the dynamic rupture propagation, earthquake nucleation is promoted and modeled either by means of time–weakening or by an appropriate state variable configuration in a predetermined nucleation patch, as explained for the 2–D case by Bizzarri et al. (2001) and for the 3–D one by Bizzarri and Cocco (2005a). Thus, in our modeling strategy the latest stages of rupture nucleation and the dynamic rupture onset are controlled by the adopted constitutive law. We consider the effect of temperature increase caused by frictional heating. Once slip starts to accelerate and slip velocity increases, temperature changes due to frictional heating. The temperature increase modifies the pore fluid pressure, which in turn changes the modulus of the effective normal traction according to the Terzaghi effective stress law (Terzaghi et al., 1996; Wang, 2000): \( \sigma_n^{\text{eff}} = \sigma_n - p_{\text{fluid}} \), where \( \sigma_n \) is the value of the normal stress and \( p_{\text{fluid}} \) is the value of the pore fluid pressure. The modulus \( \tau \) of the shear traction is related to \( \sigma_n^{\text{eff}} \) through the well known relation: \( \tau = \mu \sigma_n^{\text{eff}} \), \( \mu \) being the friction coefficient. Therefore, we aim to model the shear traction evolution by means of the temporal variations of both the friction coefficient and the effective normal stress. In other words, we aim to simulate the effects of thermal pressurization of pore fluids on the earthquake rupture propagation. In this context, the adopted constitutive law controls the traction evolution at low slip rates, while at large slip velocities it is mainly controlled by thermal pressurization caused by frictional heating. This issue will be
discussed in detail in the companion paper (Bizzarri and Cocco, 2005b). Our simulations differ from those presented by Fialko (2004) with a 2-D model, because we account for the spontaneous evolution of the breakdown process in the cohesive zone (governed by the constitutive law). They also differ from those presented by Andrews (2002), since we include different constitutive laws and we consider in the analytical solution for temperature and pore pressure proper elementary solutions and an appropriate heat source for a fault of finite thickness. Moreover, we do not introduce any artificial fading memory in the value of the effective normal stress: in this study we implicitly assume that changes in normal stress produce variations in shear strength on the same time scale. The use of a sudden time evolution of $\sigma_n^{\text{eff}}$ is also consistent with the experimental results of Prakash (1998), who showed that the delayed temporal response of shear stress due to a change in normal stress is less than few microseconds.

We consider a fault model consisting of a narrow zone having a prescribed thickness ($2w$) where the slip is localized, and we refer to this as the slipping zone (see Figure 1). The shear stress, slip and slip velocity used in the constitutive formulation should be considered as macroscopic averages of complex processes occurring within the slipping zone (asperity fractures, gouge formation and evolution, etc...). In other words, we should regard as these physical quantities as macroscopic variables (Ohnaka, 2003). Frictional heating is generated within the slipping zone and the rate of frictional heat generation depends on the macroscopic frictional slip rate, total dynamic traction and it is inversely proportional to the thickness ($2w$) of the slipping zone (see Fialko, 2004, and references therein). If the thickness of the slipping zone (i.e., the gouge layer) is constant along the rupture plane and the shear strain rate ($\dot{e}$) is uniform across the slipping zone, then slip rate ($v$) and strain rate scale through the thickness.
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2\(w\) (\(v = 2w \dot{\varepsilon}\), see Sleep et al., 2000). In this case we can consider the macroscopic slip rate as well as the dynamic traction and the slip acting on both walls of the slipping zone as the main physical quantities characterizing the constitutive laws. In order to perform our numerical calculations, in this study we compute the temperature, friction and effective normal stress changes on a specified fault plane and we implicitly assume that they are representative of the macroscopic behavior of the whole slipping zone. The effective normal stress is altered by the thermally induced pore pressure changes occurring perpendicularly to the fault plane as we will discuss in detail in the following Sections.

In reality, it is common to consider that the slipping zone is embedded within a highly fractured damage zone (see Figure 1a and Chester et al., 1993; Sibson, 2003) characterized by a higher permeability than the fault core. However, in this study we use a simpler configuration than that shown in Figure 1a and we represent the damage zone as a homogeneous and isotropic medium. Because we aim to solve a thermal pressurization problem, we have to assign several parameters in the 3–D medium, such as thermal diffusivity, porosity and permeability. In this work we take these values constant throughout the medium (see Figure 1b). The modeling of spatial and temporal variations of permeability is beyond the goals of the present study. This also because there exist serious difficulties in constraining the temporal evolution of permeability within the fault and the damage zones caused by shear dislocations. This topic will be considered in future research. In the next Sections we will describe the analytical solution of the problem and we discuss the constitutive laws governing fault friction.

3. Modeling the breakdown processes

As mentioned above, in this study we use both slip–weakening (SW hereinafter) and rate–
and state–dependent constitutive relations (RS hereinafter). The former is represented by the linear relation between total traction and fault slip (Ida, 1972; Andrews, 1976a, 1976b):

\[ \tau = \begin{cases} 
\tau_u - (\tau_u - \tau_f) \frac{u}{d_0}, & u < d_0 \\
\tau_f, & u \geq d_0 
\end{cases} \tag{1} \]

where \( u \) is the fault slip, \( \tau_u \) is the upper yield stress (i.e. the maximum value that the rock can endure), \( \tau_f \) the kinetic (or residual) frictional level and \( d_0 \) is the characteristic SW distance. The two frictional parameters appearing in (1) can be written as: \( \tau_u = \mu_s \sigma_{n_{\text{eff}}} \) and \( \tau_f = \mu_d \sigma_{n_{\text{eff}}} \) (where \( \mu_s \) and \( \mu_d \) are the static and dynamic coefficient of friction, respectively), and therefore they explicitly depend on the pore pressure value.

Rate– and state–dependent constitutive relations have been used to model the spontaneous dynamic propagation either in 2–D (Okubo, 1989; Bizzarri and Cocco, 2003 and references therein) and 3–D (see Bizzarri and Cocco, 2005a). These constitutive laws have been derived from the results of laboratory experiments (Dieterich, 1978; Ruina, 1980, 1983; Beeler et al., 1994; Roy and Marone, 1996; Marone, 1998, among many others) performed at low slip rates (< 1 mm/s) and constant normal stress. The analytical formulation of these constitutive laws consists of two equations: a governing equation, which relates total traction to slip velocity and the state variable, and an evolution equation for the state variable. It is well known, however, that the assumption of a constant normal stress is not always appropriate to model real faulting episodes. Previous investigations (Hobbs and Brady, 1985; Lockner et al., 1986; Olsson, 1988) showed that the frictional resistance to sliding raises suddenly in response to an abrupt increase in normal stress and then continues to rise toward a new steady state value.
In the present study, we aim to couple the temporal variations of friction coefficient and effective normal stress. To this goal, we need to use a constitutive law for fault friction that takes account of the normal stress variations. Linker and Dieterich (1992) performed quasi–static laboratory experiments with blocks of Westerly granite in a double–direct shear apparatus and interpreted the data by introducing the following generalization of the evolution law:

\[
\frac{d\Psi}{du} = \left(\frac{\partial \Psi}{\partial u}\right)_{\sigma_{\text{eff}}} \cdot u - \left(\frac{\alpha_{LD} \Psi}{b \sigma_{\text{eff}}^{\alpha}}\right) d\sigma_{\text{eff}}
\]

(2)

where \(\Psi\) is the state variable, the dimensionless coupling parameter \(\alpha_{LD}\) is estimated from experiments, \(b\) is the constitutive parameter (defined below) and the first derivative is computed taking the effective normal stress constant. This model incorporates the changes in normal stress in the evolution of the state variable and these sudden changes in \(\Psi\) are inversely proportional to the sudden change in \(\sigma_{\text{eff}}\). In (2) it has been assumed that, for small changes in normal stress, the steady state coefficient of friction is independent of normal stress and that the steady–state value of state variable \((\Psi^{\text{ss}})\) is consistent with the earlier models. According to Linker–Dieterich (1992, LD92 hereinafter) the constitutive relation for the ageing model becomes:

\[
\tau = \left[\mu_{\ast} + a \ln \left(\frac{\nu}{u_{\ast}}\right) + b \ln \left(\frac{\Psi' \nu}{L}\right)\right] \sigma_{\text{eff}}^{\alpha}
\]

\[
\frac{d\Psi}{dt} = \frac{\Psi' \nu}{L} - \left(\frac{\alpha_{LD} \Psi}{b \sigma_{\text{eff}}^{\alpha}}\right) \frac{d\sigma_{\text{eff}}}{dt}
\]

(3)

where \(a\) and \(b\) and \(L\) are the constitutive parameters, which we consider here being independent.
of the normal stress and temperature. In other words, we assume that these parameters are constant during the short durations of coseismic processes studied in this paper. $\mu_*$ and $v_*$ are reference values for the friction coefficient and for the slip velocity $v$, respectively. Although equation (3) includes normal stress changes, we will refer in the following of the paper to this law as the Dieterich–Ruina law (DR hereinafter).

