Probability distribution of the macroseismic intensity attenuation in the Italian volcanic districts

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We present the probabilistic version of the analysis performed in Azzaro et al. (2006a) on the attenuation of the seismic intensity in Italian volcanic districts. The main results are the estimate of the probability distribution of the intensity at site I_S, conditioned on the site-epicenter distance d and on I_0, and then, assuming the mode of this distribution as estimator of I_S, the forecasting of future macroseismic fields given I_0. To this end we have modified the method presented in Rotondi and Zonno (2004) by inserting the following innovative elements: identification of possible different trends and exploitation of knowledge from prior experience or data.

Data set. The intensity dataset considered in the present analysis is the same used in the study by Azzaro et al. (2006a), based on a deterministic approach. We consider a total of 38 earthquakes located in the Italian volcanic areas, so distributed: Etna region (24 events), Aeolian Islands (6 events), Vesuvius-Ischia (3 events) and Albani Hills (5 events). The CMTE local earthquake catalogue (Azzaro et al., 2000, 2002, 2006b) has been used for the Etna region while for the other Italian volcanic districts (Aeolian Islands, Ischia, Vesuvius and Albani Hills) the CPTI04 Italian seismic catalogue (Gruppo di lavoro CPTI, 2004) and the DBMI04 associated database (Stucchi et al., 2007) have been considered (Tab. 1). For the analysis, subsets of earthquakes with epicentral intensity I_0 ≥ VII MCS and I_0 ≥ VI MCS were used for the Etna region and for the other Italian volcanic districts, respectively.

Tab. 1 - Dataset of earthquakes located the Italian volcanic districts.

N° eqs	Area	I_0	EQ reference code	N° eqs	Area	I_0	EQ reference code
12	Mt. Etna	VII	19091021.70__7__ord	12	Mt. Etna	VII-VIII	19520319.75__20_ord
		VII	19730803.70__9__ord			VII-VIII	19200926.75__24_ord
		VII	19890129.70__10_ord			VIII	18650819.80__27_ord
		VII	19861029.70__11_ord			VIII	19841025.80__29_ord
		VII	19860202.70__12_ord			VIII	20021027.80__30_ord
		VII	19851225.70__13_ord			VIII	20021029.80__31_ord
		VII	19730818.70__14_ord			VIII	19710421.80__32_ord
		VII	19841019.70__15_ord			VIII-IX	18940808.85__33_ord
		VII	19840619.70__16_ord			VIII-IX	19111015.85__34_ord
		VII-VIII	19071207.75__17_ord			VIII-IX	18790617.85__35_ord
		VII-VIII	18980514.75__18_ord			IX	18650719.90__36_ord
		VII-VIII	18891225.75__19_ord			IX-X	19140508.95__38_ord
6	Aeolian Islands	VII	18941227.70__8__ord	5	Albani Hills	VII-VIII	18060826.75__22_ord
		VIII	18920316.80__28_ord			VI-VII	18761026.65__3__ord
		VI-VII	19160703.65__5__ord			VI-VII	18920122.65__4__ord
		VII-VIII	19260817.75__23_ord			VII	18990719.70__6__ord
		VII-VIII	19300326.75__21_ord			VII-VIII	19271226.75__25_ord
		VI	19950723.60__2__ord				
2	Vesuvius -Ischia	VIII	18810304.80__26_ord	1	Vesuvius -Ischia	VI	19990910.60__1__ord
		IX	18830728.90__37_ord				

