

Probabilistic procedure to estimate the macroseismic intensity attenuation in the Italian volcanic districts

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ABSTRACT :

In this work we apply a probabilistic procedure to estimate the macroseismic intensity attenuation in the volcanic areas of Italy which allows to exploit additional information on historical earthquakes following the Bayesian approach. The method starts from the intensity data points of the selected earthquakes and arrives at the assessment of the probability distribution for the intensity at a site given the epicentral intensity and the site-epicenter distance. The CMTE local earthquake catalogue has been used for the Etna region while for the other Italian volcanic districts (Aeolian Islands, Ischia, Vesuvius and Albani Hills) the CPTI04 Italian seismic catalogue and the DBMI04 associated database have been considered. For the analysis, subsets of earthquakes with epicentral intensity $I_0 \geq VII$ MCS and $I_0 \geq VI$ MCS were used for the Etna region and for the other Italian volcanic districts, respectively. Only earthquakes with more than 10 felt observations have been considered. The results show a specific attenuation trend for the Etna region compared with the other Italian volcanic areas.

KEYWORDS: Attenuation, macroseismic intensity, probability of the intensity at a site, seismic hazard assessment, Italian volcanic districts

1. INTRODUCTION

The intensity attenuation and its variation as a function of the distance is still a critical problem in the seismic hazard assessment, especially in volcanic areas. In the latest years, this problem was analysed by both deterministic and probabilistic procedures. The deterministic approach was applied on both a national (Pasolini et al., 2007) and local scale in the Etna region and other Italian volcanic districts (Azzaro et al., 2006), deriving different regression relationships for the intensity attenuation from a wide set of intensity data. A probabilistic method was developed both on a local scale in central Italy (Rotondi and Zonno, 2004), and on a national one (Rotondi and Zonno, 2006). In this work we apply this probabilistic procedure to the volcanic areas in Italy obtaining first results. The method starts from the intensity data points of the selected earthquakes and arrives at the assessment of the probability distribution for the intensity at a site given the epicentral intensity and the site-epicenter distance. The semi-qualitative character of the intensity is emphasized by the application of the binomial distribution for the site intensity .

The CMTE local earthquake catalogue (Azzaro et al., 2000, 2002, 2006) has been used for the Etna region while for the other Italian volcanic districts (Aeolian Islands, Ischia, Vesuvius and Albani Hills) the Italian seismic catalogue CPTI04 and the associated database DBMI04 (Stucchi et al., 2007) have been considered. The analysis shows different attenuation trends for the Etna region compared with the other Italian volcanic areas. The method has been validated on some earthquakes by applying the predictive and the binomial probability function $\text{Bin}(I_0; g(d))$ for the intensity at a site, where $g(d)$ denotes an inverse power function of the d site-epicenter distance. Finally, the new probabilistic intensity attenuation evaluations obtained in this study have been compared with those previously derived for the same areas.

2. DATASET

The intensity dataset considered in the present analysis is the same of previous study by Azzaro et al. (2006) on the Italian volcanic districts. We consider a total of 38 earthquakes located in the Etna region (24 events) and other Italian volcanic areas: Aeolian Islands (6 events), Vesuvius-Ischia (3 events) and Albani Hills (5 events).

Table 1. Dataset of used earthquakes concerning the Italian volcanic districts

#	Name	I_0	EQ reference code	#	Name	I_0	EQ reference code
12	Etna area	VII	19091021.70_7_ord	12	Etna area	VII-VIII	19520319.75_20_ord
		VII	19730803.70_9_ord			VII-VIII	19200926.75_24_ord
		VII	19890129.70_10_ord			VIII	18650819.80_27_ord
		VII	19861029.70_11_ord			VIII	19841025.80_29_ord
		VII	19860202.70_12_ord			VIII	20021027.80_30_ord
		VII	19851225.70_13_ord			VIII	20021029.80_31_ord
		VII	19730818.70_14_ord			VIII	19710421.80_32_ord
		VII	19841019.70_15_ord			VIII-IX	18940808.85_33_ord
		VII	19840619.70_16_ord			VIII-IX	19111015.85_34_ord
		VII-VIII	19071207.75_17_ord			VIII-IX	18790617.85_35_ord
		VII-VIII	18980514.75_18_ord			IX	18650719.90_36_ord
		VII-VIII	18891225.75_19_ord			IX-X	19140508.95_38_ord
6	Aeolian Islands	VII	18941227.70_8_ord	5	Albani Hills	VII-VIII	18060826.75_22_ord
		VIII	18920316.80_28_ord			VI-VII	18761026.65_3_ord
		VI-VII	19160703.65_5_ord			VI-VII	18920122.65_4_ord
		VII-VIII	19260817.75_23_ord			VII	18990719.70_6_ord
		VII-VIII	19300326.75_21_ord			VII-VIII	19271226.75_25_ord
3	Vesuvius-Ischia	VI	19950723.60_2_ord				
		VIII	18810304.80_26_ord				
		IX	18830728.90_37_ord				
		VI	19990910.60_1_ord				

