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Modal Decomposition of Magnetic Maps

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Introduction

Singular value decomposition (SVD) is a computational technique widely used in many geophysical fields such as oceanographic (Fukumori & Wunsch 1991) and seismic (Freire & Ulrych 1988) and also in image processing (Prasantha et al. 2007). In the last few decades this technique was extensively used in image coding applied to image transmission over nationwide computers network.
This work explores the feasibility of the SVD as a separation tool between signal and noise, applied on magnetic maps. In detail, the magnetic image is decomposed and then represented into a number of orthogonal basis thereby obtaining a new image formed by fewer dimensions. This decomposition seems to be efficient in noise removal and acts as an enhancement technique able to highlight hidden features. Due to the intrinsic implementation simplicity the procedure can also be used as an interactive tool.

Singular Value Decomposition

SVD is a over a hundred years old matrix factorization technique; it was discovered for square matrix independently by Beltrami in 1873 and Jordan in 1874. Eckart and Young theorem (Eckart & Young, 1936) extends the technique to rectangular matrix.

The decomposition of a rectangular matrix X (mxn) can be expressed as follows

$$X = USV^{T}$$

where $U[m \times m]$ and $V[n \times n]$ are two orthogonal matrices and S is an $[m \times n]$ diagonal matrix. U columns are eigenvectors of XX^T and represent the singular vectors of X spanning the column space; the column of V are eigenvectors of X^TX and represent the singular vectors of X, spanning the row space.

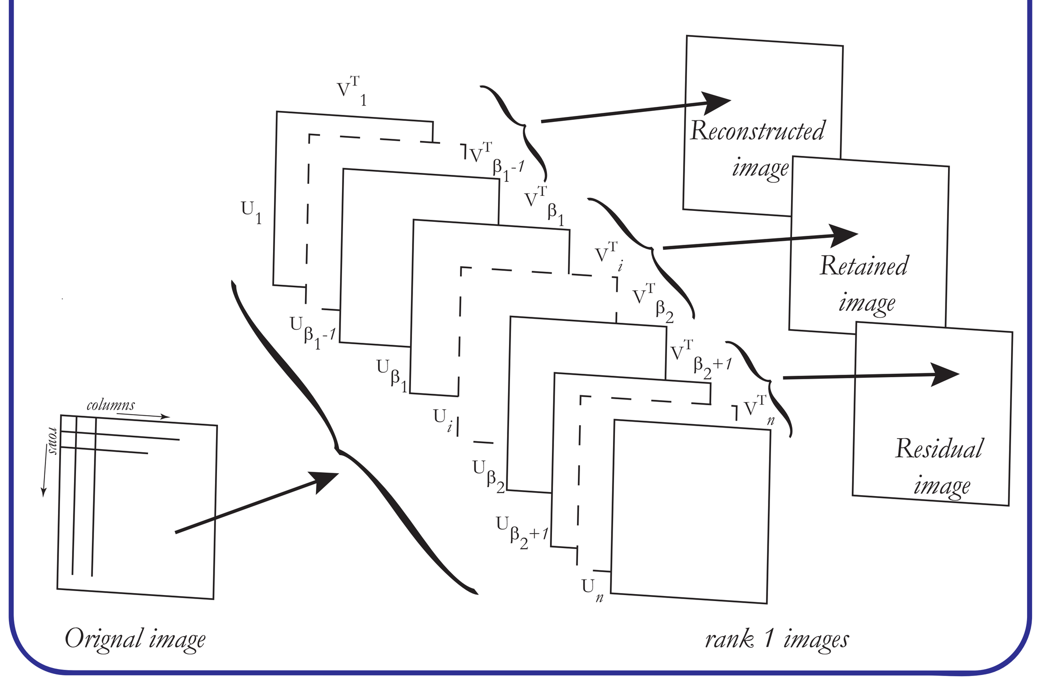
If k diagonal elements of S are null the dimension of X becomes $p = n-k$, thus the matrices of decomposition get the dimension $U[m \times p]$, $S[p \times p]$, $V[p \times n]$. In real cases the sorted set $\lambda = \{S_{11}, S_{22}, \ldots S_{ii}, \ldots, S_{nn}\}$ of the diagonal elements of S shows a general character with nearly vanishing component as i increases.
Retaining k elements of λ with $k \ll n$, we can reduce the dimensionality of data (X) and thus enhance subtle features hidden by the noise. Since each element of λ explains a portion of the total variance of X, the definition of a "modal band" β with $1 \le \beta_1 \le \beta_2 \le n$ allows a variance driven filtering technique, analogous with spectral band filtering. The "*retained dataset*" will be extracted using λ elements accordingly with :

$$\lambda_{1..(\beta_1-1)} = 0; \qquad \lambda_{\beta_1} .. \lambda_{\beta_2} = S_{(\beta_1,\beta_1)} .. S_{(\beta_2,\beta_2)}; \qquad \lambda_{(\beta_2+1)..n} = 0;$$

