Bayesian source inference of the 1993-97 deformation at Mount Etna (Italy) by numerical solutions

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**SUMMARY**

Deformation data collected at Mount Etna from 1993 to 1997 show that the volcano edifice inflation is accompanied by instability of the eastern flank. We propose a 3D finite element model including lateral variations of material properties and topography. Source parameters of the inflating source are constrained by a direct search followed by an appraisal stage of the sampled solutions. The instability of the eastern flank is here addressed using a kinematic approach consisting in a rigid translation of a prescribed area. Our aim is to evaluate how the inflating source inference is affected by eastern flank sliding. Our results show that when sliding is accounted for, the inferred source location is shifted by \(\sim 1\) km toward SE and its strength decreased by \(\sim 20\%\).

**Key words:** Numerical solutions, Inverse theory, Volcano monitoring, Radar interferometry, Crustal structure.
1 INTRODUCTION

The structure of Mt Etna, located in NE Sicily (Italy), is characterized by complex fault systems extending from the summit craters (NE Rift and S Rift) to its flanks (a simplified map is shown in Fig.1a). Several studies show evidence of instability in the SE flank of the volcano, described by a south-eastward sliding movement. The sliding is commonly ascribed to gravitational instability and/or magma intrusion (e.g., Lo Giudice and Rasà 1992; Borgia et al. 1992; Rust and Neri 1996; Bonforte and Puglisi 2003; Neri et al. 2004; Walter et al. 2005). However, the relationship between these processes are poorly understood and still subject of debate. The surface structures delimiting the SE flank are the left-lateral Pernicana Fault System (PFS), which borders the northern margin, and the Ragalna Fault (RF) and the S rift at the SW margin. Within the unstable sector at least two sliding blocks, separated by NW-SE trending faults (Santa Venerina, SV, and Timpe Fault System, TFS, Fig. 1a) can be recognized (Neri et al. 2004). The slip recorded on the PFS is related to the sliding of the SE flank but also to the summit activity (Puglisi and Bonforte 2004). For example, during the 2002-2003 event, the opening of eruptive fractures along the NE Rift enhanced the seaward displacement of the eastern flank. The sliding movement showed a southward progression from the PFS, reaching the TFS and the Trecastagni Fault, TF (Neri et al. 2004). The estimated slip rates on PFS range from 0.8±0.2 to 2.2±0.4 cm/year, even though large and long-lasting creep episodes occurred in 2002-2003, with a maximum displacement of 1.25 m in its western segment (Neri et al. 2004). During quiescent or recharge phases (such as 1993-1997) slip rates on PFS can be considered as lower bounds because the sliding movement can also be accommodated by different strike slip faults mapped in the SE sector (e.g., Neri et al. 2004; Monaco et al. 2005; Rust et al. 2005). Even during the deflation in 2004-2005, a continuous and independent eastward movement of the eastern sector is observed, while geodetic data on the summit of Mt Etna show general contraction (Bonforte et al. 2008). All these studies show evidence of the complex temporal and spatial evolution of the sliding mechanism.

The inflation is commonly described in terms of a pressurized cavity embedded in a homogeneous half-space such as the isotropic Mogi model (Mogi 1958) or the triaxial ellipsoid model (Davis 1986). Few studies have shown, however, that the retrieved source parameters are strongly
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biased if the source finiteness, surface topography and elastic heterogeneities of the underground are neglected (e.g., Mc Tigue 1987; Yang et al. 1988; Cayol and Cornet 1998; Trasatti et al. 2005; Manconi et al. 2007). The inflation observed at Mt Etna during 1993-1997 has been studied by Lanari et al. (1998); Lundgren et al. (2003); Puglisi and Bonforte (2004); Bonaccorso et al. (2005); Palano et al. (2008); Trasatti et al. (2008). The additional source of deformation due to flank instability has been modelled by Lundgren et al. (2003); Puglisi and Bonforte (2004); Palano et al. (2008) using models of sub-horizontal dip-slip faults based on the elastic dislocation theory. Moreover, Walter et al. (2005) studied the interaction between magma emplacement and eruptions at Mt Etna, testing the consistency between the temporal evolution of flank instability and static stress field changes. However, the assumptions of homogeneous elastic medium, of a priori defined inflation source shape and the large number of parameters required to describe multiple sources may deeply affect the results of these investigations.

