A REVISED FOREST FIRE MODEL NON-QUASISTATICALLY DRIVEN FOR THE SPORADIC ACTIVITY OF THE EARTH'S MAGNETOTAIL

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ABSTRACT/RESUME

The energy release during magnetospheric substorms in response to solar wind changes consists of two main physical processes: the directly-driven and the unloading processes. Rencent analysis on the sporadic activity related to the unloading process seems to indicate that the magnetospheric response to solar wind changes might resemble the behaviour of an out-of-equilibrium system near a marginally stable point (critical point).

Here, we present a modified version of the well-known forest-fire cellular automaton (FFM) not quasistatically driven for the sporadic activity of the energy release in the geotail regions as revealed by the auroral electrojet index.

1. INTRODUCTION

One of the still not-completely understood mechanisms in the magnetospheric dynamics is how the energy is transported and released during the magnetospheric substorms. Recent analyses demonstrated that the magnetospheric dynamics and the energy transport in the magnetotail regions are characterized by highly complex nonlinear processes [1]. Moreover, an accurate analysis of the auroral electrojet (AE) index statistical features revealed the lack of a characteristic time and/or length scale for the energy release during the auroral magnetospheric substorms [2]. These results suggested that the magnetotail dynamics could be similar to that of an out-of-equilibrium system near "forced and/or self-organised criticality" (FSOC) [2,3]. Further evidences of this near criticality dynamics have been found both in the scale-free behaviour of bursty-bulk flows (BBFs) [4] and in the "global energy deposition rate" as observed by Polar UVI measurements [5]. However, this is still an open question.

In the last years, a new scenario, involving the "generation, dispersing and merging of multiscale localized coherent plasma structures", has been proposed by Chang [3] to model the intermittent behaviour of the magnetotail dynamics. In this scenario, the coupling among micro-, meso- and macro-scale fluctuations and the topological features of the coherent magnetic and plasma structures might be responsible for the observed near criticality dynamics [3, 6, 7]. Moreover, it has been proposed that the dynamics of such coherent structures in the magnetotail plasma sheet could be similar to that of a "stirred colloidal suspension" showing topological phase transitions in the evolution from a critical state to another one [8]. Here, the coalescence process among these coherent structures could be the origin of the observed localised sporadic activity in the neutral sheet region that was termed as BBF. In this framework, the understanding of the highly intermittent character of the energy and plasma transport in the geotail requires to investigate the topological behaviour of the magnetic and plasma structures.

In an our previous work [9], we have tried to model the magnetotail intermittent dynamics using a revised version of the FFM model, chaotically driven with a non-stationary signal that resembles the solar wind spectral features. This model seems to be able to capture many of the statistical features of the magnetosperic response to solar wind changes. In detail, the global turbulent activity of our model shows power law distributions that resemble those observed in the case of AE-index, and a power spectral density, which agrees with the previous results by Tsurutani et al. [10].

Here, we present an upgraded version of our previous model [9] to better mimic the AE-index dynamics.

2. THE REVISED FOREST FIRE MODEL

The forest-fire model (FFM) [11, 12] was introduced as a possible realisation of a simple extended, non-conservative stochastic system showing self-organised criticality (SOC). Nevertheless, it was successively understood [13] that criticality, shown by FFM, cannot be called SOC strictly speaking, since the model requires the tuning of an appropriate parameter in order to show criticality.

The FFM consists of a cellular automaton defined on a d-dimensional lattice, characterised by a discrete space-time dynamics within a lattice of cells with a finite number of states. This discrete dynamics is based on a set of rules, corresponding to a simplified and coarse-grained representation of the physics. We choose the FFM to simulate the occurrence of sporadic localised relaxation phenomena, such as localised reconnection and/or current disruption events, which can occur in the geotail neutral plasma sheet, and to investigate the relationship among local events and global dynamics.

In the FFM each site of the lattice can be empty, occupied by a green tree or a burning tree. The state of the

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system is update at each time step i according to the following rules:

1) a tree can grow on an empty site with a time-depending probability p_i;
2) a burning tree becomes an empty site at the next time step;
3) a green tree catches fire with a time-depending probability f_i, or if one of its nearby trees is burning.