3. PROBABILISTIC METHOD

In order to assess the seismic hazard in terms of macroseismic intensity we have faced the problem of the intensity attenuation. Many studies on this topic have appeared in the literature; in most cases the key role is played by the deterministic function which expresses the link between the ΔI intensity decay and factors such as epicenter intensity, site-epicenter distance, depth, site types, and styles of faulting. In some cases a normally distributed random error is added to take into account the scatter of the observations around the site intensity value (I_s) predicted by the attenuation relationship. More emphasis is given to the uncertainty when the decay is considered as an *aleatory* variable: for the intensity decay normalized on I_0 , a Beta distribution with a mean proportional to an attenuation law and varying deviation, was first proposed by Zonno et al. (1995). On the other hand, a logistic model was proposed by Magri et al. (1994) to estimate the probability that the attenuation exceeds a given (i.e. threshold) value.

In this study we apply a complete probabilistic analysis of the attenuation issue, by integrating the aforementioned methods and avoiding the use of any deterministic attenuation relationship (Rotondi and Zonno, 2004). We provide the binomial distribution of the intensity at a site, conditioned on the I_0 epicentral intensity and d distance from the epicenter, for both discrete and continuous d values.

Since the variable ΔI is discrete and belongs to the domain $[0, I_0]$, it is reasonable to choose for $I_s = I_0 - \Delta I$, at a fixed distance, the binomial distribution $Bin(I_0, p)$ conditioned on I_0 and p :

$$Pr\{I_s = i \mid I_0 = i_0, p\} = Pr\{\Delta I = I_0 - i \mid I_0 = i_0, p\} = \binom{i_0}{i} p^i (1-p)^{i_0-i}$$

where $p \in [0, 1]$ and $i \in \{0, 1, \dots, i_0\}$. To account for the variability of the ground shaking even among sites located at the same distance, the parameter p has been considered as a Beta distributed random variable in the

Bayesian paradigm

$$Be(p; \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^p x^{\alpha-1} (1-x)^{\beta-1} dx$$

with mean and variance given respectively by:

$$E(p) = \frac{\alpha}{\alpha+\beta} \quad \sigma^2(p) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \quad (3.1)$$

The value of the prior hyperparameters α, β expresses our initial state of information on the decay process obtained by the examination of the earthquakes with an attenuation trend similar to that of the events under analysis. In Zonno et al. (2008), three different attenuation trends were identified for the Italian seismicity by partitioning a set of 55 earthquakes, representative of the Italian seismicity, into three classes $\mathcal{C}_A, \mathcal{C}_B, \mathcal{C}_C$ of decreasing attenuation (Fig. 1, on the right).

Let us draw L distance bins $\{R_1, R_2, \dots, R_L\}$ of width Δr around the epicenter of every earthquake of fixed I_0 intensity belong to the class \mathcal{C}_A , that is characterized by the quichest attenuation. Taking a sufficiently small step, we may assume that the decay process behaves at the same way within each R_j band, $j = 1, \dots, L$; moreover

we denote by $\mathcal{D}_{j,0} = \{i_s^{(n)}\}_{n=1}^{N_{j,0}}$ the set of $N_{j,0}$ felt intensities in the j -th band. As the probability of null decay ($I_s = I_0$) is $p_j^{I_0}$ in each band, we assign the initial mean value of p_j using simply the frequency of null decay,