In the same way a reconstructed and a residual dataset, can be obtained.

$$\lambda_i = S_{(i,i)} \textit{ with } i < \beta_1 \text{ and } \lambda_i = 0 \textit{ with } i >= \beta_1$$

$$\lambda_i = S_{(i,i)} \textit{ with } i > \beta_2+1 \text{ and } \lambda_i = 0 \textit{ with } i < \beta_2$$



Conclusions

The proposed method seems to be efficient in removing noise, enhancing subtle features and also in revealing hidden leveling errors. This may significantly assist magnetic anomalies interpretation. The implementation of the filtering procedure is quick and the calculus, relying on both open source and commercial packages, requires very little resources. Moreover, the choice of coefficients can be implemented as an interactive tool. The empirical character of the filtering procedure requires a trial and err approach. Keeping in mind the method's empirical nature, the results should be used only to infer (verify) the presence of magnetic lineaments and/or discover areas where some hidden leveling error may affect the final magnetic anomaly map.
The modal decomposition works only on square/rectangular not sparse matrices.

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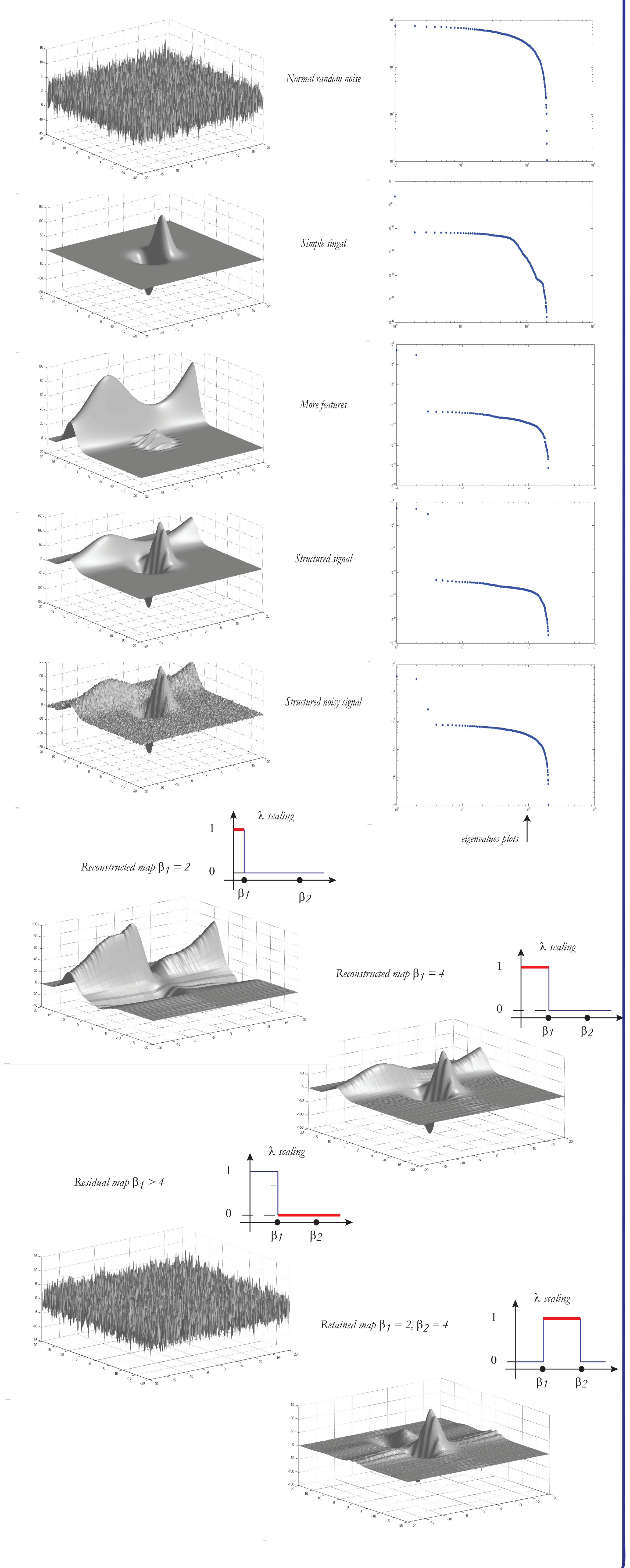
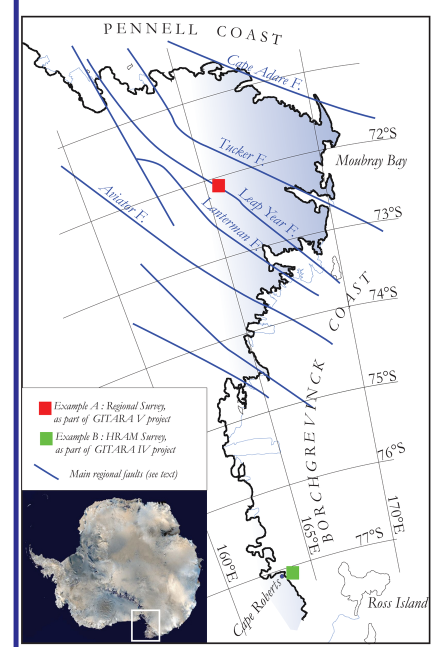
SVD on sinthetic data