In this study (results are shown in the companion paper, Bizzarri and Cocco, 2005b) we also use the slip law (see Beeler et al., 1994), which differs from (3) only for the state variable evolution law:

$$
\tau = \left[ \mu_* + a \ln \left( \frac{v}{v_*} \right) + b \ln \left( \frac{\Psi v_*}{L} \right) \right] \sigma_n^{\text{eff}}
$$

$$
\frac{d}{dt} \Psi = -\frac{\Psi v}{L} \ln \left( \frac{\Psi v}{L} \right) - \left( \frac{\alpha_{LD} \Psi}{b \sigma_n^{\text{eff}}} \right) \frac{d}{dt} \sigma_n^{\text{eff}}
$$

In the following we will refer to this relation as the Ruina–Dieterich (RD) law. The state variable $\Psi$ included in (3) and (4) should be also considered as a macroscopic variable, similarly to shear stress and slip. According to Dieterich (1978) and Linker and Dieterich (1992) the state variable depends on the slip rate, that we consider here a macroscopic frictional slip velocity. Although the state variable has been originally interpreted to represent the state of the sliding surface, in this study we should intend $\Psi$ to be representative of the mechanical state of the whole slipping zone (see also Sleep, 1997).

We emphasize here that the temporal variation of the effective normal stress, included both in (3) and (4), can be due to: (i) an imposed load condition, as in LD92 and Dieterich and Linker (1992); (ii) a previous seismic event that modifies the state of the stress along the actual fault (see, for instance, Perfettini et al., 1999; 2003; Antonioli et al., 2005); (iii) a bi–material
fault in which the two halves of the fault plane have different elastic properties (Andrews and Ben–Zion, 1997; Harris and Day, 1997) or finally (iv) a consequence of pore fluid pressure variation due thermal pressurization, which is controlled by different physical mechanisms. In the following of the paper we will focus on the last phenomenon.

4. The thermal pressurization model

4.1 Frictional heating

We compute the temperature changes caused by frictional heating within a slipping zone whose thickness is $2\nu$. To this goal, we solve the 1–D Fourier’s equation of heat conduction for a thermally isotropic medium, when the state changes (Stefan, 1981) are not considered:

$$\frac{\partial}{\partial t} T = \chi \frac{\partial^2}{\partial \zeta^2} T + \frac{1}{c} q$$

where $\zeta$ is the spatial coordinate normal to the fault (see Figure 1b), $\chi$ is the thermal diffusivity ($\chi = \kappa/\rho_{\text{bulk}} C_{\text{bulk}}$, where $\kappa$ is the thermal conductivity, $\rho_{\text{bulk}}$ is the cubic mass density of the bulk composite and $C_{\text{bulk}}$ is the specific heat of the bulk composite at constant pressure), $c \equiv \rho_{\text{bulk}} C_{\text{bulk}}$ is the heat capacity for unit volume of the bulk composite and $q$ is the heat generated for unit volume and for unit time ($[q] = J/(m^3 \cdot s) = W/m^3$). Fialko (2004) solved the same equation for his 2–D calculation of heat flow caused by frictional sliding. The solution of equation (5) is given in Appendix A. We report here the solution for the temperature evolution on the fault plane ($\zeta = 0$):
where \( T^f_0 \) is the initial temperature distribution on the fault plane (i.e. \( T^f_0 \equiv T(\xi_1,0,\xi_3,0) \)), erf(.) is the error function and \( \varepsilon \) is an arbitrarily small positive number. The temperature change on the fault plane is therefore given by \( \Delta T = T^f_w - T^f_0 \).

We have simulated the dynamic propagation of an earthquake rupture obeying to a SW law accounting for the temperature changes due to frictional heating by using the solution shown in (6) and derived in Appendix A. We show the results of these calculations in Figure 2. This Figure illustrates the spatial pattern of slip velocity and the temperature change time histories in a target fault point for different fault strength values. The temperature changes with time are plotted for several values of the slip zone thickness \( 2w \). For the same slip zone thickness, the temperature change associated with the low fault strength \( (S = (\tau_u - \tau_0)/(\tau_0 - \tau_f) = 0.8; \) Das and Aki, 1977a, 1977b) are higher than those generated by the high fault strength \( (S = 1.5) \). This happens because slip velocity is higher for the low strength than for the high strength fault. As expected decreasing the slip zone thickness \( 2w \) increases the temperature on the fault plane. These simulations point out that thermal pressurization affects the dynamic rupture propagation. We will discuss this issue later in this paper.

### 4.2 Thermal pressurization of pore fluids

In this Section we will compute the pore pressure evolution by solving constitutive
equations for fluid flow. The formulation of this problem and the solution of the constitutive equations are described in detail in Appendix B. Pore pressure changes are associated with the temperature variations caused by frictional heating, the porosity evolution and the fluid transport through the equation:

\[
\frac{\partial}{\partial t} p_{\text{fluid}}(x, t) - \frac{1}{\beta_{\text{fluid}} \phi} \frac{\partial}{\partial t} \phi + \frac{\partial^2}{\partial z^2} p_{\text{fluid}}(x, t) \quad (7)
\]

where \( \omega \) is the hydraulic diffusivity, which is defined as:

\[
\omega = \frac{k}{\eta_{\text{fluid}} \beta_{\text{fluid}} \phi} \quad (8)
\]

where \( k \) is the permeability of the medium, \( \eta_{\text{fluid}} \) is the dynamic fluid viscosity, \( \phi \) is the porosity, \( \beta_{\text{fluid}} \) is the coefficient of the compressibility of the fluid and \( \alpha_{\text{fluid}} \) is the coefficient of thermal expansion of the fluid. As shown in Appendix B, for a constant porosity \( \Phi(\xi_1, \xi_2, \xi_3, t) = \Phi_0(\xi_1, \xi_2, \xi_3) \), the solution for pore pressure evolution in a generic fault point \( (\xi_1, \xi_2, \xi_3) \) and at a time \( t \) is obtained by solving equations (5) and (7); this gives:

\[
p_{\text{fluid}}(\xi_1, \xi_2, \xi_3, t) = p_{\text{fluid}}(x, t) + \frac{\gamma}{2 \omega(\xi_1, \xi_2, \xi_3)} \int_0^t \frac{1}{\omega - \chi} \left\{ \frac{w(\xi_1, \xi_2, \xi_3)}{2 \sqrt{\omega(t-t')}} - \right. \left. \frac{w(\xi_1, \xi_2, \xi_3)}{2 \sqrt{\omega(t-t')}} \right\} dt' \quad (9)
\]

where \( p_{\text{fluid}}(x, t) \) is the initial fluid pressure distribution (i.e. \( p_{\text{fluid}}(x, 0) \equiv p_{\text{fluid}}(\xi_1, 0, \xi_2, 0) \)) and \( \gamma \equiv \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \). The effective normal stress can be expressed as:
\[ \sigma_n^{\text{eff}} = \sigma_n - p_{\text{fluid}}^{w_f} = \sigma_n - p_{\text{fluid}0}^f - \Delta p_{\text{fluid}}^{w_f}, \]  

where \( \Delta p_{\text{fluid}}^{w_f} \) is the second term in the right member of equation (9). In this work both normal stress and pore fluid pressure are assumed to be negative for compression: this means that, if \( \hat{n} \) is the unit vector normal to the fault surface (see Figure 1b), thus the normal traction acting on the fault is \( \Sigma = -\sigma_n^{\text{eff}} \hat{n} \). In the following of the paper we will consider as “dry” a fault in which there are no pore fluid pressure changes and therefore \( \Delta p_{\text{fluid}}^{w_f} = 0 \). In this case the effective normal stress is constant and equal to its initial value \( \sigma_n - p_{\text{fluid}0}^f \). On the contrary, when fluid flow is allowed \( (\Delta p_{\text{fluid}}^{w_f} \neq 0) \) outside the fault plane, we will refer to a “wet” fault: in this case \( \sigma_n^{\text{eff}} \) is evolving through time.

We will present in the following Sections the results of several numerical experiments performed with different governing laws and for various configurations. In all these simulations the porosity is taken constant and we use the solutions described above in our numerical algorithm. We have also considered in this study a case in which the porosity evolves with time and we present the analytical solutions for this case study in Appendix C. The results of several numerical simulations with variable porosity will be presented and discussed in the companion paper (Bizzarri and Cocco, 2005b).

5. Temperature changes caused by frictional heating

We consider now a dry fault governed by the linear SW law (equation (1)); in these simulations and in the others presented and discussed in this paper we assume a dimensional
set of parameter typical of a real-world fault: models and constitutive parameters are listed in Table 1. We plot in Figure 3 the temperature change history in a fault point, considering three different slipping zone thicknesses \( w = 35 \text{ mm}, 1 \text{ mm} \) and \( 10 \mu \text{m} \). For each \( w \) solid curves refer to a crack-like models with no healing (i.e. infinite slip duration), since the rupture does not arrest but propagates indefinitely. On the contrary, dashed curves refer to numerical experiments with heterogeneous faults, for which the slip duration is finite and therefore we have healing of slip. We observe that, before the rupture onset, temperature is equal to its initial value \( T_0^f \), as expected from equation (6) for zero slip velocity; after the crack onset the temperature increases, depending on the value of the slipping zone thickness. Figure 3 shows that the initial sudden increase of temperature is the same with or without healing, because it is driven by the slip velocity evolution. After the initial change, temperature further increases before to decrease when slip starts healing. Moreover temperature changes are relevant only for thin slipping zones: for a slipping zone thickness larger than 0.01 m the temperature changes are negligible. In the heterogeneous cases of Figure 3, we indicate the time step at which the maximum fault slip is reached and we report the associated temperature change value. For the homogeneous configuration (solid curves in Figure 3) we indicate the time step at which the fault has developed the same cumulative slip and the associated temperature change value. We note that in general the temperature changes are larger and faster for the crack-like solutions: for the same slip value, temperature change is larger than that inferred for the finite slip duration models (i.e. with healing of slip).