$N_{j,0}(I_0)/N_{j,0}$ ($N_{j,0}(I_0)$ being the number of any sites in which no intensity decay has occurred), and deduce from this value the hyperparameters $\alpha_{j,0}$ and $\beta_{j,0}$ by inverting (1). Where there is no report of null decay, we have smoothed the valuable p_j 's through the function $f(d) = (c_1/d)^{c_2}$ and estimated the coefficients c_1, c_2 by the method of least squares. Now let us consider all the earthquakes of I_0 intensity in our dataset;

$\mathcal{D} = \bigcup_{j=1}^L \mathcal{D}_j = \bigcup_{j=1}^L \{i_s^{(n)}\}_{n=1}^{N_j}$ denote the set of their intensity data points, subdivided into L subsets.

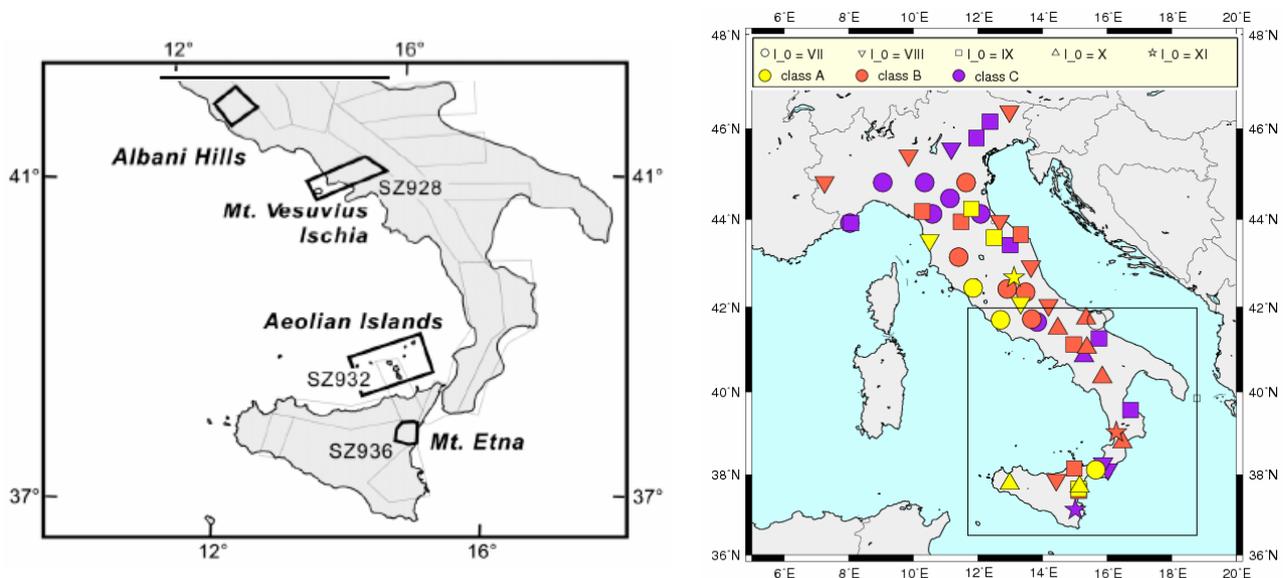


Figure 1 Left: location of the volcanic districts in the frame of the Italian seismogenic zoning (from Azzaro et al., 2006). Right: classification of 55 Italian earthquakes according to their attenuation trends (from Zonno et al., 2008).

On the basis of this new information we can update our knowledge on the attenuation process. We use Bayes' theorem to compute the posterior distribution $Be(p_i | \mathcal{D}_i)$, and estimate p_i through its posterior mean

$$\hat{p}_j = E(p_j | \mathcal{D}_j) = \frac{\alpha_{j,0} + \sum_{n=1}^{N_j} I_s^{(n)}}{\alpha_{j,0} + \beta_{j,0} + I_0 \cdot N_j}$$

where $I_s^{(n)}$ is the intensity felt at the n -th site inside the R_j band, N_j is the total number of data points assumed at d_j distance from the epicenter, and $\alpha_{j,0}, \beta_{j,0}$ are the parameters of the prior **Beta** distribution for p_j . In order to let the p parameter of the binomial distribution for the intensity I_s at a site may vary with continuity, we smooth the estimates $\hat{p}_j, j = 1, \dots, L$ with the method of least squares, again using an inverse power function $g(d) = (\gamma_1/d)^{\gamma_2}$. In this way it is possible to assign the probability of the intensity decay $Pr\{\Delta I | I_0, g(d)\}$ at any distance from the epicenter.