In order to test the decomposition, a square 200x200 px image (*structured noisy signal*) has been created. The plots below show a sinthetic signal divided into its components : a normal random noise, a dipolar anomaly and some more complex features.

The images below propose different separations varying the β coefficents. As it is clearly visible the proposed method acts as an effective separation tool able to distinguish between signal and noise and between different signal components (see retained map).



Applications



In order to test the method two cases were selected from the large database of PNRA surveys performed in the last decade over Victoria Land, East Antarctica (see location).

1) a small portion on the GITARA V aeromagnetic regional survey (Bozzo, et al. 1999) (Chiappini, et al. 2002)

2) a subset of the first HRAM survey in Antarctica, namely the GITARA IV (Bozzo, et al. 1997). Further details concerning the Cape Roberts (GitaraIV) can be found in last ISAES workshop proceedings (Armadillo, et al. 2007)

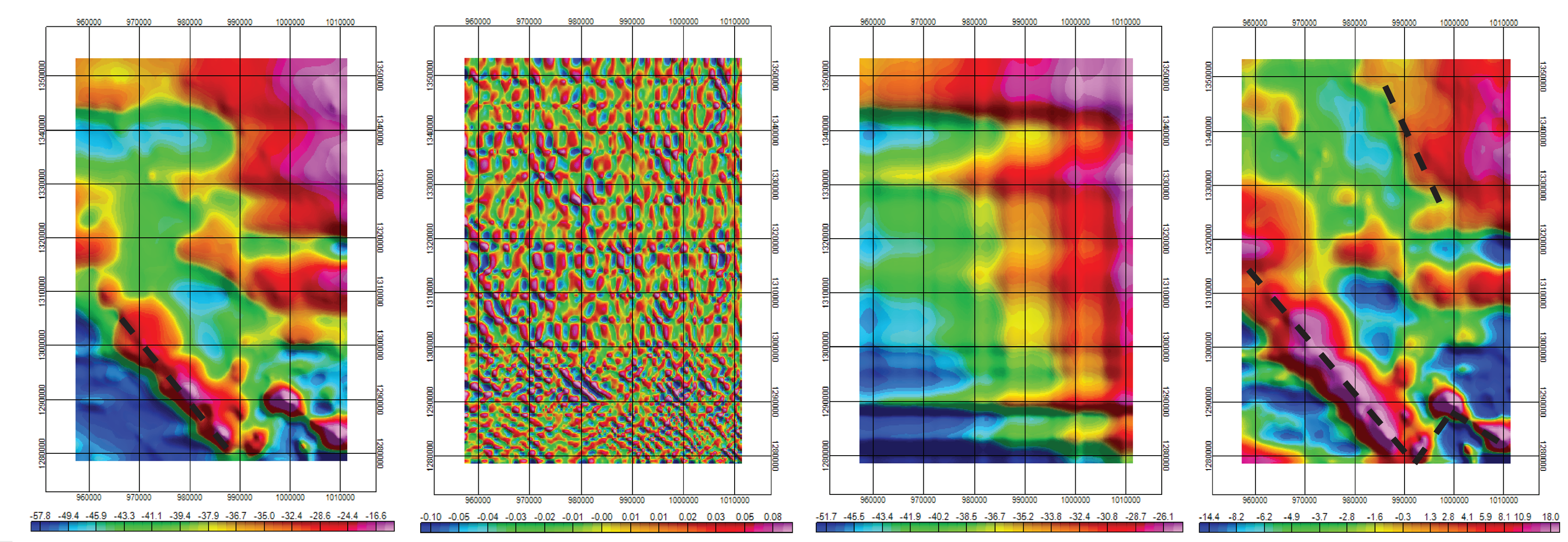
Regional Aeromagnetic Survey

Concerning the regional aeromagnetic anomaly map, the prominent positive and elongated NW-SE anomaly (see original map - bottom left) has been interpreted as the magnetic evidence of the Leap Year fault, which juxtaposes Robertson Bay and Bowers Terrane. (Chiappini, et al. 2002)

The modal decomposition $\beta_1 = 2$, $\beta_2 = 20$ reveals :

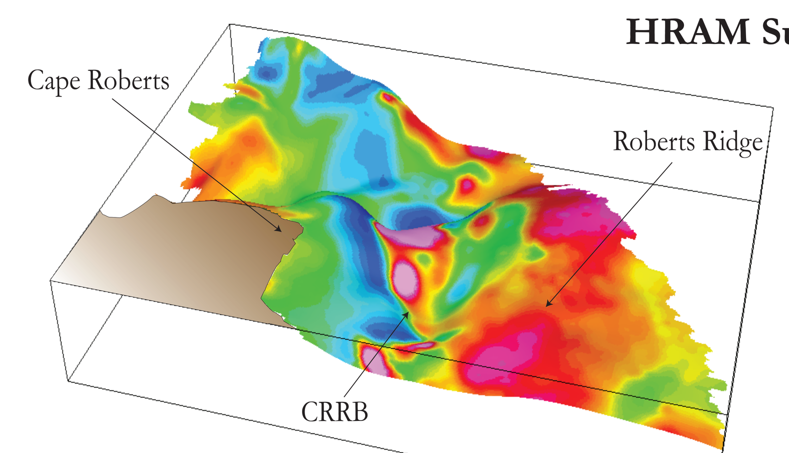
1) Recostructed map : some E-W trending features that seem to mimic the profile lines and have been interpreted as residual levellin error.

2) Retained map : . a clean map where the LYF anomaly appears much more persistent.