We show in Figure 4 the spatial pattern of temperature changes (still with respect to the initial value \( T_0^f \)) on the fault plane computed for a crack model obeying to a SW friction law. In this set of simulations we also account for the thermal pressurization of pore fluids. In Figure
4a we plot the temperature evolution at a fixed time step as function of the spatial coordinate along the strike at the hypocentral depth: full symbols refer to simulations with thermal pressurization (the effective normal stress $\sigma_{n}^{\text{eff}}$ changes with time), while empty symbols refer to dry configurations (normal stress is constant and equal to its initial value, $\sigma_{n} - p_{\text{fluid}} = 30$ MPa). We observe that thermal pressurization increases the effects of frictional heating: this is due to the effective normal stress reduction caused by pore pressure increase. We also display in Figure 4b the temperature changes on the fault plane for the simulation with 2 mm thickness ($w = 1$ mm) and thermal pressurization: this snapshot is taken at 0.38 seconds from the rupture nucleation. The estimates of temperature changes obtained in our 3–D model for dry configurations are in agreement with the values reported by Fialko (2004).

In order to further discuss the effect of thermal pressurization, we show in Figure 5 the comparison between the temporal evolutions for dry and wet faults in the same target fault point for the same three values of slipping zone thickness used in Figure 3. We compare dry and wet faults (black and gray curves, respectively), as well as crack–like and healing–like cases (solid and dashed curves, respectively). The solutions for the wet case, which includes the effects of thermal pressurization, display a trend very similar to those inferred for dry conditions, also shown in Figure 3, for both finite or infinite slip durations (healing and crack–like). The temperature changes for wet faults begin earlier than those for dry faults, because thermal pressurization of fluids makes the fault more unstable, as we will discuss in the next Section. For this reason, the temperature change can be faster at the very early stage for wet faults than for dry ones. However, the general trend is that thermal pressurization reduces the temperature increase on the fault caused by frictional heating.
A very interesting outcome arises from Figure 5: the effect of the thickness of the slipping zone is much more important than the contribution of thermal pressurization. For a thick fault zone ($w = 35$ mm, panel a) the temperature changes ($\Delta T$) are of the order of $30$ °C (30% of the initial temperature value) for both dry and wet faults and independently of slip duration. If the thickness of the slipping zone is smaller ($w = 1$ mm, panel b), the temperature for a dry fault with finite slip duration increases nearly up to $1000$ °C; when accounting for thermal pressurization the temperature is still close to $850$ °C. In both cases the temperature changes are larger than eight times the initial temperature value. A further decrease in thickness of the slipping zone ($w = 10 \mu$m, panel c) further rises the temperature nearly up to $2000$ °C and $1600$ °C for dry and wet faults, respectively. These temperatures might produce melting of fault gouge; Di Toro and Pennacchioni (2004) suggest that rock–composite melts at temperatures between $1100$ °C and $1550$ °C. Otsuki et al. (2003) estimated melting temperature for the Nojima fault ranging between $750$ °C and $1280$ °C. According to our calculations to have gouge melting we need $w = 1$ mm. In these simulations we have used a value of hydraulic diffusivity equal to $0.02$ m$^2$/s and the maximum slip is slightly less than a meter.

It is important to note that the difference between the temperature changes computed accounting for thermal pressurization (wet faults) and those in adiabatic conditions (see Kanamori and Heaton, 2000; Sibson, 2003) depends on the thickness of the slipping zone and on the value of cumulative slip. This is evident from Figure 6a, which illustrates the temperature changes as a function of slipping zone half thickness ($w$) and compares the results of our numerical simulations (solid curves) with the temperature changes computed for uniform adiabatic shearing (dashed curves). In the latter case the temperature change is expected to increase linearly with the cumulative fault slip $u$ and decrease inversely with $2w$.
according to the relation: \( \Delta T^{(\text{adiab})} \equiv \tau_f u/2cw \). Moreover, Figure 6a shows that: (i) the temperature changes obtained using our solution are in agreement with the inverse dependence on \( 2w \); (ii) \( \Delta T^{(\text{adiab})} \) are smaller than \( \Delta T \) resulting from thermal pressurization, and this difference depends on the cumulative slip value: the larger the slip the smaller the difference. (iii) only for relatively small values of the slipping zone thickness (\( w \leq 1 \) mm) thermal pressurization reduces the temperature changes. These results can be explained if we consider that for small slip values thermal pressurization generates larger stress drop and larger slip velocity yielding higher temperature changes than the adiabatic predictions. Figure 6b reveals that the temperature changes do not vary with the adopted values of hydraulic diffusivity. We will discuss the implications of these results for melting of fault gouge materials in the discussion Section.

6. Rupture propagation with thermal pressurization

In the previous Section we have shown the results of several numerical simulations performed using a SW constitutive law in which the traction evolution is prescribed within the cohesive zone. In this Section we aim to discuss the effects of thermal pressurization of pore fluids caused by frictional heating using different constitutive laws, as the RS constitutive formulation. The assumed constitutive law controls the initial stages of dynamic fault weakening and slip acceleration; once slip velocity increases, temperature changes (as discussed in the previous Section) and produces an increase of pore pressure that modifies the effective normal stress. Figure 7 shows the pore pressure evolution for different fault zone thickness and hydraulic diffusivity values obtained by modeling thermal pressurization in fault
models governed either by SW (upper panels) and DR (bottom panels) constitutive laws. The adopted parameters are listed in Table 1; we emphasize that we have chosen a characteristic length scale parameter $L$ for RS that is one order of magnitude smaller than $d_0$ in SW: this agrees with the scaling between the equivalent SW distance $d_0^{eq}$ and $L$ proposed by Cocco and Bizzarri (2002) and Bizzarri and Cocco (2003). This Figure illustrates that decreasing the slipping zone thickness as well as the hydraulic diffusivity value pore pressure increases with time, causing a temporal reduction of the effective normal stress. The shape of the pore pressure temporal evolution depends on the adopted friction law. RS laws always produce a nearly constant pore pressure evolution after the end of the breakdown process (when the traction is at the kinetic stress level). Looking at the pressure increase, we can observe that, for the same values of $w$ and $\omega$, SW produces a more abrupt increase at the rupture onset but a lower final level with respect to DR.

In Figure 8 we compare the rupture evolution obeying to SW law with and without thermal pressurization effects. As in the previous numerical experiments, we assume that the porosity $\Phi$ does not change in time and its value equals the initial one. Panels (a) and (b) refer to a reference configuration in which fluid effects are ignored (dry case) and therefore the effective normal stress is constant over time. On the contrary, in panels (c) and (d) we plot the solutions for a fault in which thermal pressurization causes time variations of the effective normal stress (wet case). As we will discuss in more detail in the following and in the companion paper (Bizzarri and Cocco, 2005b), the wet fault is more unstable than the dry fault: at the same time step the wet fault has ruptured a broader region than the dry one. We emphasize that the inclusion of fluid pressurization causes more abrupt stress drop: we can observe from Figures 8b and 8d that the weakening phase is shorter for the wet fault and the weakening rate is higher.
than that inferred for the dry fault. This implies that the temperature change is faster. Figure 8 points out that thermal pressurization modifies the dynamic propagation of the earthquake rupture: for the same strength value (given by the value of the $S$ parameter) a wet fault displays higher rupture velocities than the dry one and the rupture front acceleration to super–shear speeds along the direction of the imposed pre–stress (Figure 8c).

In Figure 9 we compare the solutions obtained adopting the Dieterich–Ruina law for dry and wet faults; in the former case the governing model is the original ageing law (because normal stress is constant), while the latter one is the modified DR law (defined in equation (3)), which includes the LD92 evolution law. This Figure shows that, also in the framework of RS laws, thermal pressurization produces higher rupture velocity and slip rates, although the rupture front does not accelerate to a super–shear speed as shown in Figure 8. In both cases the peak slip velocity does not corresponds to the equivalent kinetic frictional level, but it is reached before, especially in the dry case, confirming the findings of Tinti et al. (2004).

The temporal evolution dynamic traction and slip velocity illustrated in Figure 9 reveals that thermal pressurization produces a shorter duration of the breakdown process (of about 60%), similarly to what observed for a SW law. The evolution of the state variable when thermal pressurization affects the effective normal stress is very different. This is evident in Figure 10, where we plot the state variable time evolution for both the dry and wet cases; for the dry fault (black curves) the state variable starts from its initial configuration (assumed to be the steady state) and then smoothly decreases reaching a final state, that is still a steady state (in agreement with previous 2–D numerical simulations (Bizzarri and Cocco, 2003) and with laboratory observations (see for instance Wang and Scholz, 1983)). Conversely, for the wet fault (gray curves) the fluid pressure changes causes an initial abrupt increase of the state
variable during the breakdown time and then a fast decrease to the steady state. We will discuss in the companion paper how this different evolution of the state variable affect the dynamic traction evolution.