4. EVALUATION OF INTENSITY ATTENUATION

To summarize the information contained in each macroseismic field, we have chosen some measures of location and dispersion of each set for epicenter-site distances with the same ΔI : median, mean, and 3rd quartile. A graphical representation of one of these quantities is provided in Figure 2 for three of the 38 earthquakes in our dataset (Table 1).

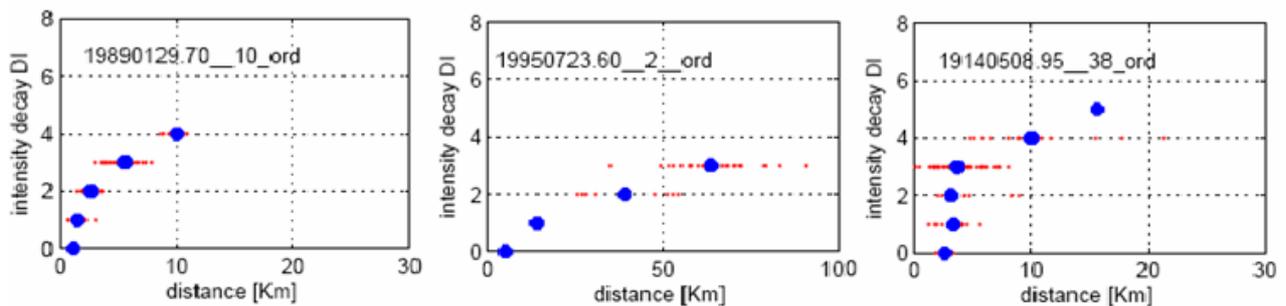


Figure 2 Example of different types of identified intensity decay. The red points indicate the epicentre-site distances vs intensity decay ΔI , while the blue dots represent the median values. On the top, the identification by year, month, day, intensity I_0 and reference number of the represented earthquakes.

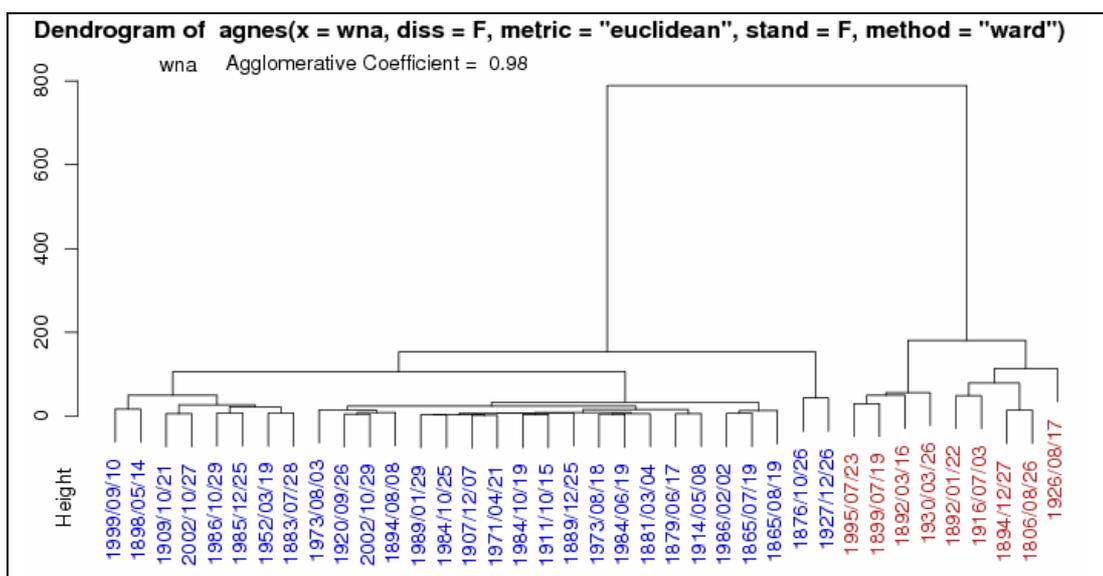


Figure 3 The 38 earthquakes of the dataset, subdivided into two clusters through the classification algorithm AGNES of the free package R: SET 1 (blue) and SET 2 (red).