The analogy between this model (FFM) and the local plasma and magnetic field conditions in the geotail is based on the following considerations. We can associate the empty sites to those regions where plasma conditions are locally stable, the green trees to the locally unstable regions, and the burning trees to the regions where a relaxation phenomenon is taking place.

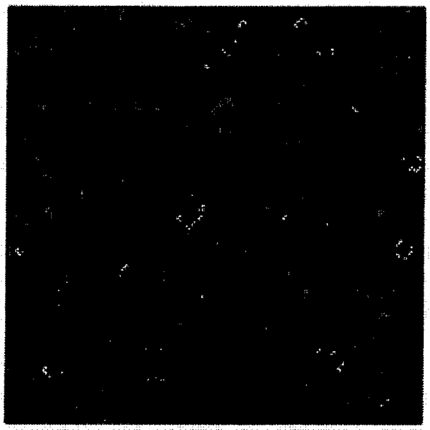

Fig.1 A snapshot of the FFM. Black pixels refer to stable sites (empty sites), grey pixels to locally unstable sites (green trees), and white pixels to sites where a relaxation event is taking place (burning trees).

In our model the growing probability p_i has been chosen, at each time step, to be proportional to a chaotic driven signal y_{i-1}, while the lightning probability f_i is a function of the density both of the green trees (ρ^t) and of burning ones (ρ^f). In particular, we have:

$$\begin{cases} p_i = \alpha + \beta y_{i-1} + \eta \rho^f_{i-1} \\ f_i = \gamma \exp[\delta \rho^t_{i-1}] \end{cases} \tag{1}$$

where the parameters α, β, γ and η have been set according to the following condition:

$$\theta = \frac{\langle f \rangle}{\langle p \rangle} < 1 \tag{2}$$

The choice of a growing probability p_i proportional to the external driving and to the number of burning trees may be justified as follows. According to *Chang*'s model [3, 6, 7] we might expect that the number of the energy relaxation events could influence the growing probability. In other words, the occurring of a relaxation event may induce new unstable sites and so on.

At the end, as a driving signal y_i, we used the following 1-d coupled map [14, 15]:

$$\begin{cases} z_{i+1} = C z_i [1 - z_i] \\ y_{i+1} = \dfrac{B}{2\pi} z_i sin[2\pi y_i] \end{cases} \tag{3}$$

where C and B are two constant value and (z_i, y_i) are in the interval [0,1]. This choice must be related to the fact that this 1-d coupled map has spectral features similar to the solar wind driver vB_s.

3. RESULTS AND DISCUSSION

In our simulation we adopted square lattice, consisting of 202 × 202 sites, with fixed boundary conditions. Fig. 1 shows a snapshot of the lattice where some non-overlapping relaxation events can be observed.

At each time step the density of the unstable sites ρ^t_i and the total number of relaxing sites $n^f_i = \rho^f N^2$ (where N^2 is the total number of the lattice sites) has been monitored. In detail this last quantity (n^f_i), which is a measure of the extension of the regions where a relaxation phenomenon is taking place, can be regarded as an indirect measure of the energy dissipation rate; i.e. as a synthetic representation of AE-index.