The present work addresses the simultaneous effect of both the inflation source and the sliding of SE flank, following the approach outlined in Trasatti et al. (2008). An inversion scheme is adopted, employing a realistic deformation model based on the finite element (FE) method, which takes into account topography and heterogeneous elastic structure. Including these features is suggested by the asymmetry found in the deformation that might, in principle, be ascribed to the asymmetric shape of the edifice (with a prominent mass deficit in the eastern flank) and/or to the internal structure imaged by seismic tomography, evidencing a high rigidity body below the SE sector (Chiarabba et al. 2000).

The inflation source is represented by a general moment tensor, so that the source geometry is not prescribed a priori in terms of a pressurized cavity with pre-assigned shape (e.g., a sphere, an ellipsoid, a sill or a dike). The sliding of the SE flank is described by a simple kinematic rigid translation of a defined sector. In particular, we employ GPS and EDM measurements and we take advantage of the large spatial coverage provided by Differential Synthetic Aperture Radar Interferometry (DInSAR) data acquired during the 1993-97 time period.
Mount Etna had experienced a continuous inflation since the 1991-1993 strong eruptions. This recharging phase was detected by several geodetic techniques. In particular, we adopt geodetic data recorded between 1993 and 1997, consisting of 20 GPS (with errors ranging between 3 mm and 17 mm) and 147 EDM measurements (with formal uncertainty of 1 cm). These datasets have already been used by Bonaccorso et al. (2005) and Trasatti et al. (2008) to perform inversions of analytical and numerical models, respectively. Fig. 1b shows the GPS velocities and the coverage of the EDM network (translucent yellow areas). Although the GPS sites are not uniformly distributed over the volcano, the observed deformation is characterized by a radial pattern with increasing amplitude in the eastern sector of the volcano with respect to the western part: geodetic sites located along the coast move faster than those closer to the volcano summit (e.g., Borgia et al. 1992).

This evidence is also strongly supported by independent ascending and descending DInSAR data, which are processed via the Small BAseline Subset (SBAS) algorithm (Berardino et al. 2002), that permits to generate mean deformation velocity maps and the corresponding time series in the sensor line of sight (LOS). These data, due to their large and homogeneous surface coverage in zones where GPS and EDM data are lacking, constitute a valid complement to understand involved deformation mechanisms. The SBAS dataset has been processed and published by Lundgren et al. (2004). By combining the ascending and descending mean velocity maps in correspondence to the common pixels, we discriminate the E-W and vertical velocity components which we convert to the displacement maps shown in Fig. 1c, d, respectively. The obtained DInSAR dataset is composed of 6863 pixels. The error associated with the SBAS-DInSAR mean velocity is of about 2 mm/year, corresponding to 8 mm for the ascending, descending and vertical components of the displacement, while it is about twice for the E-W component (Casu et al. 2006). DInSAR data generally show eastward and downward movement of the eastern flank of the volcano.
3 MODELLING

We perform inversions of data described in Section 2 focusing on two different models: INFL (INFLation), in which the deformation is caused by a moment tensor applied to a single element of the FE grid; INSL (INflation and SLiding), in which an additional rigid translation of the eastern flank of Mt Etna is included. With respect to Trasatti et al. (2008), we have the following twofold aim: on one side to strengthen the source inference using the DInSAR dataset; on the other side to understand how the sliding process affects the inferred source parameters.

