Modal decomposition of magnetic maps: the case of Cape Roberts aeromagnetic survey, Antarctica.

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Abstract

This paper proposes a digital enhancement tool for magnetic anomaly maps. The magnetic anomaly map is decomposed by means of Singular Value Decomposition into a number of orthogonal basis. The dataset is then filtered, after eigenvalues evaluation, accordingly to a specific variance pattern. The filtering product is efficient in enhancing subtle and hidden features. The proposed filtering procedure is first tested with a synthetic signal, then it has been applied to the Cape Roberts (Antarctica) aeromagnetic survey, flown over an off-shore rift basin. The proposed method appears to be efficient as noise removal and act as a digital enhancement tools able to provide TMI anomaly maps which can reveal hidden lineaments, otherwise not visible. The results have been checked against independent data.

Key words: Magnetic maps production, Digital Enhancement, SVD, HRAM, Antarctic Magnetic Surveys
Introduction

Singular value decomposition (SVD) is a computational technique widely used in many geophysical fields such as oceanographic (Fukumori and Wunsch 1991) seismic (Freire and Ulrych 1988) deep electromagnetic soundings (Egbert and Booker 1989) and also in image processing (Prasantha et al. 2007). In the last few decades this technique was extensively used in image coding and compression applied to image transmission over nationwide computers network. This work explores the feasibility of the SVD as a separation tool between signal and noise, used with magnetic maps. In detail, the magnetic image is decomposed and then represented into a number of orthogonal basis thereby obtaining a new image formed by fewer dimensions. By means of this technique it is possible to separate the meaningful signal from the noise. This decomposition seems to be efficient in noise removal and acts as an enhancement technique able to highlight hidden features. Due to the intrinsic implementation simplicity the procedure can also be used as an interactive tool.

Singular Value Decomposition

SVD is a over a hundred years old matrix factorization technique; discovered for square matrix independently by Beltrami in 1873 and Jordan in 1874. Eckart and Young theorem (Eckart and Young 1936) extends the technique to rectangular matrix. The decomposition of a rectangular matrix $X \ [m \times n]$ can be expressed as follows:

$$X = U S V'$$

where $U [m \times m]$ and $V [n \times n]$ are two orthogonal matrices and $S$ is an $[m \times n]$ diagonal matrix. $U$ columns are eigenvectors of $XX'$ and represent the singular vectors of $X$ spanning the column space; the column of $V$ are eigenvectors of $X'X$ and represent the singular vectors of $X$, spanning the row space. The matrix $S$ is diagonal and holds the eigenvalues of $X$. If $k$ diagonal elements of $S$ are null the dimension of $X$ becomes $p = n-k$, thus the matrices of decomposition get the dimension $U[m \times p]$, $S[p \times p]$, $V[p \times n]$. In actual cases the sorted set $S = \{S_{11},..,S_{22},..S_{nn}\}$ of the diagonal elements of $S$ shows a general character with nearly vanishing component as $i$ increases. Retaining
$k$ elements of $S$, with $k << n$ we can reduce the dimensionality of data ($X$) and thus enhance subtle features hidden by the noise. Since each element $S_{ii}$ explains a portion of the total variance of $X$, the definition of a modal band with $1 \leq \beta_1 \leq \beta_2 \leq n$ allows variance driven filtering technique, analogous with spectral band filtering. The reconstructed $\lambda_c$, retained $\lambda_r$, and residual $\lambda_r$ dataset can be extracted using $S_{ii}$ elements accordingly with:

$$\lambda_c = \{S_{ii}: i < \beta_1\}; \lambda_r = \{S_{ii}: \beta_1 \leq i \leq \beta_2\}; \lambda_r = \{S_{ii}: i > \beta_2\}$$

Application

In order to check the performance of the decomposition, a square 200x200 px image matrix has been created. The input signal is composed by a large ($\mu = 0$; range=270) perturbated dipolar anomaly, some elongated features and a normal random noise ($\mu = 0$; $\sigma = 3$). Figure 1 shows the modal decomposition of a synthetic signal using $\beta_1 = 3$ and $\beta_2 = 10$. The reconstructed dataset singles out the main features both from the perturbation, visible in the retained dataset, and from the incoherent noise, which is confined in the residual dataset.

In order to perform a test of the proposed method the dataset of the HRAM survey performed in the Cape Roberts area (Victoria Land, Antarctica) was used. The choice is supported by the fact that Cape Roberts is a key area to address in further detail the geometry and kinematics of strike-slip faulting and its control on Cenozoic magmatism at the boundary between the Transantarctic Mountains and the Ross Sea Rift. (Ferraccioli & Bozzo 2003)

The CRRB HRAM (Cape Roberts Rift Basin High Resolution Aero Magnetic) survey was performed in the framework of the Cape Roberts Drilling Project (Davey et al. 2001). This was the first HRAM helicopter borne survey in Antarctica performed with towed caesium magnetometer, sampling at 10 Hz with DGPS positioning (Bozzo et al 1997). The survey was carried out in November 1994 during the GITARA IV (German Italian Aeromagnetic Research in Antarctica) campaign, at fixed altitude (150 m a.s.l.) with a line spacing of 500m, covering over 900 km$^2$. 
The CRRB is a V-shaped trough about 500 m deep, interpreted as a graben, bounded to the east by a bathymetric high (~ 100 m bsl) known as the Roberts Ridge (RR), and, to the west, by the foothills of the Transantarctic Mountains (TAM). The CRRB startigraphy is provided both by seismic cruises (Behrendt et al. 1987, Hamilton et al. 2001) and three drillings (CRP1, CRP2, CRP3) located along the eastern flank of the graben. The graben is truncated to the north by the Pleistocene trough of the McKay sea valley, about 800 m deep.