In Figure 11 we show the results of several simulations performed by using either a SW law (left panels) or a DR friction law (right panels); solid squares and open circles identify dry and wet fault conditions, respectively. We plot in this Figure the friction coefficient \( \tau / \sigma_n^{\text{eff}} \) as a function of time (panels (a) and (b)), of the fault slip (panels (c) and (d)) and, only for the wet cases, of the ratio between pore fluid pressure and nominal normal stress (i.e. Stribeck curve, panels (e) and (f)). All parameters are those listed in Table 1, with the exception of slipping zone thickness that has been chosen equal to 2 mm \( (w = 1 \text{ mm}) \). The solutions are calculated at the same target fault point used in previous Figures. This Figure further confirms that the breakdown time for the wet simulations is shorter than that inferred for the dry ones and this effect is more pronounced for RS laws (see panels (a) and (b)). Moreover, while for the SW solutions the drop of the friction coefficient does not change for the dry and wet faults, for the DR solutions it is larger for a wet fault. This is easily explained considering that only in the RS formulation the friction coefficient evolves in time and it can be also affected by normal stress variations through the state variable evolution (see equation (2)). On the contrary, in the SW case the variation of the effective normal stress caused by thermal pressurization changes the value of the frictional stress level \( \tau_f = \mu_d \sigma_n^{\text{eff}} \), but not the value of the coefficient of friction \( \mu_d \), which in this case is an assigned value. This is a fundamental difference between SW and RS friction laws. Furthermore, panels (c) and (d) in Figure 11 point out that, while the characteristic slip–weakening distance is fixed \textit{a priori} when the SW law is adopted (and in this case it does not change in dry and wet numerical experiments), it changes dramatically
between dry and wet faults for the RS formulation. We have obtained an equivalent SW distance $d_0^{eq}$ of about 10 cm in the dry case and about 50 cm in the wet one. This larger value does not follow the scaling law between $d_0^{eq}$ and $L$ proposed in dry conditions by Bizzarri and Cocco (2003), because in that work they did not account for the effects of thermal pressurization caused by frictional heating. We will discuss this issue more in detail in the companion paper (Bizzarri and Cocco, 2005b).

We have drawn in panels (e) and (f) of Figure 11 the Stribeck curve (Spikes, 1997), which represents the friction coefficient $\mu$ as a function of the Sommerfeld number (Sommerfeld, 1950). The Sommerfeld number is given by the ratio between the pore pressure and the normal stress, that is: $So \equiv \frac{p_{fluid}^{w}/\sigma_n}{(p_{fluid}^{f} + \Delta p_{fluid}^{w})/\sigma_n}$. (If normal stress $\sigma_n$ is equal to the lithostatic (overburden) pressure, $So$ is often called pore fluid pressure factor and it is indicated with the symbol $\lambda$). We observe that $So < 1$ for both the constitutive formulations; this means that pore fluid pressure is always smaller than normal stress (then $\sigma_n^{eff} > 0$). In this case we are in the boundary lubrication regime and we do not enter in the hydrodynamic lubrication range (Brodsky and Kanamori, 2001). This plot also reveal that the two constitutive formulations yield a completely different evolution of the friction coefficient. In fact, while for the SW the friction coefficient always decreases after the initial increase due to the loading from other points on the fault (see panel (e)), in the DR case the Stribeck curve shows a nearly linear increase of friction coefficient with the Sommerfeld number up to its peak value, which is followed by a fast drop. However, RS yields a larger pore pressure variation and therefore the values of the Sommerfeld number are greater than those obtained adopting the SW governing law. In other words, the effective normal stress is smaller in the simulations performed using
the RS formulation than those which use the SW law. We will discuss in detail in the
companion paper (Bizzarri and Cocco, 2005b) how the variation of the fluid pressure affects
the behavior of the dynamic traction evolution within the breakdown zone.

7. Discussion and conclusive remarks

In this paper we have analytically solved the 1–D thermal pressurization problem and
applied it to model the fully dynamic, spontaneous propagation of a 3–D earthquake rupture on
a planar strike–slip fault. The frictional heating is represented by the temperature increase
caused coseismic slip episodes on the fault plane; this generates a fluid migration in the
direction normal to the fault plane, which we model by coupling the thermal conduction
equation and the Darcy’s law for fluid flow. We consider only coseismic frictional heating (see
equation (A.2)), but in full of generality the heat sources can be also modified to represent the
heat generated during time–dependent compaction or ductile processes (Beeler and Tullis,
1997). This in turn causes modifications of the effective normal stress affecting fault friction.
We have extended to a 3–D fault framework the frictional heating model proposed by Cardwell
et al. (1978) and recently applied to a non–spontaneous 2–D crack problem by Fialko (2004).
Our analytical solution of the thermal pressurization problem differ from that proposed by
Andrews (2002), because we include a heat source appropriate for a fault of finite width and
different constitutive relations: a time–weakening or a linear slip–weakening laws, as well as
rate– and state–dependent friction laws with temporally varying effective normal stress.

We have demonstrated that thermal pressurization can play a very important role in the
dynamic rupture propagation. Frictional heating and consequent pore fluid pressure changes
are able to alter the traction evolution during the breakdown process, by modifying (with
respect to a dry fault) the shape of the crack tip and the extension of the fractured zone at the same time step, complicating the time evolution of the friction coefficient. We will further investigate some of these issues in the companion paper (Bizzarri and Cocco, 2005b). We point out here that pore fluid pressure changes driven by coseismic frictional heating increases the breakdown stress drop and reduces fault friction.

Our simulations suggest that the effect of the adopted fault zone thickness is more important than thermal pressurization in controlling the temperature changes caused by dynamic sliding episodes. Fialko (2004) have shown that the effective thickness of the slipping zone controls the temperature changes and the place where this perturbation occurs on the fault plane. This author concluded that thermal effects are expected to be significant for fault with highly localized slipping zones: if the slipping zone thickness is of the order of 1 mm, the temperature changes can be even larger than 1000 °C, thus causing melting of fault gouge. Thermal pressurization has been proposed in the literature as a viable mechanism to reduce the temperature increase and to prevent melting (Sibson, 2003; Fialko, 2004). The results of our study confirm that thermal pressurization reduces the temperature rise. However, if the thickness of the slipping zone is extremely thin, the increase of temperature is still large also when thermal pressurization is taken into account: for half meter of slip in 1 mm of slipping zone thickness, the temperature change is still of the order of 800 °C. In these simulations the thickness of the thermally boundary layer is of the order of 6 mm (corresponding to a thermal diffusivity of $10^{-6}$ m$^2$/s and 10 s of slip duration). It is important to remark here that this might be due to the fact that we consider constant permeability and porosity values, which do not change within the damage zone. We will show in the companion paper the results of several
simulations performed considering the temporal evolution of porosity, as described in Appendix C of this paper.

Our numerical simulations show that thermal pressurization can reduce the fault resistance resulting in a substantial dynamic fault weakening. Moreover, temperature changes of the order of 800 °C can be still sufficient to generate melting of fault gouge materials. Two distinct thermally activated mechanisms can produce a drop in the friction coefficient and melting: thermal pressurization and flash heating of fault–contact asperities (Rice, 1999). We emphasize that these two mechanisms are substantially different: in fact, melting caused by thermal pressurization should be considered as a macroscopic phenomenon involving the whole effective slipping zone thickness. On the contrary, flash heating is due to the highly localized heating created by a fast slip episode in the region near the moving asperity contacts on the slipping surface; in other words it is generated on a relatively small contact area that sustains high shear strength. Under these conditions, the sudden and local increase of temperature causes a diminution of the contact’s shear strength and a friction drop with slip rate. This might explain the experimental results of Di Toro et al. (2004). Another difference between thermal pressurization and flash heating is that while the former strongly depends on the effective normal stress changes, the latter does not strongly change with normal stress. This because in the latter case the affected zone is very small and extremely thin, thus the capacity to support normal stress and the net area of contact might not be affected (Rice and Cocco, 2004).

The numerical experiments presented in this study show that the hydraulic diffusivity strongly affects the traction evolution and pore pressure changes, in agreement with the results shown by Andrews (2002). This is expected because the pore pressure increase depends on the fluid flow outside the fault plane. Our results show that, if the hydraulic diffusivity value is
similar or slightly larger than thermal diffusivity, the pore pressure rise causes a relevant reduction of the effective normal stress (see the simulations shown in Figure 7). This would produce large rupture velocities, which might become super–shear, and a nearly complete stress drop, as we will discuss in detail in the companion paper (Bizzarri and Cocco, 2005b). The pore fluid pressure changes as well as their effects on the friction evolution strongly depend on the assumed constitutive law. In the framework of rate– and state–dependent friction laws, the state variable evolution is strongly affected by thermal pressurization, showing a preliminary increase followed by a sudden drop to the new steady state value. Thermal pressurization produces a faster and larger drop of the state variable affecting the evolution of the friction coefficient. We emphasize here that the use of a rate– and state–dependent constitutive law has the advantage to include the effects of effective normal stress variations in the temporal evolution of the friction coefficient, which is not allowed if a slip–weakening law is adopted. However, further investigations are needed to understand the best approach to include temporal changes of effective normal stress in the state variable evolution. We will further investigate the effects of thermal pressurization on the main physical parameters controlling the traction evolution and the dynamic rupture propagation (breakdown stress drop, critical slip–weakening distance and fracture energy) in the companion paper. We conclude by pointing out that the analytical solution of the frictional heating problem implies the assumption that all the work spent in allowing the crack advance and fault sliding is heat. This raises a further question on the meaning of fracture energy when thermal pressurization controls the dynamic rupture propagation. We leave this issue for a more detailed discussion in the companion paper and for future research.
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Appendix A. Solution of the thermal conduction problem

In this Appendix we will derive the solution of the 1-D thermal conduction problem defined by equation (5). We denote with \( w \) the fault zone half width or, analogously, the slip zone half thickness where the faulting process takes place (see Figure 1). The quantity \( 2w \) can also be interpreted as the slipping zone thickness and can be associated to the quantity \( 2D \) in Andrews (2002). We also indicate with \( \xi_1 \) and \( \xi_3 \) the spatial coordinates on the fault plane and with \( \zeta \) (namely \( \xi_2 \)) the coordinate normal to the fault.