The inversion method used is the same described and tested in Trasatti et al. (2008) and briefly outlined as follows: i) a FE model of the medium is designed taking into account the topography of the volcanic edifice and the elastic heterogeneities derived from seismic tomography (Chiarabba et al. 2000). The shear modulus is assigned to each FE, as computed from the $v_p$ value interpolated at the centroid. A reference density of 2500 km m$^{-3}$ and a Poisson ratio of 0.25 are used. The whole computational domain spans a volume of $140 \times 140 \times 60$ km$^3$, discretized into $\sim 150,000$ isoparametric 8-node brick elements; ii) the central volume below Mt Etna is discretized into regular cubic elements of length $l = 400$ m; a subset extending $8 \times 8 \times 8$ km$^3$ is assumed to include the potential sources of deformation; iii) the ground deformation, caused by unitary perturbation of the tractions $\sigma_{ij}$ on the opposite faces of each potential source, is computed and stored into a library. This library consists of 48,000 entries for each observation point at the surface; iv) linear combinations of these elementary solutions are used as forward models for the Neighbourhood Algorithm (Sambridge 1999a), performing a direct search of the parameter space. The ensemble of models is then evaluated by Bayesian inference to determine the statistical distribution of the inverted parameters (Sambridge 1999b). With this approach we recover the best fitting and the most statistically significant position of the source and its moment tensor $M_{ij} = l^3 \sigma_{ij}$. Finally, we show that the retrieved moment tensor can be approximately interpreted in terms of a pressurized ellipsoidal cavity (Davis 1986), even if such an interpretation is not always possible.

The inversion is constrained by geodetic data discussed in Section 2. For EDM and GPS measurements, the FE predictions are computed at the geodetic sites, while the resolution of the FE grid (400 m at best, in the central area of Mt Etna) is coarser than the coverage of DInSAR pixels.
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The DInSAR dataset is down-sampled using a bilinear interpolation to deal with only one value within each FE face at surface. With this assumption, the DInSAR data effectively used in the inversion are reduced to 1141 points out of the 6863 original pixels. Different interpolation rules yield negligible differences on the re-sampled DInSAR results.

The second model INS1 takes into account the sliding on the SE flank. A few models of flank sliding based on the dislocation theory have been published (Lundgren et al. 2003; Puglisi and Bonforte 2004; Walter et al. 2005; Palano et al. 2008), but here we propose a simple kinematic model, consisting in a rigid translation of a specific area, shown in Fig. 1c, d by the dashed line. The deformation of this area is computed as the sum of the displacements caused by the inflation source and a uniform sliding vector whose components are three new parameters included in the inversion procedure. The rigid sliding is only a first order approximation of the deformation occurring in this area, which may be due to several active tectonic structures accommodating the flank instability in a much more complex pattern. However, this simple assumption allows us estimating the effects of the sliding with only three new unknowns. A mechanical decoupling model would be more appropriate, but it would require additional speculations about the geometry of the detachment surface, its extent at depth, the frictional properties of the contact surfaces, and the rheology of the rocks above and below this discontinuity. In this case, the large number of free parameters prevents the possibility to include such a model in the framework of our inverse approach. Moreover, the present knowledge of the internal structure below the SE flank is not sufficient to constrain any of these unknowns. By considering the kinematic description of the SE flank sliding we have, however, to delimit the sliding sector which is, to some extent, an arbitrary task. Our choice is based both on the geodetic data considered and the mapped faults; this avoids any further assumption about decoupling structures at depth. The northern border of the sliding sector corresponds to the NE Rift and to the surface trace of the PFS. In this case, the activity of these structures is corroborated by the GPS and DInSAR data (Fig. 1). The definition of the S and W borders of the sliding sector is much more problematic: our first choice was to follow the strike of S Rift (summit area) and the TF (flank area), respectively. However, after a trial and error process aimed to match the active deformation evidenced by the DInSAR data, a better solution is found
extending the sliding area W of the S Rift at mid altitudes and toward the TFS at low altitudes. We acknowledge that multiple sliding blocks have been proposed (separated by the SV-TFS, e.g., Neri et al. 2004; Puglisi and Bonforte 2004; Walter et al. 2005) but, again, the increasing number of parameters, required to model two or more separate blocks, could not be eligible for the resolving power of the dataset considered.

