Depth and morphology of reflectors from the 2-D non-linear inversion of arrival-time and waveform semblance data: method and applications to synthetic data

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Abstract: We propose a two-dimensional, non-linear method for the inversion of reflected/converted traveltimes and waveform semblance designed to obtain the location and morphology of seismic reflectors in a lateral heterogeneous medium and in any source-to-receiver acquisition lay-out. This method uses a scheme of non-linear optimisation for the determination of the interface parameters where the calculation of the traveltimes is carried out using a finite-difference solver of the Eikonal equation, assuming an a priori known background velocity model. For the search of the optimal interface model, we have used a multiscale approach and the Genetic Algorithm global optimization technique. During the initial stages of inversion, we used the arrival times of the reflection phase to retrieve the interface model that is defined by a small number of parameters. In the successive steps, the inversion is based on the optimization of the semblance value determined along the calculated traveltime curves. Errors in the final model parameters and the criteria for the choice of the bestfit model are also estimated from the shape of the semblance function in the model parameter space. The method is tested and validated on a synthetic dataset that simulates the acquisition of reflection data in a complex volcanic structure.

Introduction

The reconstruction of geological discontinuities from active seismic data is usually carried out by migration techniques that generally demand an a priori known background velocity model in order to place a reflection event at a particular subsurface location. The success of these migration methods strongly depends on the data quality and the accuracy of the adopted background velocity model. In complex geologic structures, the presence of unmodelled multiples, very energetic principal phases, and diffraction and scattering phenomena can often produce poor quality reflected signals. In these complex envi-

ronments, the estimation of a reliable velocity distribution in the subsurface is extremely difficult when significant lateral velocity variations are present.
Recently, Improta et al. (2002) [7] used a two-step procedure for the joint 2-D inversion of the first-P and reflected P-to-P traveltimes. In the first step, a background P-velocity model was determined by the first-arrival, non-linear, traveltime tomography. Then this model was used as a reference model to perform an inversion of reflected traveltimes for retrieve the interface parameters. Both of these inversion methods used techniques for solving the forward problem solution based on the finite difference solver of the Eikonal equation combined with a multiscale approach [2] and a non-linear optimization scheme for model-space exploration. The method proposed in the present study follows a similar approach to that of Improta et al. (2002) [7] but, regarding this last one, the inversion is based on a combined optimization of traveltimes and waveforms semblance data. The location and reconstruction of a two-dimensional (2-D) seismic interface geometry is performed in an a priori known background velocity model using a non-linear optimization scheme. The reference medium is obtained by the first-arrival tomography or the integrated tomography-velocity analysis procedures.
Thus we propose a method for parameter estimation of multiple irregular interfaces embedded in the subsurface medium. This is based on an iterative, non-linear, inversion scheme that follows a two-step procedure to combine with this the information from picked traveltimes and waveform semblance data. No data reduction is performed before the inversion (i.e. normal move-out and/or stack), while the effects of lateral heterogeneities are taken into account by the 2-D background velocity medium. The inversion problem is solved by the use of the Genetic Algorithm (GA) optimisation technique [5] [4], through searching in the global parameter space for the minimum of a cost function that depends on the theoretical and observed data. The theoretical arrival times of the P-to-P arrivals are calculated using the Podvin and Lecomte (1991) [11] solver of the 2-D finite difference equation, and a multiscale strategy of inversion is adopted, as used by Improta et al. (2002) [7]. Moving from the shallowest interface, we follow a layer-stripping approach [9] to determine the location and morphology of the deeper interfaces, by accurate re-picking of the deep-reflection traveltimes on zero-time move-out (ZTMO). This method is specifically designed for geophysical investigations in complex geological environments, in order to obtain the morphology and positions of embedded discontinuities.

Method

Forward modelling

The reflection interface is described by a 2-D cubic-spline function, where the control points are equally spaced in the horizontal direction at fixed horizon-

tal positions, and they can move vertically with continuity within an assigned depth range. The vertical coordinates of the nodes of the spline-function are the parameters of the interface model. For a given interface model, the reflection traveltimes are calculated following a four-step procedure:

1. The medium is discretized as a fine grid, with nodes equally spaced along the vertical and horizontal directions. The grid dimension depends on the accuracy required for the traveltime calculations (see step 2).
2. The first-arrival times from each source and receiver at the nodes of the-grid are calculated using the 2-D Eikonal equation and the finite difference solver of Podvin and Lecomte (1991) [11].
3. The one-way traveltimes for a source/ receiver to each point of the discretized interface are calculated by performing an interpolation among the nearest four grid nodes.
4. For a given source-receiver pair, the reflection location point and the total traveltime are calculated according to the Fermat principle: the reflection point will be the one providing the minimum total traveltime.