For an elementary heat source (i.e. point–source in space and impulsive in time) of intensity \( h (q^e (\zeta, t)) \equiv h \delta (\zeta) \delta (t) \), where \([h] = J/m^2\) and \([q^e (\zeta, t)] = J/(m^3 s) = W/m^3 = Pa/s\) the solution of equation (5) is (see Carslaw and Jaeger, 1959):

\[
T^e (\zeta, t) = \frac{h}{2c \sqrt{\pi \chi} t} e^{-\frac{\zeta^2}{4 \chi t}} \tag{A.1}
\]

The heat source (or the rate of frictional heat generation within the slipping zone) in a generic point of a fault \((\xi_1, \xi_3)\) is expressed as the product of the shear stress \( \tau \) and the shear strain rate. Accordingly to Cardwell et al. (1978) and Fialko (2004), we assume that the shear strain rate is constant within \( 2w \). Mair and Marone (2000) have shown with laboratory experiments that this hypothesis might be adequate. Therefore, the shear strain rate becomes the ratio of the total slip velocity \( v \) over the thickness of the slipping zone \( 2w \). More complicated models in which the slip velocity profile may be non–linear across the slipping zone can be considered, but they are not corroborated by the actual state of knowledge of the behavior of the slip zone. Therefore the actual heat source \( q ([q] = \{q^e (\zeta, t)\}) \) is:
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In this relation it is implicitly assumed that all the work spent to allow the fault sliding is converted into heat (Cardwell et al., 1978; Scholz, 2002; Fialko, 2004; Di Toro et al., 2005). This assumption maximizes the effect of the heat flux during the dynamic instability. Because we do not know how total work is partitioned between frictional heating and surface energy (see also Tinti et al., 2005), we cannot consider in our study alternative heat source models.

The integral of \(q(\xi_1, \xi, \xi_3, t)\) over the normal coordinate \(\xi\) gives the heat flux, i.e. the heat produced per unit area on the fault and per unit time. In (A.2) we explicitly state that the slipping zone may depend on the local coordinates and therefore it can change spatially; in our numerical experiments, however, we assume for simplicity that \(w\) is constant in space.

The solution of equation (5) for this heat source is therefore determined by the Green’s kernel corresponding to (A.1). This leads to (e.g. Morse and Feshbach, 1953; Cardwell et al., 1978; Fialko, 2004):

\[
T^w(\xi_1, \xi, \xi_3, t) = T(\xi_1, \xi, \xi_3, 0) + \int_0^t \int_{-\infty}^{\infty} d\xi' \int_{-\infty}^{\infty} d\xi'' \left\{ \frac{1}{2c\sqrt{\pi\chi}} \frac{1}{\sqrt{t-t'}} e^{- \frac{(\xi-\xi')^2}{4(t-t')}} \right\} q(\xi_1, \xi_3, \xi', t')
\]

(A.3)

Assuming that quantities \(c\) and \(\chi\) are homogeneous over the normal local coordinate \(\xi\), putting (A.2) into (A.3) we can write:
\[ T^w(\xi_1, \zeta, \xi_3, t) = T_0 + \frac{1}{4c \mu(\xi_1, \xi_3)} \int_0^{t-\epsilon} d t' \left\{ \text{erf} \left( \frac{\zeta + \mu(\xi_1, \xi_3)}{2 \sqrt{\chi(t-t')}} \right) - \text{erf} \left( \frac{\zeta - \mu(\xi_1, \xi_3)}{2 \sqrt{\chi(t-t')}} \right) \right\} \tau(\xi_1, \xi_3, t') \nu(\xi_1, \xi_3, t') \]  

(A.4)

where \( T_0 \equiv T(\xi_1, \zeta, \xi_3, 0) \) is the initial temperature distribution (i. e. the host rock temperature prior to faulting), \( \text{erf}(.) \) is the error function ( \( \text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z dx e^{-x^2} \) ) and \( \epsilon \) is an arbitrarily small positive real number. Simple algebra shows in fact that in the limit of \( t \to t' \) the Green function in (A.3) is null and therefore the contribution at \( t = t' \) in the temporal convolution is null. This has important consequences because it states that, in calculating the temperature change \( T^w - T_0 \), we can stop the temporal integration at an instant immediately before the actual time \( t \). In other words, the contribution at time \( t \) in the temperature change depends only on the previous traction and slip velocity time histories.

Equation (A.4) is a generalization to a 3–D case of the solution proposed by Fialko (2004) for a 2–D crack. On the fault surface, i. e. for \( \zeta = 0 \), equation (A.4) becomes:

\[ T^w(\xi_1, \xi_3, t) = T_0^f + \frac{1}{2c \mu(\xi_1, \xi_3)} \int_0^{t-\epsilon} d t' \text{erf} \left( \frac{\mu(\xi_1, \xi_3)}{2 \sqrt{\chi(t-t')}} \right) \tau(\xi_1, \xi_3, t') \nu(\xi_1, \xi_3, t') \]  

(A.5)

where \( T_0^f \) is the initial temperature distribution on the fault plane (i. e. \( T_0^f \equiv T(\xi_1, 0, \xi_3, 0) \)).

It is interesting to note that in the limit of zero–thickness slipping zone, taking into account the properties of the error function, we have:
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\[ T^{0f}(\xi_1, \xi_3, t) = T_0^f + \frac{1}{2c\sqrt{\pi\chi}} \int_0^{t-f} \frac{\tau(\xi_1, \xi_3, t')v(\xi_1, \xi_3, t')}{\sqrt{t-t'}} \, dt' \]  
(A.6)

which is analogue to the limit for \( D \to 0 \) of the solution obtained by Andrews (2002).

The analytical relations presented above have been obtained by solving the 1–D heat flux problem considering only one spatial coordinate (i.e. the normal to the fault). This implicitly means that we do not account for the possible contributions to the temperature changes coming from the neighboring points. It is clear from equations (A.4) and (A.5) that the temperature value at time \( t \) in a specified fault point does not explicitly depends on the values of total traction and slip velocity on the whole fault plane, but only on their values in the local position \((\xi_1, \xi_3)\). Because we are solving a 3–D elasto–dynamic problem, we have verified this assumption.

Let us consider now the 3–D thermal conduction problem

\[ \frac{\partial}{\partial t} T = \chi \left( \frac{\partial^2}{\partial \xi_1^2} + \frac{\partial^2}{\partial \xi_2^2} + \frac{\partial^2}{\partial \xi_3^2} \right) T + \frac{1}{c} q \]  
(A.7)

having the elementary solution

\[ T^{el}(\xi_1, \xi_2, \xi_3, t) = \frac{h}{c\sqrt{(4\pi\chi)}^3} e^{-\frac{\xi_1^2 + \xi_2^2 + \xi_3^2}{4\chi t}} \]  
(A.8)

where the intensity \( h \) has now dimension of Joule ([\( h \)] = J). Assuming the actual heat source \( q \) as in equation (A.2), proceeding as in the derivation of equation (A.4), we can write:
where $T_0 \equiv T(\xi_1, \zeta, \xi_3, 0) \equiv T(\xi_1, \zeta, 0)$ is the initial temperature as in equation (A.4). However, considering the short time window (the coseismic time scale) of the dynamic rupture process and the value of thermal diffusivity $\chi$, we comfortably write:

\[
T^w (\xi_1, \zeta, \xi_3, t) = T_0 + \frac{1}{4 c} \int_0^{t-\epsilon} dt' \int_{-\infty}^{+\infty} d\xi_1 \int_{-\infty}^{+\infty} d\xi_3 \sqrt{\frac{4 \pi}{\chi}} \left\{ \text{erf} \left( \frac{\zeta + \omega(\xi_1, \xi_3)}{2 \sqrt{\chi(t-t')}} \right) - \text{erf} \left( \frac{\zeta - \omega(\xi_1, \xi_3)}{2 \sqrt{\chi(t-t')}} \right) \right\} \tau\left( \xi_1, \zeta, \xi_3, t' \right) v\left( \xi_1, \zeta, \xi_3, t' \right)
\]

and an analogous expression for the integral involving the second on–fault coordinate ($\zeta_3$).