4 RESULTS

Fig. 2 shows the Posterior Probability Density functions (PPD, see Sambridge 1999b) for parameters of models INFL (red solid line) and INSL (blue solid line), respectively. The vertical dashed lines show the mean value of each probability distribution and identify the most likely parameter values, summarized in Table 1. The width of the PPD curves can be considered as a measure of parameter uncertainties: source position (Fig. 2a-c) is well constrained in both models. In detail, the INFL source is located ∼ 1100 m NW with respect to model INSL. A possible explanation for this difference is that model INFL tries to reproduce simultaneously the large horizontal displacement and the negative vertical displacement recorded in the eastern flank by the DInSAR, moving the source NW to lower the vertical and to increase the horizontal deformation. In opposition, model INSL reproduces the subsidence of the eastern sector with the additional negative vertical displacement provided by the sliding vector. Source depth is practically the same for both models, which means that the correction to the uplift pattern due to sliding in case of model INSL does not affect significantly this parameter.

In Fig. 2d-i, the moment tensor $M_{ij} = l^3 \sigma_{ij}$ is analyzed in terms of principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$) and orientations of the corresponding eigenvectors $\hat{v}_1$, $\hat{v}_2$, $\hat{v}_3$, respectively. The orientation of the eigenvectors is described splitting the complete rotation of a Cartesian coordinate system into three rotations about single axes of the system (in analogy with Euler’s angles). Supposing that initially $\hat{v}_1$, $\hat{v}_2$, $\hat{v}_3$ are oriented along the x, y, z-axes (corresponding to E, N, up, respectively), we compute a first rotation about $z \equiv \hat{v}_3$ by an angle $\phi$ (strike angle of $\sigma_3$), then a second rotation by an angle $\delta$ about $\hat{v}_1$ (dip angle of $\sigma_3$) and finally a third rotation by an angle $\vartheta$ about $\hat{v}_3$ (self-rotation angle of $\sigma_3$).
Principal stress values are well constrained in both models. For INFL, $\sigma_1$ is much larger than $\sigma_2$ and almost twice $\sigma_3$. For INSL, instead, $\sigma_1$, $\sigma_2$ and $\sigma_3$ are closer to each other and $\sigma_1 - \sigma_2 \simeq \sigma_2 - \sigma_3$. Stress intensities for model INSL are smaller than for INFL because the deformation is partially accommodated by the sliding and this yields an overall reduction of the strength of inflation. For models INFL and INSL, the dip angles $\delta$ of $\sigma_3$ are 66° and 47°, respectively, while the orientation is, in both cases, NW-SE, being $\phi \simeq 145°$. The self-rotation angles $\vartheta$ exhibit two different patterns. While for model INFL we find a unimodal distribution around $\vartheta = 24°$, for model INSL we retrieve a bimodal distribution with two relative maxima around $\vartheta = \pm 45°$. This difference is probably due to the fact that for model INSL the ratio $\sigma_1/\sigma_2$ is closer to 1 than for model INFL: this could indicate an approximate axial symmetry about $\hat{v}_3$ which cannot be resolved by the inversion. In Fig. 2j-l the PPD distributions for $L_x$, $L_y$ and $L_z$ define a rigid translation vector of 6.3 cm oriented toward SE. This displacement, averaged over the 4 years, corresponds to a rate of 1.6 cm/yr, a value compatible with the slip rate on the PFS recorded during quiescent/recharge phases (Neri et al. 2004).