The CRRB is filled with a complex sedimentary sequences ranging from latest Eocene to Miocene, which have been coded after Cooper (Cooper et al, 1987). Six seismic stratigraphic units (V1 – V5 sedimentary and V6 volcanic) and an acoustic basement (V7) are known in the CRRB-RR area (Davey et al. 2001). Magnetic susceptibility measurements have been carried out giving mean \( k \) values for the stratigraphic units filling the pull-apart basin (after Sagnotti et al 2001). Table 1 reports the \( k \) values.

<table>
<thead>
<tr>
<th>Depth range [m b.s.f.]</th>
<th>Mean ( k ) [10^{-5} \text{ SI}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-243</td>
<td>270</td>
</tr>
<tr>
<td>243-440</td>
<td>132</td>
</tr>
<tr>
<td>440-627</td>
<td>40</td>
</tr>
<tr>
<td>627-790</td>
<td>223</td>
</tr>
</tbody>
</table>

Table 1: Mean \( k \) values of CRP3 drill hole (Fig 3).

The acoustic basement is composed of Beacon supergroup sandstone with mean \( k \) \( 1 \times 10^{-5} \) SI

The current shape and size of CRRB has been inferred (Ferraccioli and Bozzo 2003) relying on seismic data (Behrendt et al. 1987, Hamilton et al. 2001) combined with the aeromagnetic survey results.

The modal decomposition proposed revises the aeromagnetic data. The leveled and micro leveled (Ferraccioli et al. 1998) TMI anomaly draped (Armadillo et al. 2007) map has been used as input for the proposed method. The decomposition has been performed using \( \beta_1 = 3; \beta_2 = 14 \).
Figure 3 shows in the upper panel TMA (Total Magnetic Anomaly) and Tilt Derivatives (Verduzco et al. 2004) data along a profile crossing the CRRB. The schematic geological cross section has been redrawn after Hamilton (2001). The TDR profile data calculated using the “retained” dataset (red line) shows a prominent maximum (M) corresponding to the right basin flank, not marked by the TMI anomaly map. TDR performance is much more efficient if used on the “retained dataset” instead of the original TMI. I assume that the very low TM anomaly is due to the uplift of block 1 (fig 3) which places the high susceptibility sediments (V5) closer to the sea bottom. Since the V5 mean susceptibility is not high (mean $k \sim 220 \times 10^{-5}$ SI), the absolute value of the magnetic anomaly and the feature may not be visible using standard digital enhancement techniques.

The lower panel shows the original TMA map (A) and the TDR calculated using the original dataset (B). Box C show the modal decomposed (retained) map (rTMA) and the TDR calculated using the “retained” dataset (rTDR). This representation dramatically exhibits the efficiency of the method. The spatial persistence of low amplitude feature (M in fig 3 lower panel) has been singled out.

Figure 4 shows the interpretation of the CRRB boundary; the yellow dashed line reports the boundary proposed by Ferraccioli (Ferraccioli & Bozzo, 2003) and the solid black line draws the traced using the rTDR map. The decomposition shows also a number of intra-basin faults (dotted lines, fig 4) that seem to be the paths along which the known Cenozoic intrusions (Bozzo et al. 1997) took place. These lineaments were not visible from the original TMI anomaly map.

**Conclusions**

It has been shown that the proposed method is efficient in removing noise and when used in synergy with TDR digital enhancement is able to reveal concealed features: the general shape of CRRB has been confirmed and seismic lineaments have been matched with magnetic lineaments. A new set of magnetic lineaments it has also been highlighted. The proposed decomposition leads to TMI anomaly maps that may significantly assist magnetic anomalies interpretation. The implementation of the filtering procedure is quick and the calculus, relying on both open source and
commercial packages, requires very little resources. The empirical character of the filtering procedure requires a trial and err approach for a proper tuning of $\beta$ coefficients so, it is convenient implement the procedure as an interactive tool. As practical rule, the user can assume that then low amplitude noise is confined in the upper modes and the lower modes often carry broad features. Keeping in mind the method’s empirical nature, the results should be used only to infer (verify) the presence of magnetic lineaments. The modal decomposition works only on square/rectangular not sparse matrices.

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References


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Figure captions

Figure 1. Test of the modal decomposition with a controlled input

Figure 2. Location of Cape Roberts area. Red dashed line: McMurdo sound fault system

Figure 3. Modal decomposition CRRB HRAM. Above: TMI and tilt derivative profile data with a schematic geologic cross section (modified after Hamilton 2001). Below: a) Original TMI anomaly, b) Tilt derivatives calculated using the original data, c) Modal decomposition – “retained map” (rTMA), d) Tilt derivatives calculated using the “retained” dataset (rTDR)

Figure 4. The Cape Roberts Rift Basin inferred using the Tilt derivatives map calculated using the “retained” dataset. Yellow dashed line: basin boundary (Ferraccioli et al. 2003)