We have collected all these new values together in a matrix of as many rows as the events listed in Table 1, and we have then applied on it a clustering algorithm (the AGNES routine of the free R library), based on the evaluation of the distance between each pair of rows of the matrix, in order to quantify their degree of dissimilarity. The result is depicted in Figure 3. The dataset has been partitioned into two groups of events according to their attenuation trends: the blue one mainly formed by the earthquakes of Mt. Etna and Vesuvius-Ischia areas, the red one formed by the earthquakes of the Aeolian Islands and Albani Hills. In the following we indicate the former set as SET 1 and the latter as SET 2.

The probabilistic analysis described in Section 3 has been applied separately to the two groups discriminating the events of $I_0 \leq VII$ from those of $I_0 \geq VIII$, and using as *a priori* distributions for the hyperparameters α 's and β 's those produced by Zonno et al. (2008) for the class of earthquakes with the highest attenuation. In this way we have estimated through their posterior means the parameters $p_j, j = 1, \dots, 25$, on 25 bins of width 1 km for the set (B) and 25 km for the set (R), and the corresponding smoothing inverse power functions $p = g(a)$ (see Section 3). These results are shown in Figure 4. The updated hyperparameters α 's and β 's were used to evaluate the predictive distributions and the plug-in binomial distributions of I_s in the different cases (Figure 5).