Data inversion

For a given reflection event identified on seismic sections, the data space is subdivided into two subsets: one containing the arrival-time pickings, and the other the complete waveform of the reflection phase considered. The data inversion proceeds first with the modelling of the traveltime data, to retrieve a smooth, low-frequency model of the interface; then the waveforms are used to obain a more refined model. The non-linear inversion of the traveltime data is based on the search for the minimum of the RMS function computed using computed and the observed raveltimes of reflection events.

According to the multiscale inversion approach [2] we adopt an optimization strategy whereby several optimization runs are performed by progressively increasing the density of cubic-spline nodes describing the interface reflector. At the very early stages (where the interface is described by a very small number of parameters) the optimization is performed over a quite wide range of possible depth values for the interface nodes. Furthermore, for runs where the interface is parametrized by a large number of nodes, the search is performed with smaller allowed depth variations around the model estimated in the previous run. The search for the model parameter vector that minimizes the RMS cost function is performed by GA optimization technique [5] [4] [13].

In our inversion scheme, the picked arrival times of the reflection phase are initially used to retrieve a smooth interface model, i.e the interface is defined by a small number of cubic-spline nodes. In the successive steps a more refined interface model is obtained using the reflection waveform information. In this case, the optimal interface model is determined through a global search for the maximum of the semblance objective function [10] computed using

the amplitudes of waveforms in a time window chooses to bracket the calculated reflection arrival time for a given source and receiver couple. The window length is arbitrarily defined based on the dominant period of the reflection events considered. The semblance function is therefore calculated in the selected time windows for all of the records along the whole Common Receiver or Common Shot seismic gathers. Since the semblance is a measure of the waveform similarity and coherence, the main advantage of using the semblance [10] objective function is that at this stage, the inversion does not require phase picking, thus improving the time performance of the whole procedure and removing the effects which might be introduced by the subjectiveness of the picking.

The reliability of the retrieved model can be verified a posteriori by the construction of the seismic panel offset-two-way-time (TWT), where each seismogram is shifted back in time according to the theoretical reflection traveltime calculated for the optimal interface model retrieved by the time-picks and waveform semblance inversion. Following on form Improta et al. (2002) [7], these panels are designated as zero-time move-out (ZTMO) sections.

The inversion procedure described above is iteratively applied to a sequence of reflection events identified on seismic sections. Starting from the shallowest interface, an approach similar to layer-stripping [9] is used to determine the location and morphology of the deeper interfaces, by accurate re-picking of deep reflection traveltimes.

The uncertainty in the depth values of each node describing the interface is estimated by local exploration of the semblance function in the neighbourhood of the model which maximizes the semblance value. We use the second-order Akaike Information Criterion (AICc) [1] [6] for determines when the multistep process should be stopped.

Synthetic data modelling

To validate the proposed methodology we have applied it to a synthetic dataset calculated on a complex velocity model (Figure 1a) constructed based on the images of the Campi Flegrei (Southern Italy) caldera structure that were recently obtained through high resolution tomographic inversion of the first P-arrival times from active seismic experiments [15] [8]. Inside each layer below the water the P-wave velocity is assumed to increase linearly with depth while the Vp/Vs ratio and the density are constant is constant. Using this model we simulated a seismic experiment with an acquisition lay-out consisting of 55 seismic stations with a horizontal spacing of 500 m (at an equal depth of 270 m). Each receiver records the seismic signals generated by 21 shots spread around the receiver, with an offset of between -200 m and 200 m, and a horizontal spacing of 10 m. The synthetic seismograms were computed using a 2-D, elastic, velocity-stress finite-differences algorithm [3]. The