Our next goal is to image the inflation source responsible of the observed deformation. Following Davis (1986), we try to interpret the source moment tensor in terms of a triaxial point-like ellipsoidal cavity with volume $V$ and internal overpressure $\Delta p$, which can be described by an equivalent system of double forces and double couples. The ellipsoid orientation is directly related to the principal stress axes $\hat{v}_i$ orientation. Furthermore, the axes of the ellipsoid ($a > b > c$) are inversely related to the principal stresses $\sigma_i$: the semi-major axis $a$ is associated to the minimum stress $\sigma_3$, and so on. Once the ellipsoid orientation is assigned, Davis’ solution describes the ellipsoidal source in terms of its strength $V \Delta p$ and the two ratios $b/a$ and $c/a$. We point out that our inversion does not prescribe any a priori constraint on the principal stress ratios since only upper and lower bounds for each stress component are fixed. As a consequence, the inverted stress tensor describes a general point source but its unambiguous interpretation as a pressurized cavity is not always possible. An additional drawback is that the analytical expressions provided by Davis (1986) allow us to compute the moment eigenvalues $M_1, M_2, M_3$ knowing $a, b, c$, but contain elliptic integrals that cannot be backward substituted.
To find the best fitting source, we perform a grid-search over the semi-axes ratios $b/a$ vs $c/a$ and we compute the misfit function

$$E = \frac{1}{2} \sqrt{\left( \frac{M_3}{M_1} - \frac{M'_3}{M'_1} \right)^2 + \left( \frac{M_2}{M_1} - \frac{M'_2}{M'_1} \right)^2}$$

where $M'_i = l^3 \sigma_i$ ($i = 1, 2, 3$), $\sigma_i$ are the mean values obtained from PPD functions for models INFL and INSL (Table 1) and $M_1, M_2, M_3$ are calculated for a pressurized ellipsoidal cavity from Davis (1986). If an ellipsoidal cavity exists, which provides the same moment tensor inferred from the observed deformation, $E$ vanishes for the appropriate values of the ratios $b/a$ and $c/a$.

Fig. 3 shows, in a logarithmic scale, $E$ as a function of $M_3/M_1$ vs $M_2/M_1$ and $b/a$ vs $c/a$, respectively. Before going into model details, it is interesting to note that among all the possible combinations of stress ratios only those enclosed within colored areas in Fig. 3a, c are compatible with ellipsoidal cavities. The moment ratios inferred from our inversions, depicted by crosses in Fig. 3a, c, are very close but they do not lie within the filled areas: for this reason they can be considered only approximately as ellipsoidal cavities and furthermore the ratios $b/a$ and $c/a$ are ill resolvable. This is shown in Fig. 3b, d, where the low $E$ zones are elongated, suggesting a linear trade-off between $b/a$ and $c/a$ that can be used to constrain only the $b/c$ ratio. For model INFL, acceptable values of $b/a$ and $c/a$ are in the ranges 0.05-0.3 and 0.05-0.15, respectively, and $b \simeq 2c$. For model INSL, $b/a$ and $c/a$ are in the ranges 0.05-0.2 and 0.05-0.2, respectively, and $b \simeq c$: this confirms that model INSL is characterized by a large degree of axial symmetry around the semi-major axis $a$.

A further analysis of retrieved source properties can be carried out computing the associated volume change $\Delta V$ and overpressure $\Delta p$. The source volume change can be computed from the moment tensor as follows:

$$\Delta V = \frac{M'_1 + M'_2 + M'_3}{3(\lambda + 2\mu)}$$

where $\lambda, \mu$ are the Lamé constants. Note that this value is the actual $\Delta V$ of the element-source and it scales linearly with the initial volume $l^3$. The volume variation of the INSL source is reduced by 20% with respect to model INFL, as suggested by the 20% reduction of the trace of the best fitting tensor $\sigma_{kk} = \sigma_1 + \sigma_2 + \sigma_3$ (see Table 1). As pointed out before, this is due to the sliding
vector accommodating part of the deformation. The relationship between the real dimensions of
the sources and the overpressure acting inside them is discussed in Trasatti et al. (2008). The ratio
between the overpressure and the maximum principal stress $\Delta P/\sigma_1$ can be calculated, following
Davis (1986), knowing the principal stress ratios $\sigma_3/\sigma_1$ and $\sigma_2/\sigma_1$. Assuming a source volume
$V \sim 3 \text{km}^3$ (Bonaccorso et al. 2005), we obtain $\Delta P = 25 \text{ MPa}$ for model INFL and 20 MPa for
model INSL. Note that the $\Delta P$ reduction between INFL and INSL amounts to 20%, according to
the lowering of $\Delta V$.