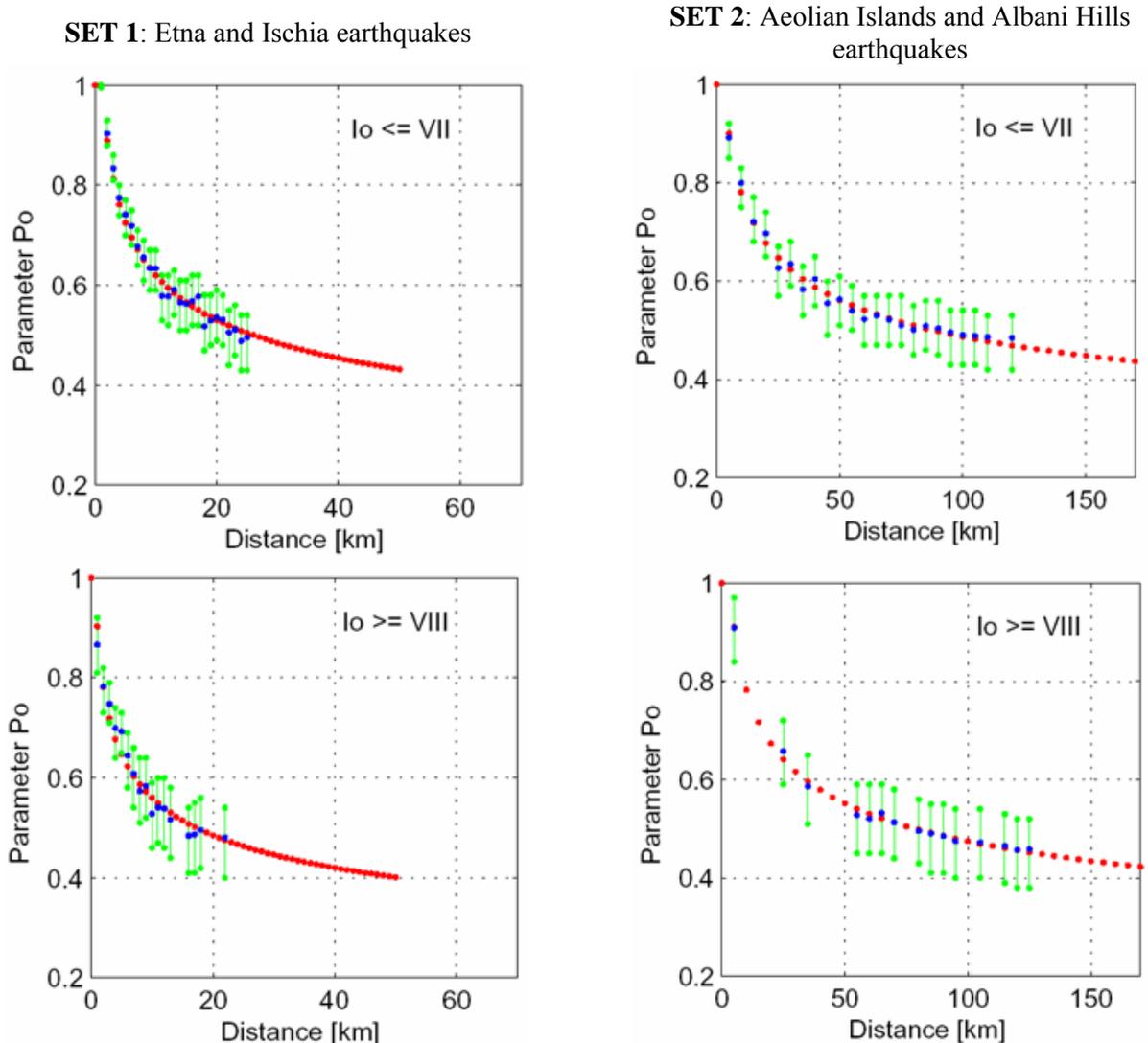


Figure 4 Estimates of p_j (blue dots, section 3) concerning the SET 1 and SET 2 (Fig. 3). 90% highest posterior density (HPD) intervals of the Beta distribution for each p_j (green bars). Smoothing inverse power function (red dots).

For each bin, the values of the predictive probability function of I_s for the Etna area and Aeolian islands district, are shown in Figure 6. We can consider to compare the mode (blue coloured) that gives the intensity I_s , and the intensity decay given I_0 , with the results obtained with deterministic method. The intensity decay vs distance (red dots) plotted in Figure 6 have been evaluated by the deterministic relationships (Table 2) coming from the logarithmic regressions based on the same dataset used in this study.

Table 2. Intensity decay vs distance obtained by the logarithmic regressions (from Azzaro et al., 2006)

Area	Logarithmic regression	R ²	Valid for d (km) ≥
Etna	$\Delta I = 0.98\ln(d)+1.01$	0.92	0.4
Aeolian Islands	$\Delta I = 1.28\ln(d)-2.39$	0.89	6.5

The comparison between the results obtained by the probabilistic method (this study) and the deterministic approach is done only through the mode. In the deterministic relationships the intensity decay $\Delta I = 0$ is not reached and the valid range of relationships is defined only for distance greater of a given threshold (Table 2).

5. CONCLUSIONS

In this study we present a probabilistic procedure to estimate the macroseismic intensity attenuation in the volcanic areas of Italy. The semi-qualitative character of the intensity is emphasized by the application of the predictive and binomial distribution for the site intensity I_s . The adopted probabilistic method provides a matrix of values of probability function of I_s that can be directly applied for the computation of probabilistic seismic hazard at the site using a numerical procedure implemented in the software 'SASHA' (D'Amico and Albarello, 2007).

The preliminary results here presented (for $I_0 = VII$ MCS), together with the comparison with the previous attenuation model obtained by the deterministic approach, represent a first step for more detailed analyses in the Etna area, in particular by defining the predictive and the binomial probability function of I_s also for other earthquakes having higher epicentral intensity I_0 .