Therefore, taking into account these results, from (A.9) we obtain equation (A.4), and in the limit of $\zeta = 0$ we obtain equation (A.5). In other words we have demonstrated that, in the coseismic temporal scale, the 1–D (normal to fault plane) approximation of the thermal conduction problem is acceptable and that the temperature in a fault point mainly depends on the fault slip velocity and traction time histories in that point.
Appendix B. Solution of the simplified thermal pressurization problem

The flow of fluids in the Earth crust occurs through a matrix of interconnected passages, represented either by small fractures in rocks and by voids of naturally porous rocks. According to Turcotte and Schubert (2002) we define porous media materials where the scale of the flow system is large with respect to the scale of the passages. The fraction of the material’s volume made up of pore is known as porosity ($\Phi = V_{void}/V_{tot}$). The resistance of a porous media to fluid flow depends on the permeability $k$ that represents the tortuosity of the fluid pathways through the solid matrix (the dry skeleton); the more permeable the medium is, the larger $k$ is. It can be determined by laboratory experiments or by in situ observations (Lockner et al., 2000; Wibberley and Shimamoto, 2003).

Fluids can flow through a porous material under the influence of the applied pressure gradient. If we assume that (i) the scale of the porosity is small compared with the other characteristic dimensions of the flow and (ii) the flow in the individual channels is laminar (and therefore no advective terms are considered), the fluid flow in a porous medium can be described by an analytical relationship known as Darcy’s law (Darcy, 1856), that states that the flow through the porous material is linearly proportional to the difference between the pressure gradient and the hydrostatic pressure and inversely proportional to the viscosity of the fluid. If permeability in the damage zone is high enough (and this is a likely condition), it is reasonable to assume that fluids flow in the direction perpendicular to the fault; consequently, we have considered the 1–D version of the Darcy’s law:
where $q_\zeta$ is the volumetric flow rate per unit area, $\frac{\partial}{\partial \zeta} P_{\text{fluid}}$ is the fluid pressure gradient along the direction perpendicular to the fault and $\eta_{\text{fluid}}$ is the dynamic viscosity of the fluid. (In the case of a dipping fault equation (B.1) contains also the projection on $\zeta$ of the hydrostatic term $\frac{k}{\eta_{\text{fluid}}} \rho_{\text{fluid}} g \hat{x}_3$, that is equal to zero in the case of vertical fault considered in this work, as $\zeta$ is perpendicular to the $x_3$ axis; see Figure 1b). The volumetric flow rate $q_\zeta$ has the dimension of a velocity (and therefore it is also named Darcy’s velocity) and expresses the average velocity per unit area and not the fluid particle velocity.

In this Appendix we will derive constitutive equations for fluids, that relate the fluid pressure changes to the temperature changes and the heat generated on a fault. McKenzie and Brune (1972) and Richards (1976) consider the frictional heating of a planar fault with zero width; Delaney (1982) investigated the fluid flow and the pressurization in country rocks due to a rapid magmatic intrusion; Lee and Delaney (1987) solved the 1-D temperature–driven fluid pressure equation for zero–thickness fault and with different unit sources.

Continuity equations for pore fluid and solid (rock) mass in Eulerian formulation imply that

$$\frac{\partial}{\partial t} m_{\text{fluid}} + \frac{\partial}{\partial \zeta} (\rho_{\text{fluid}} q_\zeta) = 0 \tag{B.2}$$

where $m_{\text{fluid}}$ is the fluid mass content (Batchelor, 1967), i.e. the fluid mass per unit volume of
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dry skeleton), and \( \rho_{\text{fluid}} \) is the pore fluid cubic mass density.

As its rate is expressed as

\[
\frac{\partial}{\partial t} m_{\text{fluid}} = \rho_{\text{fluid}} \frac{\partial}{\partial t} \phi + \phi \frac{\partial}{\partial t} \rho_{\text{fluid}},
\]

assuming that permeability \( k \), density \( \rho_{\text{fluid}} \) and dynamic viscosity \( \eta_{\text{fluid}} \) are independent on the position (i.e. spatially homogeneous), from equations (B.1) and (B.2) we have:

\[
\rho_{\text{fluid}} \frac{\partial}{\partial t} \phi + \phi \frac{\partial}{\partial t} \rho_{\text{fluid}} = k \rho_{\text{fluid}} \frac{\partial^2}{\partial \zeta^2} P_{\text{fluid}}
\]

((B.3))

The fluid density \( \rho_{\text{fluid}}(P_{\text{fluid}}, T) \) is related to fluid pressure and temperature through the definition (see for instance Batchelor, 1967) of the coefficients of compressibility of the fluid (i.e. the inverse of the bulk modulus of elasticity of the fluid, \( \beta_{\text{fluid}} = \frac{1}{\rho_{\text{fluid}} \frac{\partial \rho_{\text{fluid}}}{\partial P_{\text{fluid}}}} \), where the partial derivative is made in isothermal conditions) and of thermal expansion of the fluid

\[
(\alpha_{\text{fluid}} = -\frac{1}{\rho_{\text{fluid}} \frac{\partial \rho_{\text{fluid}}}{\partial T}}), \text{ where the partial derivative is made in isobaric conditions):}

\[
\frac{1}{\rho_{\text{fluid}}} \frac{\partial}{\partial t} \rho_{\text{fluid}} = \beta_{\text{fluid}} \frac{\partial}{\partial t} P_{\text{fluid}} - \alpha_{\text{fluid}} \frac{\partial}{\partial t} T
\]

((B.4))

(see also Lee and Delaney, 1987). From equations (B.3) and (B.4) we have:

\[
\frac{\partial}{\partial t} P_{\text{fluid}} = \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \frac{\partial}{\partial t} T - \frac{1}{\beta_{\text{fluid}} \phi} \frac{\partial}{\partial t} \phi + \omega \frac{\partial^2}{\partial \zeta^2} P_{\text{fluid}}
\]

((B.5))

where \( \omega \) is the hydraulic diffusivity, which is defined in equation (8).
Let us assume that porosity $\Phi$ is constant in time ($\Phi(\xi_1, \zeta, \xi_3, t) = \Phi_0(\xi_1, \zeta, \xi_3)$). Under this assumption equation (B.5) becomes:

$$\frac{\partial}{\partial t} p_{\text{fluid}} - \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \frac{\partial}{\partial t} T = \omega \frac{\partial^2}{\partial \zeta^2} p_{\text{fluid}}$$  \hspace{1cm} (B.6)

where the hydraulic diffusivity $\omega$ is constant over time, since porosity and permeability are constant. For an elementary heat source (see Appendix A), taking into account the solution (A.1) for the temperature, the solution of equation (B.6) coupled with equation (5) is:

$$p_{\text{fluid}}^e(\zeta, t) = \frac{\gamma h}{2\sqrt{\pi} t} \left[ -\frac{\sqrt{\chi}}{\omega-\chi} e^{-\frac{\zeta^2}{4\omega t}} + \frac{\sqrt{\omega}}{\omega-\chi} e^{-\frac{\zeta^2}{4\omega t}} \right]$$  \hspace{1cm} (B.7)

where the dimensionless parameter $\gamma$ is $\alpha_{\text{fluid}}/\beta_{\text{fluid}}$. If the heat source is expressed as in equation (A.2), we obtain the solution for pressure to the simplified 1-D thermal pressurization problem (i.e. of the coupled equations (5) and (B.6)) integrating over the normal coordinate $\zeta$ the temporal convolution between the Green kernel corresponding to (B.7) and the actual heat source (A.2):

$$p_{\text{fluid}}^w(\xi_1, \zeta, \xi_3, t) = p_{\text{fluid}_0}^w + \frac{\gamma}{4u(\xi_1, \xi_3)} \int_0^t dt' \left\{ -\frac{\chi}{\omega-\chi} \left[ \text{erf} \left( \frac{\xi + w(\xi_1, \xi_3)}{2\sqrt{\chi(t-t')}} \right) - \text{erf} \left( \frac{\xi - w(\xi_1, \xi_3)}{2\sqrt{\chi(t-t')}} \right) \right] + \frac{\omega}{\omega-\chi} \left[ \text{erf} \left( \frac{\xi + w(\xi_1, \xi_3)}{2\sqrt{\omega(t-t')}} \right) - \text{erf} \left( \frac{\xi - w(\xi_1, \xi_3)}{2\sqrt{\omega(t-t')}} \right) \right] \right\} \tau(\xi_1, \zeta, \xi_3, t')$$  \hspace{1cm} (B.8)

where $p_{\text{fluid}_0}$ is the initial fluid pressure distribution (i.e. $p_{\text{fluid}_0}^w(\xi_1, \zeta, \xi_3, 0)$) and we have
assumed that the dimensionless parameter $\gamma$ is constant over the slipping zone $\Delta v$. As in equation (B.7), in (B.8) terms containing $\chi$ account for thermal contribution and terms containing $\omega$ for hydraulic one.