Probability model. We cite here the key-elements of the probabilistic method, referring to Rotondi and Zonno (2004) for a detailed description. Instead of adding a gaussian error to deterministic relationships which express the ΔI intensity decay as a function of some factors (epicentral intensity, site-epicenter distance, depth, site types, and styles of faulting), we treat the decay as an *aleatory* variable defined on the domain $\{0, I_0\}$. Consequently, we assume that the intensity I_S is a discrete binomial distributed variable $Bin(I_0, p)$ where p^{I0} means the probability of null decay, and p belongs to [0,1]. According to the Bayesian approach, p is considered as a random variable following the beta distribution $Beta(\alpha, \beta)$. Since mean and variance of p are functions of the α, β hyperparameters, we can express our initial knowledge on the decay process through these parameters. To do this, we have divided each macroseismic field in bins of fixed width and the intensity data points in subsets according to this spatial subdivision. For each bin we have repeated the following procedure: a) assessing the prior values to α, β, that is a prior distribution for p; b) updating, through Bayes' theorem, the hyperparameters on the basis of the current observations; c) estimating the p parameter through the mean of its posterior distribution. By substituting this estimate in the distribution $Bin(I_0, p)$, we obtain an updated binomial distribution indicated as plug-in distribution. Its mode has been assumed as the expected value of the intensity at the sites within the corresponding bin. To predict the intensity at any distance we have smoothed the p's estimated in the different bins through a monotonically decreasing function; the lowest mean squared error was given by the inverse power function $g(d) = (\gamma_1/d)^{\gamma_2}$.
Hence, the mode of the plug-in distribution obtained by setting $p=g(d)$ provides an expected value for I_S at any distance. If, on the contrary, we assume that, from the attenuation viewpoint, the sites inside any bin behave in the same way, we can average over the domain [0,1] of p by integrating the product of the likelihood with respect to the posterior Beta distribution of p. In this way we have obtained the so-called *predictive* distribution for every bin and its mode is taken as expected value for I_S at any site inside that bin.

Trends in the intensity decay. We have analysed the macroseismic field of the 38 earthquakes constituting our dataset (Tab. 1) by drawing the decay versus the site-epicenter distance of each data point. A quick look at these graphical representations suggests that these earthquakes do not show an homogeneous decay. To identify different trends in the decay, we have synthetized the information contained in each field by collecting, in a matrix, median, mean, and 3^{rd} quartile of each set of distances from the epicenter of the points with the same ΔI. Then we have applied to this matrix a clustering algorithm based on the evaluation of the distance between each pair of rows of the matrix. The dataset has been thus partitioned into two groups of events according to their attenuation trend: the first set mainly formed by the earthquakes of Mt. Etna and Vesuvius-Ischia areas, the second one including the events of the Aeolian Islands and Albani Hills.
The set 1 shows an higher decay than the set 2, so two different spatial scales are required: bins of width 1 km for the set 1 and of width 25 km for the set 2. A similar classification analysis was performed in Zonno et al. (2008) on 55 earthquakes representative of the Italian territory; in that case three classes were identified.
The probabilistic analysis above described has been separately applied to the two sets, discriminating the events of $I_0 \le VII$ from those of $I_0 \ge VIII$, and using as *a priori* distributions for the parameters p's those indicated in Zonno et al. (2008) for the class of earthquakes with the highest attenuation. The hyperparameters α's and β's have been then updated through the observed intensity data points according to the expressions $\alpha=\alpha_0 + \Sigma^N_{n=1} I_S^{(n)}$ and $\beta= \beta_0 + \Sigma^N_{n=1} (I_0 - I_S^{(n)})$.

Some results. For each bin the values of the predictive probability function of I_S for the Etna area and Aeolian Islands, are shown in Fig. 1; the squares indicate the values of the intensity decay computed through the logarithmic regressions (Tab. 2) obtained by Azzaro et al. (2006) with the same dataset. These values can be compared with the mode of the predictive function in each bin.

Tab. 2 - Intensity decay vs distance obtained through the logarithmic regressions (from Azzaro et al., 2006).

Area	Logarithmic regression	R^2	Valid for d (km) ≥
Mt. Etna	ΔI = 0.98ln(d)+1.01	0.92	0.4
Aeolian Islands	ΔI = 1.28ln(d)-2.39	0.89	6.5

The fit between the two methods is good but much more information is provided by the probabilistic approach. In addition to the estimate of the intensity at any site, the probability distribution of I_S provides a measure of the uncertainty and its values can be directly used in the software “SASHA” (D’Amico and Albarello, 2007) to calculate the probabilistic seismic hazard at the site.