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REFERENCES

- Albarello D., Azzaro R., Barbano M.S., D'Amico V., D'Amico S., Rotondi R., Tuvè T. and Zonno G. (2007). Valutazioni di pericolosità sismica in termini di intensità macrosismica utilizzando metodi di sito. Progetto DPC-INGV S1, Deliverable D9, <http://esse1.mi.ingv.it/d9.html>.
- Albarello D., D'Amico V., Gasperini P., Pettenati F., Rotondi R. and Zonno G. (2007). Nuova formulazione delle procedure per la stima dell'intensità macrosismica da dati epicentrali o da risentimenti in zone vicine. Progetto DPC-INGV S1, <http://esse1.mi.ingv.it/d10.html>
- Albarello D. and Mucciarelli M. (2002). Seismic hazard estimates using ill-defined macroseismic data at site, *Pure Appl. Geophys.*, **159**, 1289–1304.
- Azzaro R., Barbano M.S., D'Amico S. and Tuvè T. (2006). The attenuation of the seismic intensity in the Etna

- region and comparison with other Italian Volcanic districts, *Annals of Geophysics*, **49**, **4/5**, 1003-1020.
- Azzaro R., D'Amico S., Mostaccio A. and Scarfi L. (2006): Terremoti con effetti macrosismici in Sicilia orientale nel periodo Gennaio 2002-Dicembre 2005, *Quad. Geofis.*, **41**, pp. 62.
- Azzaro R., Barbano M.S., Antichi B. and Rigano R. (2000): Macro seismic catalogue of Mt. Etna earthquakes from 1832 to 1998, *Acta Volcanol.*, **12** (1), 3-36 (with CD-ROM).
- Azzaro R. and Barbano M.S. (2000). Analysis of the seismicity of Southeastern Sicily: a proposed tectonic interpretation, *Annals of Geophysics*, **43**, **1**, 171-188.
- Azzaro R., D'Amico S., Mostaccio A. and Scarfi L. (2002). Terremoti con effetti macrosismici in Sicilia orientale – Calabria meridionale nel periodo Gennaio 1999 – Dicembre 2001, *Quad. Geofis.*, **27**, pp. 59.
- D'Amico V. and Albarello D. (2007). Codice per il calcolo della pericolosità sismica da dati di sito: SASHA (Site Approach to Seismic Hazard Assessment). Progetto DPC-INGV S1, Deliverable D12, <http://esse1.mi.ingv.it/d12.html>
- INGV-DPC 2004-2006 Agreement, Project S1 (2007). Continuation of assistance to DPC for improving and using the seismic hazard map compiled according to the Prime Minister “Ordinanza” 3274/2003 and planning future initiatives, Final Report, coord. Calvi G.M., Meletti C., Stucchi M., <http://esse1.mi.ingv.it>.
- Magri, L., Mucciarelli, M. and Albarello, D. (1994). Estimates of site seismicity rates using ill-defined macro seismic data, *Pageoph.*, **143**, **4**, 617-632.
- Pasolini C., P. Gasperini, D. Albarello, B. Lolli, and V. D'Amico (2008). The Attenuation of Seismic Intensity in Italy, Part I: Theoretical and Empirical Backgrounds, *Bull Seism. Soc. Am.*, Vol. **98**, No. **2**, pp. 682-691.
- Rotondi R., Tertulliani A., Brambilla C. and Zonno G. (2008). The intensity attenuation of Colfiorito and other strong earthquakes: the viewpoint of forecasters and data gatherers, *Annals of Geophysics*, accepted.
- Rotondi R. and Zonno G. (2006). Seismic scenarios in terms of macro seismic intensity. Proceedings of the *Proceedings of the First European Conference on Earthquake Engineering and Seismology*, Geneva (CH), 3-8 september 2006, Abstract, 124.
- Rotondi, R. and G. Zonno (2004). Bayesian analysis of a probability distribution for local intensity attenuation, *Annals of Geophysics*, **47**, **5**, 1521–1540.
- Stucchi M. et alii. (2007). DBMI04, il database delle osservazioni macrosismiche dei terremoti italiani utilizzate per la compilazione del catalogo parametrico CPTI04, <http://emidius.mi.ingv.it/DBMI04>, *Quaderni di Geofisica*, **49**, pp.38.
- Zonno, G., Rotondi R. and C. Brambilla (2008). Mining macro seismic fields to estimate the probability distribution of the intensity at site, submitted *Bull Seism. Soc. Am*
- Zonno, G., Meroni F., Rotondi R. and Petrini V. (1995). Bayesian estimation of the local intensity probability for seismic hazard assessment, *Proceeding of the Fifth International Conference on Seismic Zonation*, Nice, October 17-19, 1995, 1723-1729.