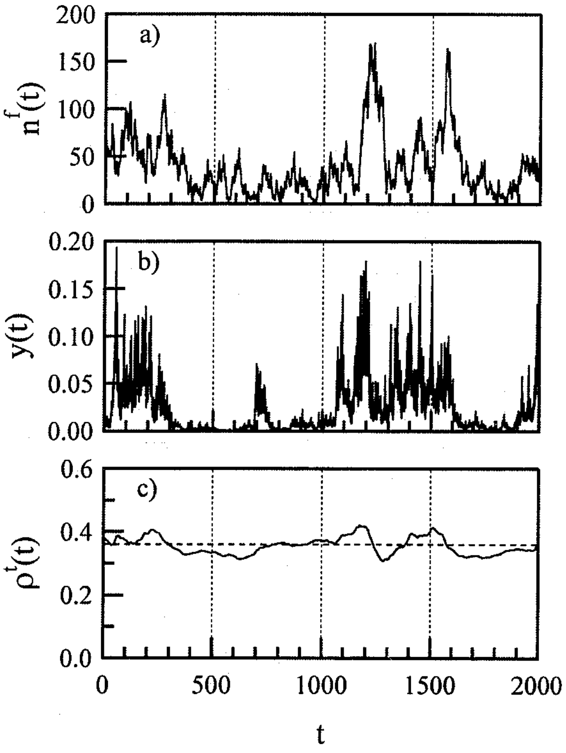

Fig. 2. A short time sequence of the global activity of the FFM model (panel a) in comparison with the driving signal (panel b) and the density of unstable sites (panel c). The horizontal dashed line in the panel c refers to the average value of the density of unstable sites.

The dynamical evolution of the system can be studied looking at the density vector $\underline{\rho} = (\rho^e, \rho^I, \rho^J)$. In Fig. 2 we show the behaviour of the global activity, in comparison with the driver y_i, and the density of the unstable sites ρ^I.

The model when continuously driven evolves towards a dynamical stationary state characterised by an average value of the number density of the unstable sites approx. $< \rho^I > \approx 0.36$. Clearly, being the number density of the unstable sites well below the percolation threshold, the dynamics of the system is the result of the superposition of many non-overlapping relaxation events. Furthermore, the global activity is characterised by intermittency in time, showing periods of relative stasis punctuated by activity bursts. This intermittent dynamics shows directly-driven, as well as, not directly-driven activity bursts. That can be read as an evidence of metastability, and resembles the global behaviour of the Earth's magnetospheric dynamics in respose to solar wind changes, where IMF-triggered and spontaneous substorms occur.

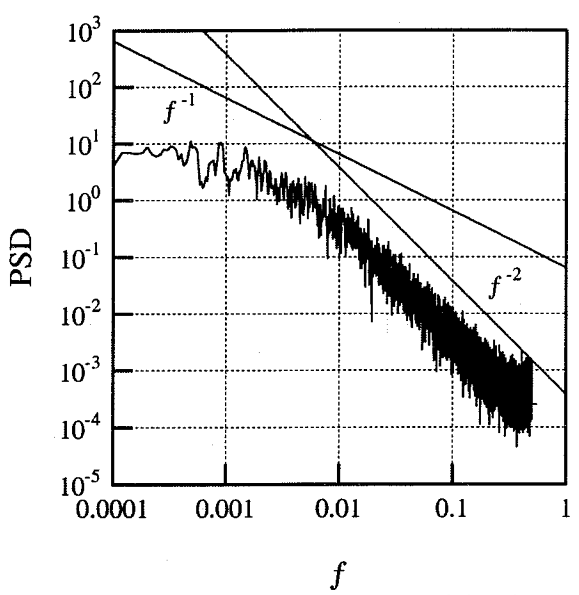

Fig. 3. The power spectral density (PSD) of the global activity n^I_i. The two solid lines refer to power-laws characterised by scaling exponents equal to –1 and –2.

In Fig. 3 we report the power spectral density (PSD) of the global activity of our revised FFM as monitored by the number of relaxing sites. While the power spectral density of the driving signal is a single power-law (as shown in our previous work [9]), the PSD of the global activity shows a broken power-law behaviour, displaying at least three different scaling regions with scaling exponents close to –2 (at high frequency), -1 (at mid frequency), and 0 (at low frequency), respectively. The shape of the activity PSD, if compared with the driving signal spectral features, evidences the nonlinear character of the response of revised FFM to the external perturbation. Moreover, as also argued by Hwa & Kardar [16], the high frequency spectral region (f^{-2} region) can be due to the random superposition of non-overlapping relaxation events, while the $1/f$ spectral region could be the signature of a cooperative dynamics due to the interference of relaxation events. In this framework, the spectral break between these two regions could be related to the maximum lifetime of a single relaxation event.