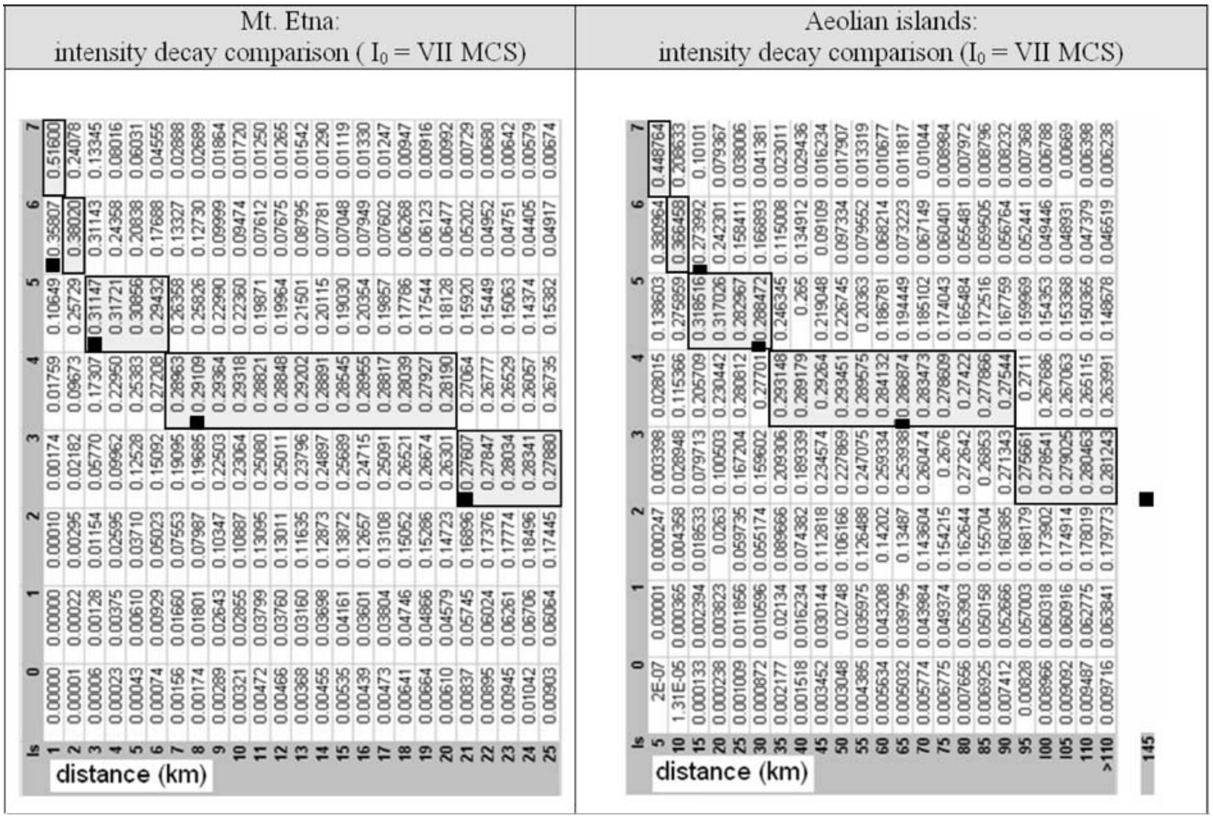

Fig. 1 - Values of the predictive probability function of I_S for the set 1 and set 2. The black squares show the intensity decay ΔI = 1,2,3 and 4 vs distance obtained by the deterministic methods for the Etna area and Aeolian Islands (Azzaro et al., 2006). The boxes in bold highlight the probability associated with the intensity at a site estimated in each bin (modified from Zonno et al., 2008).

Conclusions. The identification of different decay trends produced by the clustering algorithm matches well with that already presented in the literature (Azzaro et al. 2006), and this suggests that the method could be successfully applied to other cases. Only two earthquakes in Albani Hills - 1876/10/26, I_0 VI-VII, 1927/12/26, I_0 VII-VIII - are unexpectedly included in the set 1 together with the events of Mt. Etna and Vesuvio-Ischia areas; further, detailed analyses are required to explain such an anomaly.
Some problems are still open: a) most of the earthquakes here considered have epicentral intensity I_0 VII or VIII, so that we have evaluated the probability functions of I_S conditioned on these two values of I_0. Also other values of I_0 must be used in the analysis; b) the method should be also validated on other earthquakes not included in the dataset of Tab. 1, on the basis of probabilistic measures of the degree to which the model predicts the decay in the data points of a macroseismic field (Rotondi and Zonno, 2004).

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