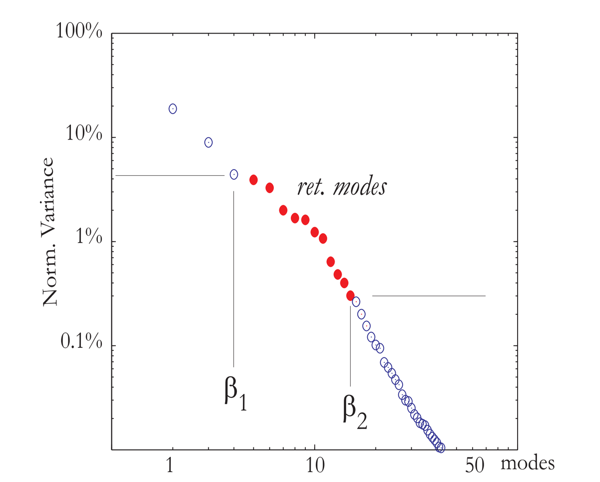


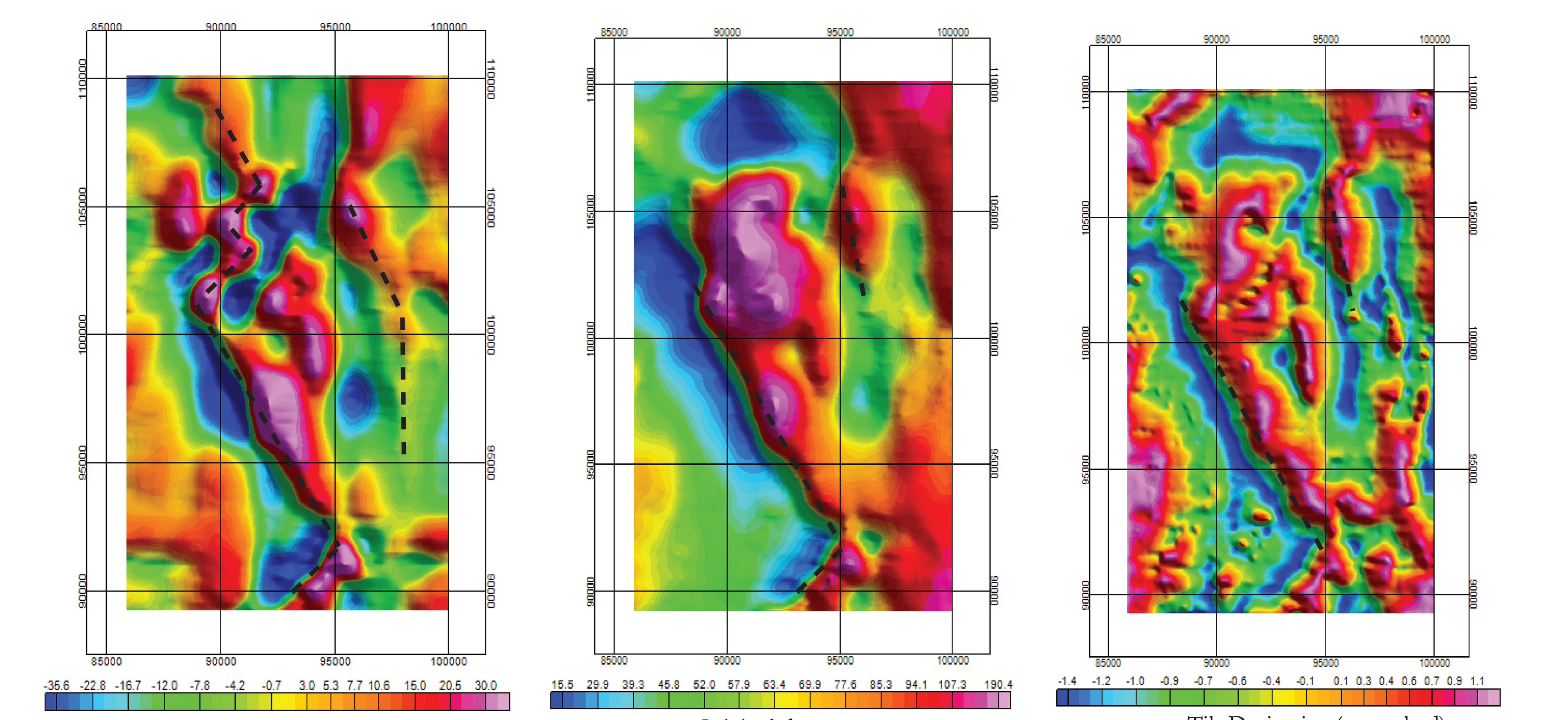
Original map | Residual | Reconstructed | Retained

HRAM Survey example



3D view of CRRB (Cape Roberts Rift Basin) with draped magnetic anomaly colouring





Modal Decomposition, "retained map" | Original dataset | Tilt Derivative (smoothed)

As latter example of modal decomposition is a subset of the HRAM survey performed in 1994 within the frame of GITARA IV project as member of the Cape Roberts Drilling Project (Barrett e Pyne 1995) (Barrett, Fielding e Wise 1998) (Fielding e Thomson 1999) (Barrett, Sarti e Wise 2000). The survey was performed during 1994 in Antarctica, offshore Cape Roberts, along the western margin of the Ross Sea Rift, where drilling information, multi-channel seismic reflection and structural results (Wilson 1995, Hamilton, et al. 2001) have been combined with aeromagnetic images. A complete and detailed synthesis of the tectonic framework together with the description of magnetic lineament was provided by Ferraccioli and Bozzo. (Ferraccioli e Bozzo 2003).
A view of Cape Roberts bathymetry is provided where the sea floor has been coloured with the draped magnetic anomaly. the black dashed lines figure out the Cape Roberts Rift Basins (CRRB) shoulders

Similarly to the above example the modal decomposition was performed. The aim was to highlight structural features flanking the oval shaped central anomaly interpreted as intrusive intrasedimentary bodies. The highlighted lineaments should draw the edges of the Cape Roberts Rift Basin (Hamilton et al. 2001).The existence of ENE-WSW oriented lineaments, interpreted as possibile transfer faults is highlited by the modal decomposition. This technique appears to be more effective than the tilt derivative in enhancing these lineaments. Independent seismic evidence is consistent with the existence of these transfer faults.

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