A similar result can be found analysing the spectral features of the AE-index. In Fig. 4 we shows the PSD of the AE-index relative to a period of 12 days (from December, 1, 1994 to December, 12, 1994). AE-index data, available on Web, come from *WDC-2*, Kyoto, Japan. Two well-defined spectral regions may be identified in the AE-index PSD characterised by power-law with scaling exponents near –1 and –2, respectively.

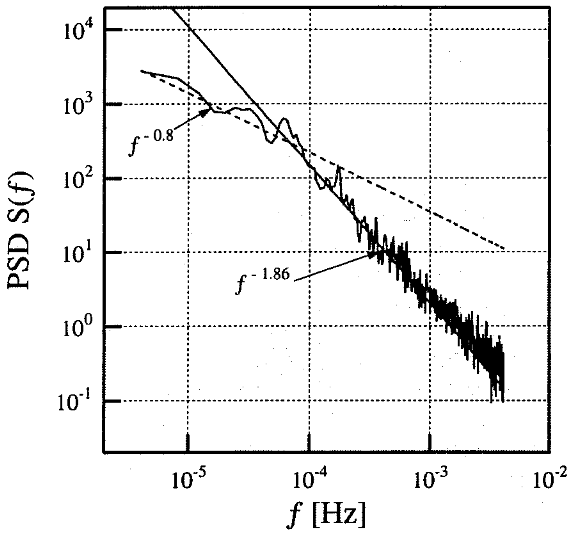

Fig. 4. The power spectral density (PSD) of the selected period of AE-index data. The solid and dashed lines are to power-law best fits.

To investigate the statistical features of the global activity bursts, we have applied the same threshold technique described in [2, 9] to discriminate active periods from quiet ones. In Fig. 5 we show the distribution $D(s)$ of the activity burst size s defined as in [9].

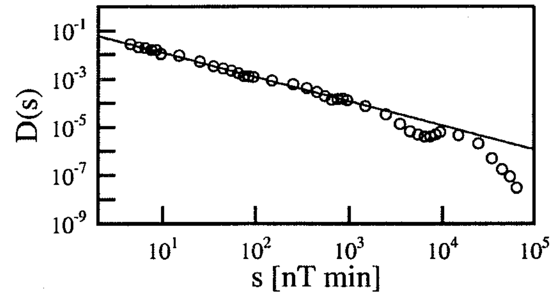

Fig. 5. The size distribution function $D(s)$ of the global activity burst events. The solid line is a power-law best fits.

The distribution function $D(s)$ of the activity burst size obeys power-law over a wide range of scales, meaning that a characteristic size is lacking in this model. The scaling exponent is about -1. The existence of power-law distribution function of the activity burst size can be the evidence of a near-criticality dynamics in the model. In other words, the model as a consequence of the continuous driving evolves towards an out-of-equilibrium marginally stable dynamical state where relaxation events are characterised by scale invariance.
This result agrees with the previous findings on AE-index statistical features [2].

4. SUMMARY AND CONCLUSIONS

In the present work, we have focused our attention to a modified version of the well-known forest-fire model in order to model the local feature of the geomagnetic tail activity. In respect of our previous version of this revised FFM, here we have introduced different rules for the growing probability in order to take into account the generation of new coherent plasma structures as a consequence of the occurrence of relaxation events.
In contrast to the extreme simplification of the dynamics, the revised FFM is able to display many features of the magnetospheric dynamics. In detail, we have found that the spectral and statistical properties of the global "turbulent" activity resemble those of the AE-index, and that the model response to the driving signal is well in agreement with the previous results obtained by Tsurutani et al. [10].
The main feature of our revised FFM is that the global activity is due to the superposition of several non-directly connected local events near an out-of-equilibrium marginally stable dynamical state. The good-agreement between the model global activity and the AE-index behaviour strongly supports the view of the geomagnetic tail dynamics in terms of a collection of sporadic localised events related to a complex magnetic field and plasma topology near criticality.

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