On the fault plane (i.e. in the limit of $\zeta \to 0$) the solution for the fluid pressure is:

$$p_{\text{fluid}}^f(\xi_1, \xi_3, t) = p_{\text{fluid}0}^f + \frac{\gamma}{2w(\xi_1, \xi_3)} \int_0^t dt' \left\{ -\frac{\omega - \chi}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2\sqrt{\chi(t-t')}} \right) \right\}_{\xi_3}^{\xi_3} +$$

$$+ \frac{\omega}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2\sqrt{\omega(t-t')}} \right) \tau(\xi_1, \xi_3, t') \nu(\xi_1, \xi_3, t')$$

(B.9)

where $p_{\text{fluid}0}^f$ is the initial fluid pressure distribution on the fault plane (i.e. $p_{\text{fluid}0}^f \equiv p_{\text{fluid}}(\xi_1, 0, \xi_3, 0)$). Also in this case it is interesting to calculate the fluid pressure response in the limit of zero–thickness slipping zone:

$$p_{\text{fluid}}^f(\xi_1, \xi_3, t) = p_{\text{fluid}0}^f + \frac{\gamma}{2\sqrt{\pi}} \int_0^t dt' \left\{ -\frac{\omega - \chi}{\omega - \chi} \frac{1}{\sqrt{\chi(t-t')}} \right\}_{\xi_3}^{\xi_3} +$$

$$+ \frac{\omega}{\omega - \chi} \frac{1}{\sqrt{\omega(t-t')}} \tau(\xi_1, \xi_3, t') \nu(\xi_1, \xi_3, t')$$

(B.10)

that represents the upper boundary of the fluid pressure distribution on the fault plane.

We emphasize that our solution is different with respect to that proposed by Andrews (2002, his equation (17)) because we consider appropriate heat source and Green’ s kernels. Andrews (2002) used a Gaussian analytical expression for the elementary heat source and neglected the term involving thermal diffusivity. Moreover, there is one important advantage in
our solution: due to the analytical properties of the proper Green’s kernel used in this work we can limit the domain of the integration of the temporal convolution that gives $p_{\text{fluid}}^{w}$ up to $t - \varepsilon$. In this case pore fluid pressure can be determined independently on the actual values (i.e. at time $t$) of the fault friction and fault slip velocity. Finally, we do not include any artificial fading memory in the calculation of the value of the effective normal stress at the actual time $t$. 
Appendix C. Solution of the generalized thermal pressurization problem

In Appendix B we have derived the solution of the 1–D thermal pressurization problem under the hypothesis of constant porosity. In full of generality porosity $\Phi$ can change in time, accordingly to an evolution law, that has to be assigned \textit{a priori}. If porosity changes, we have to consider the generalized problem described by the coupled equations (5) and (7). In this case the hydraulic diffusivity $\omega$ is also varying in time, as it explicitly depends on porosity (see equation (8)). In the following we will assume for simplicity that parameters $\gamma$ and $\beta_{\text{fluid}}$, porosity and its time derivative are constant in space and do not vary within the slipping zone.

If we calculate $\frac{\partial T}{\partial t}$ from equation (5) and we insert this expression in equation (B.6) of the simplified thermal pressurization problem, we obtain:

$$
\frac{\partial}{\partial t} p_{\text{fluid}} = \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \chi \frac{\partial^2}{\partial \xi^2} T - \omega \frac{\partial^2}{\partial \xi^2} p_{\text{fluid}} = \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \frac{1}{c} q ;
$$

(C.1)

while, if we calculate $\frac{\partial T}{\partial t}$ from equation (5) and we insert this expression in equation (7) defining the generalized thermal pressurization problem, we have:

$$
\frac{\partial}{\partial t} p_{\text{fluid}} = \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \chi \frac{\partial^2}{\partial \xi^2} T - \omega \frac{\partial^2}{\partial \xi^2} p_{\text{fluid}} = \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \frac{1}{c} q - \frac{1}{\beta_{\text{fluid}} \Phi} \frac{\partial}{\partial t} \Phi
$$

(C.2)

The comparison between (C.1) and (C.2) suggests that the solution of the fluid pressure $p_{\text{fluid}}$ for the generalized thermal pressurization problem can be obtained in the same way of
Appendix B, but calculating the time convolution and the integral over the normal coordinate $\zeta$ of the Green’s kernel $p_{\text{fluid}}^{el}(\zeta, t)$ of equation (B.7) and using the following term as the actual heat source

$$
\tilde{q}(\xi_1, \zeta, \xi_3, t) = \begin{cases} 
\tau(\xi_1, \zeta, \xi_3, t) w(\xi_1, \zeta, \xi_3, t) - \frac{1}{\gamma \beta_{\text{fluid}} \Phi} \frac{\partial}{\partial t} \Phi, & t > 0, |\zeta| \leq w(\xi_1, \xi_3) \\
0, & |\zeta| > w(\xi_1, \xi_3)
\end{cases} \quad (C.3)
$$

instead of $q(\xi_1, \zeta, \xi_3, t)$ of equation (A.2). Simple algebra shows that:

$$
\bar{p}_{\text{fluid}}^{\text{w}}(\xi_1, \zeta, \xi_3, t) = p_{\text{fluid}}^{\text{w}} + \frac{\gamma}{4 \omega w(\xi_1, \xi_3)} \int_0^{t - \xi} \partial t \left\{ - \frac{\chi}{\omega - \chi} \left[ \text{erf} \left( \frac{\zeta + w(\xi_1, \xi_3)}{2 \sqrt{\chi (t - t')}} \right) - \text{erf} \left( \frac{\zeta - w(\xi_1, \xi_3)}{2 \sqrt{\chi (t - t')}} \right) \right] + \right.
$$

$$
+ \frac{\omega}{\omega - \chi} \left[ \text{erf} \left( \frac{\zeta + w(\xi_1, \xi_3)}{2 \sqrt{\omega (t - t')}} \right) - \text{erf} \left( \frac{\zeta - w(\xi_1, \xi_3)}{2 \sqrt{\omega (t - t')}} \right) \right] \right\} \{ \tau(\xi_1, \zeta, \xi_3, t') v(\xi_1, \xi_3, t') +
$$

$$
- \frac{2 \omega w(\xi_1, \xi_3)}{\gamma} \frac{1}{\beta_{\text{fluid}} \Phi(t)} \frac{\partial}{\partial t} \Phi(\xi_1, \zeta, \xi_3, t') \}
\right. \quad (C.4)
$$

where $p_{\text{fluid}}^{\text{w}}(\xi_1, \zeta, \xi_3, 0)$, as in Appendix B.

On the fault plane (i.e. in the limit of $\zeta \to 0$) the analytical solution for the fluid pressure is:

$$
\bar{p}_{\text{fluid}}^{\text{w}}(\xi_1, \zeta, \xi_3, t) = p_{\text{fluid}}^{\text{w}} + \frac{\gamma}{2 \omega w(\xi_1, \xi_3)} \int_0^{t - \xi} \partial t \left\{ - \frac{\chi}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2 \sqrt{\chi (t - t')}} \right) + \right.
$$

$$
+ \frac{\omega}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2 \sqrt{\omega (t - t')}} \right) \right\} \{ \tau(\xi_1, \zeta, \xi_3, t') v(\xi_1, \xi_3, t') +
$$

$$
- \frac{2 \omega w(\xi_1, \xi_3)}{\gamma} \frac{1}{\beta_{\text{fluid}} \Phi(t)} \frac{\partial}{\partial t} \Phi(\xi_1, 0, \xi_3, t') \}
\right. \quad (C.5)
$$
where $p_{\text{fluid}_0}^f = p_{\text{fluid}}(\xi_1,0,\xi_3,0)$, as in Appendix B. Let us emphasize that in equations (C.4) and (C.5) the hydraulic diffusivity implicitly depends on time. Moreover, for constant porosity (i.e., $\Phi(\xi_1,\zeta,\xi_3,t) = \Phi_0(\xi_1,\zeta,\xi_3)$) solutions (C.4) and (C.5) become, as expected, the solutions derived in Appendix B (equations (B.8) and (B.9), respectively) for the simplified thermal pressurization problem of the pore fluid.
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Figure Captions

**Figure 1.** (a) Fault zone structure inferred from geological observations (e.g. Chester et al., 1993; Chester and Chester, 1998; Lockner et al., 2000; Wibberley and Shimamoto, 2003; Sibson 2003). The slipping region has a thickness $2w$ and it is indicated by the shaded area; the solid line is the principal slipping zone (Sibson, 2003). The fault core in embedded in a highly fractured damage zone (Li et al., 1994). (b) The fault model used in this work: $2w$ represents the thickness of the slipping zone and the straight line in the center is the fault plane ($\zeta = 0$). The permeability ($k$), porosity ($\Phi$) and thermal diffusivity ($\chi$) are taken constant through the medium. $O\xi_1^3\xi_3^3$ is the absolute Cartesian coordinate system, while $O\xi_1^\xi_3^\xi_3$ is the local one.

**Figure 2.** Snapshots of slip velocity on the fault plane at the last time step of the numerical simulations for a low strength ($S = 0.8$) fault (a) and a high strength ($S = 1.5$) fault (b). A linear SW law has been adopted in these simulations. (c) Temperature time histories at a fault point (9000 m, 6200 m) for different values of slipping zone thickness $2w$; open and solid symbols refer to low and high strength ($S$) configurations, respectively. Medium and constitutive parameters are listed in Table 1; the high strength fault has $\mu_u = 0.82$ and $\mu_f = 0.57$.