To complete this section, we finally compare models INFL and INSL showing the residuals, i.e.
theoretical minus observed displacements, in Fig. 4. GPS residuals of model INSL are, in general,
lower than model INFL. This is particularly true for the vertical component (Fig. 4c), where INSL
residuals are mostly very low. EDM elongations (Fig. 4b) are more difficult to evaluate since
in some cases the fit improves enormously, while in others it worsens badly. The most relevant
differences are found in the DInSAR residuals (Fig. 4d, f, for model INFL and Fig. 4e, g, for
model INSL). The fit largely improves when the sliding is included, considering that the yellow
pixels indicate that data and predictions are within error bars. DInSAR residuals strongly reduce
inside the sliding area, but also in the western side of the volcano, suggesting that, in this case,
the inflation source is better constrained. We can quantitatively evaluate model performances by
analyzing the misfit function reduction of each dataset. In the inversion procedure the evaluation
function is computed as a weighted average of the reduced $\chi^2$ of the three dataset (weight is
1/3), obtaining 9.8 for model INFL and 6.2 for INSL. Although the null tests of single datasets
emphasize the relevant role of GPS data, being 55.5 against 37.2 of EDM and 26.2 of DInSAR, the
$\chi^2$ reduction attests the importance of DInSAR data. For models INFL and INSL, respectively, the
$\chi^2$ reduces by 70% and 82% for GPS, 84% and 85% for EDM, 73% and 87% for DInSAR, so that
this dataset is the best fitted among the others. Finally, it is reasonable asking if the improved fit
of INSL with respect to INFL is significant, or it may be casually related to the increased number
of free parameters. We perform the $F$-test (Fisher 1922) for the two models, obtaining $F = \chi^2_{\text{INFL}}/\chi^2_{\text{INSL}} = 1.586$. This value is larger than the $F^c$ critical corresponding to the degrees of
freedom of the two models (total number of data points minus the number of unknowns), being $F^c$
= 1.068 (95% confidence). We can reasonably state that the $\chi^2$ reduction is a result of a significant difference between the models.

5 CONCLUSIONS

In this paper we extend the analysis of the dynamics of Mt Etna during 1993-1997. In particular, this study is complementary to Trasatti et al. (2008), improving the observational constraints with the large spatial coverage SBAS-DInSAR dataset. Additionally, it accounts for the sliding of the SE flank of Mt Etna with a simple rigid translation of a suitably defined sliding sector to estimate its influence on the retrieved source parameters.

The main topic we want to discuss is the role of extended high quality data, such as the DInSAR dataset, on source inference at Mt Etna. We consider results from model HET of Trasatti et al. (2008), shown in Table 1 for comparison; this model was characterized by the same topography and elastic heterogeneities of model INFL. The inversion setting differs only for the dataset adopted: optimization for HET is performed in a reduced dataset consisting of GPS and EDM measurements, over the same time period. If we compare INFL source to model HET, we find larger differences than those emerging from models INSL and INFL. Indeed, we observe that the source position for model HET is located $\sim 1800$ m SE with respect to INFL source and it is slightly shallower while the coordinates of the INSL source are $\sim 1100$ m SE of INFL at the same depth. This large discrepancy between source locations is due to the deformation recorded by DInSAR data, (negative vertical displacement affecting a large extent of the eastern flank of Mt Etna, as reported in Fig. 1d), a signal scarcely evidenced by the sparse EDM and GPS networks. Trying to match this additional information, the inversion performed with model INFL retrieves a source characterized by a high dip angle $\delta = 66^\circ$ similar to model HET $\delta = 62^\circ$ but located NW with respect to model HET. The shift of the source position and its more vertical dip translate into an increased ratio of the predicted horizontal vs vertical components of surface displacement. Moreover, comparing the traces of the stress tensors for models HET and INFL, we find that larger stresses are needed by model HET to fit the available data: the resulting $\Delta V$ is 13% larger than the corresponding INFL volume change. If part of the deformation is accommodated by the sliding
of the E flank of the volcano (model INSL), the volume change is further reduced by 20% with respect to INFL. The modelled sliding-rate vector amounts to 1.6 cm/yr, in agreement with the slip-rate on the PFS recorded during volcanic quiescent period.