**Figure 3.** Temperature changes (measured with respect to the initial value $T_{0f}$) as a function of time for dry faults (only caused by frictional heating) governed by the linear SW law and having different slipping zone thickness. For each $w$ solid curves refer to crack–like
models (i.e. no healing of slip), while dashed curves refer to numerical experiments in which the slip duration is finite (with healing). In the latter case, we can identify the first time step at which the maximum total fault slip is reached, which is indicated by an empty square in the dashed curves. In the solid curves, full symbols indicate the time step at which the fault has developed the same cumulative slip. Numbers indicate the temperature change values (°Celsius) at those time steps. The vertical dotted line indicates the time step at which the traction on the fault reaches the upper yield value \( \tau_u \), while vertical arrows define, for each curve, the time step at which the fault slip velocity is at its maximum. Solutions are plotted in a fault point at a distance of 900 m from the hypocenter, at the depth of the hypocenter.

**Figure 4.** (a) Temperature changes for different slipping zone thicknesses due to frictional heating on a fault governed by the SW law. Full symbols refer to rupture for which the temperature is coupled with fluid flow (i.e. numerical experiments with thermal pressurization), while open symbols refer to rupture without fluid effects. Each line is generated at the depth of the hypocenter. (b) 3-D spatial distribution on the fault plane of the temperature changes for a particular case of panel (a) \( w = 1 \) mm, with thermal pressurization effects). Both panels are at the same time step (after 0.38 seconds form the nucleation).

**Figure 5.** Temperature changes as a function of time for three different slipping zone thicknesses: \( w = 35 \) mm (panel (a)), \( w = 1 \) mm (panel (b)) and \( w = 10 \ \mu m \) (panel (c)), for a fault governed by the SW law. Black curves indicate dry faults, while gray ones refer to
wet faults. Solid curves refer to crack–like solutions and dashed curves refer to models with finite slip duration (healing of slip). For the latter cases, we have indicated with full squares the time step at which the maximum fault slip is reached and we report the temperature change values in Celsius degrees. In the dry cases with healing of slip we have also indicated with open diamonds the corresponding time at which the slip saturates in the wet case (the fault slip velocity is zero). In the wet case with no healing (grey solid curves) we have indicated the temperature change when the fault has developed a value of the cumulative slip equal to the maximum slip in the wet case with healing (open gray circle). In panel (b) we have also drawn a simulation for a dry fault with healing with hydraulic diffusivity equal to $0.01 \text{ m}^2/\text{s}$ (dotted curve). Solutions are plotted in the same fault point of Figure 3.

**Figure 6.** (a) Temperature changes as a function of the slipping zone thicknesses due to frictional heating on a fault governed by the SW law with thermal pressurization effects (solid curves). Each line represent a particular value of fault slip. Dashed lines refer to the adiabatic temperature changes $\Delta T^{(\text{adiab})} = \tau_f u/2cw$ (Sibson, 2003); solid circles in each line indicate the values of temperature change in dry conditions, modeled in the numerical experiments for the reference value of slipping zone thickness ($w = 0.035 \text{ m}$). (b) Temperature changes as a function of the hydraulic diffusivity ($\omega$) and with the reference value of slip zone thickness $w$. To change $\omega$ we modify the permeability $k$. All solutions refer to the same fault point of Figure 3.
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Figure 7. Pore fluid pressure change (with respect to the initial value of $\sigma_n - p_{fluid0}$) as a function of time for a fault obeying to the SW law (top panels) and to a DR law (bottom panels). In panels (a) and (c) we have changed the slip zone thickness $2w$ (with respect to the reference configuration listed in Table 1). In panels (b) and (d) we have changed the hydraulic diffusivity ($\omega$) (through changes in permeability $k$). The horizontal dashed lines indicate the pore pressure value equal to the normal stress, which implies $\sigma_n^{eff} = 0$. Solutions are plotted in a fault point located at a distance from the hypocenter of 900 m (panels (a) and (b)) and 1300 m (panels (c) and (d)).

Figure 8. Slip velocity snapshots at the last time step of the numerical simulation for a rupture obeying to the SW constitutive equation without (a) and with (c) thermal pressurization effects. In panels (b) and (d) we have plotted the superposition of the solutions as a function of time in the two different configurations in the same fault point of Figure 3. All quantities are normalized with respect to their absolute maximum, with the exception of traction $\tau$ that is normalized as follows: $(\tau - \tau_{max})/(\tau_{max} - \tau_{min})$. Vertical dashed lines indicate the time step at which the maximum value of the slip velocity is reached.

Figure 9. The same of Figure 8, but for dry and wet faults governed by DR friction law. Solutions are plotted in the same fault point of Figures 7c and 7d. Vertical dashed lines indicate the time step at which the maximum value of the slip velocity and of the kinetic frictional level are reached.
**Figure 10.** State variable evolution for the configurations showed in Figure 9: black curves refer to dry faults, while gray ones to wet faults. Solid curves are the results of the numerical experiments and thin lines represents the steady state value ($\Psi^{ss}$) of the state variable.

**Figure 11.** Friction coefficient ($\tau/\sigma_n^{eff}$) as a function of time (panels (a) and (b)) and fault slip (panels (c) and (d)) for dry and wet faults (open and solid symbols, respectively). For wet faults panels (e) and (f) show the Stribeck curve, which is the friction coefficient as a function of the Sommerfeld number (i.e., the ratio between pore fluid pressure and nominal normal stress). Plots refer to dry and wet faults governed by SW law (left panels) and by DR law (right panels). Dashed vertical lines in panels (e) and (f) indicates the lithostatic pore pressure value. In panels (c) and (d) the value of the characteristic SW distance is indicated by the vertical segment. Solutions are in the same fault point of Figure 3 for SW simulations and of Figures 7c and 7d for DR ones. Parameters are the same of those listed in Table 1, with the exception of slipping zone thickness that now is $w = 1$ mm.
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Table 1. Medium and constitutive parameters adopted in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
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<td>$v_P$</td>
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<td>$L$</td>
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<td>$\Psi^\infty (v_{init})$</td>
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<tr>
<td>$w$ (reference)</td>
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(a) The permeability $k$ used here may be considered as an average value in the slip and in the damage zones. Data from different fault zones show that $k$ ranges between $10^{-16}$ m$^2$ and $10^{-22}$ m$^2$ (Morrow et al., 1984; Chester et al., 1993; Ito et al., 1998; Lockner et al., 2000; Mizoguchi et al., 2000; Wibberley and Shimamoto, 2003).

(b) See Andrews (2002) and Miller et al. (2004); Miller (2002) uses $1.9 \times 10^{-4}$ Pa s.

(c) See Andrews (2002). Using the values reported in the Table, the parameter $\gamma = \alpha_{\text{fluid}}/(\beta_{\text{fluid}}c)$ is equal to 0.5 in all numerical experiments presented and discussed in this paper. The specific heat $C_{\text{bulk}}$ is 1100 J/(Kg °C).

(d) The same value used by Cardwell et al. (1978), Fialko and Rubin (1998), Andrews (2002) and Fialko (2004); McKenzie and Brune (1972) use $7 \times 10^{-7}$ m$^2$/s.

(e) This reference value is in agreement with those reported by Chester (1995) and Fialko (2004).
Figure 1

(a) Damage zone
(b) Slip zone

$2w(\xi_1, \xi_3)\]
Figure 2

(a) and (b) Illustrate the variation of slip velocity along the strike and dip directions for two different slip parameters, $S = 0.8$ and $S = 1.5$. The color scale denotes the slip velocity in meters per second.

(c) Shows the temperature increase vs. time for different temperature changes, denoted by $T - w$. The temperature changes range from 0.001 mm to 5 m.
Figure 3
Figure 4

(a) Temperature change along strike.

(b) 3D representation of temperature change with different widths and conditions.

Legend:
- red: $w = 0.5$ m
- purple: $w = 0.035$ m
- black: $w = 0.01$ m
- blue: $w = 0.001$ m
- red: $w = 0.5$ m - no pf
- purple: $w = 0.035$ m - no pf
- black: $w = 0.01$ m - no pf
- blue: $w = 0.001$ m - no pf
Figure 7

(a) SW

(b) SW

(c) DR

(d) DR

Fluid pressure change (Pa)
Figure 8

(a) Dry

(b) Normalized solutions
   Slip velocity
   Traction
   Temperature change

(c) Wet

(d) Normalized solutions
   Slip
   Slip velocity
   Traction
   Temperature change
   Fluid pressure
Figure 9

(a) Dry

(b) Normalized solutions

(c) Wet

(d) Normalized solutions

Along strike direction (m)
Figure 10

The figure shows the comparison between the dry and wet steady states over time. The x-axis represents time (s) ranging from 0.00E+00 to 1.25E+00, and the y-axis represents state (s) ranging from 1.0E-03 to 1.0E+04.

- **Dry Steady State**: The dark line represents the dry steady state, which remains relatively constant over time.
- **Wet Steady State**: The light line represents the wet steady state, showing a different behavior compared to the dry state.

The figure highlights the transition and stability of these states over time.
Figure 11

(a) Friction coefficient vs. time for dry and wet conditions. 
(b) Similar to (a) but with Sommerfeld number on the x-axis.
(c) Friction coefficient vs. slip for dry and wet conditions. $d_0 = 0.1 \text{ m}$.
(d) Similar to (c) but with Sommerfeld number on the x-axis. $d_0^{eq} = 0.093 \text{ m}$ and $d_0^{eq} = 0.532 \text{ m}$.
(e) Friction coefficient vs. Sommerfeld number. $\sigma_n^{err} = 0$.
(f) Similar to (e) but with Sommerfeld number on the x-axis. $\sigma_n^{err} = 0$. 