Another important issue raised in the present paper concerns the interpretation of the retrieved moment tensor in terms of a small pressurized cavity of ellipsoidal shape, following Davis (1986). We show that the implemented inversion procedure provides a generalized moment tensor that could not be univocally interpreted as an ellipsoidal cavity. Indeed, only few combination of principal moment tensor ratios are compatible with a pressurized ellipsoidal cavity and the moment tensors retrieved for both models INFL and INSL are very close but do not fall within the domain pertinent to ellipsoidal cavities. Furthermore, the ratios $b/a$ and $c/a$ between different semi-axes are ill constrained: as a consequence we cannot identify unambiguously the source shape.

Finally, while Trasatti et al. (2008) evidenced the importance to implement models characterized by realistic features such as topographic relief and crustal heterogeneities, in the present work we stress the relevant role of the high quality and uniform distribution of data. Both these factors (more detailed description of the medium and wider data coverage) allow addressing with greater confidence the study of complex or multiple deformation sources. Last, but not least, we would like to emphasize that an exhaustive search of the parameter space must include a Bayesian inference of the generated ensemble of models to determine the statistical significance of parameter distributions.

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Table 1. Results of the Bayesian inference following the Neighborhood Algorithm inversion for models INFL (only inflation) and INSL (inflation and sliding). For symbol details see Fig. 2. Results of model HET from Trasatti et al. (2008) are shown for comparison.

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Bayesian Source Inference at Mt Etna

Figure 1. Geodetic data collected between 1993 and 1997. (a) Simplified structural map of Mt Etna. VdB Valle del Bove; PFS Pernicana Fault System; TFS Timpe Fault System; RN Ripe della Naca Faults; SV Santa Venerina Fault; TF Trecastagni Fault; RF Ragalna Fault. Redrawn from Neri et al. (2004). (b) GPS horizontal vectors and associated errors. The translucent yellow areas identify the EDM networks. DInSAR data are shown as a combination of ascending and descending orbits and converted into (c) E-W and (d) vertical displacement components. The dashed line indicates the sector supposed to be subjected to flank instability.
Figure 2. Posterior Probability Density (PPD) distributions (red line for model INFL, blue line for model INSL) of the inverted parameters: source position \((S_x, S_y, S_z)\); principal stresses \((\sigma_1, \sigma_2, \sigma_3)\); Euler angles (see text for details): \(\delta\), dip angle of the eigenvector \(\hat{v}_3\) of \(\sigma_3\); \(\phi\), strike angle of \(\hat{v}_3\); \(\vartheta\), self-rotation, i.e. rotation of \(\hat{v}_1\) and \(\hat{v}_2\) around \(\hat{v}_3\); sliding vector \((L_x, L_y, L_z)\), only for model INSL. The mean values of each distribution are indicated with the dashed vertical lines and are summarized in Table 1.
Figure 3. $\log_{10}$ Misfit $E$ as a function of $M_2/M_1$ vs $M_3/M_1$ (a, c) and $b/a$ vs $c/a$ (b, d) for models INFL and INSL, respectively. The crosses mark the INFL and INSL principal moment tensor ratios retrieved by the NAB inversion and shown in Table 1.
Figure 4. Comparison between best fit models and observed deformation. (a) GPS horizontal residuals (i.e. predicted minus observed deformation). The squares show source positions while the green vector is the sliding vector retrieved with model INSL. (b) EDM elongations: the solid line is the perfect agreement between observations and predictions; the dashed lines are the associated measurement errors. (c) Residuals of the vertical GPS displacements. Residuals of the (d, e) E-W and (f, g) vertical DInSAR data for models INFL and INSL, respectively. The black polyline delimits the sliding sector of Mt Etna.