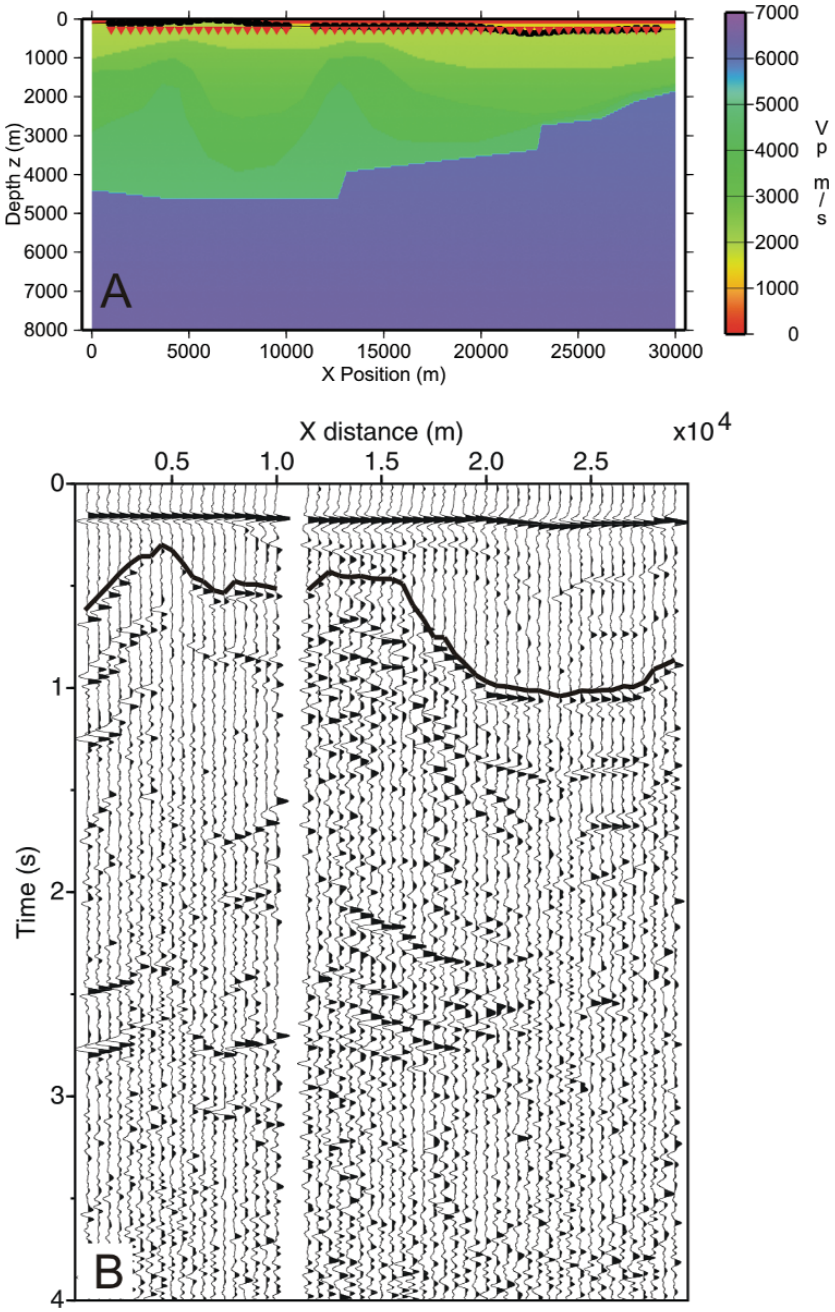

Fig. 1. Model adopted for the generation of synthetic seismograms (a) and synthetic zero offset section (b). The red triangles in upper part of the model show the positions of the receivers, and the black stars show the positions of the sources on the topographic surface. The traces in the section are normalized for the maximum trace value and are filtered with an AGC filter with a window of 1 s. No filters were applied to the traces during the inversion. The black line in the section shows the picking of a reflected phases.

synthetic seismograms were normal-move-out corrected [14] and stacked to obtain the final section used for the inversion. Gaussian noise was added to each trace to have a mean signal-to-noise ratio of 5. The resulting traces were band-pass filtered in the frequency range of 6-30 Hz. These stacked, noised and filtered traces are displayed as a function of distance in Figure 1b, after an amplitude equalization had been applied using an automatic gain control with a window of 1 s.

Data inversion

The normalised and filtered section (Figure 1b) was used to measure the arrival times of the first reflected phase. These times (Figure 1b) were taken from all of the seismograms and were inverted using the eflected traveltimes RMS as the cost function, as described above. The theoretical arrival times were calculated using a square grid with 50 m sides. In each cell, the velocity was calculated through averaging the values of the model used for the synthetic calculations in a 1000 m×1000 m surrounding the centre of the cell. We used a smooth background model, as in this way we closely approach the models provided with tomography techniques (through the inversion of the first-arrival times) that have been used as starting models for interface inversions [7]. Following the multiscale approach [2], three interface models were inverted, with parameterisations of two, three and five nodes. Increasing the number of nodes a reduction in the RMS function is seen, which goes from 0.13 s for the model defined by two nodes, to 0.07 s for that defined by five nodes.
Starting from the model defined by five nodes (Figure 2), a succession of two models parameterised by 9 and 17 nodes is progressively determined while optimising the semblance function [10]. For these inversions, the waveforms represent the data and the code searches for the interface models on the basis of the likeness of the waveforms around the arrival times calculated. The interfaces determined are shown in Figure 2 with the ZTMO sections relative to each model. In these sections, the alignment of the waveforms near 0 s becomes evident. The semblance goes from 0.18 for the model with nine nodes, to 2.9 for that defined by 17 nodes (Figure 2).
The ZTMO sections (Figure 2) show a very clear deep phase. We used this sections for the individuation and picking of the very clear secondary phase (17-nodedes ZTMO section in the Figure 2). The times from the ZTMO sections were again corrected, adding the time shifts relative to the modelled reflected phase, and the values obtained were inverted. Also for the inversion of this deeper phase, we started with an optimisation of the reflected traveltimes RMS function for three successive inversions searching the model parameterisation of two, three and five nodes. Starting from the interface parameterised by five nodes, two new models with 9 and 17 nodes were calculated using the semblance [10] optimization. The models obtained relative to five,

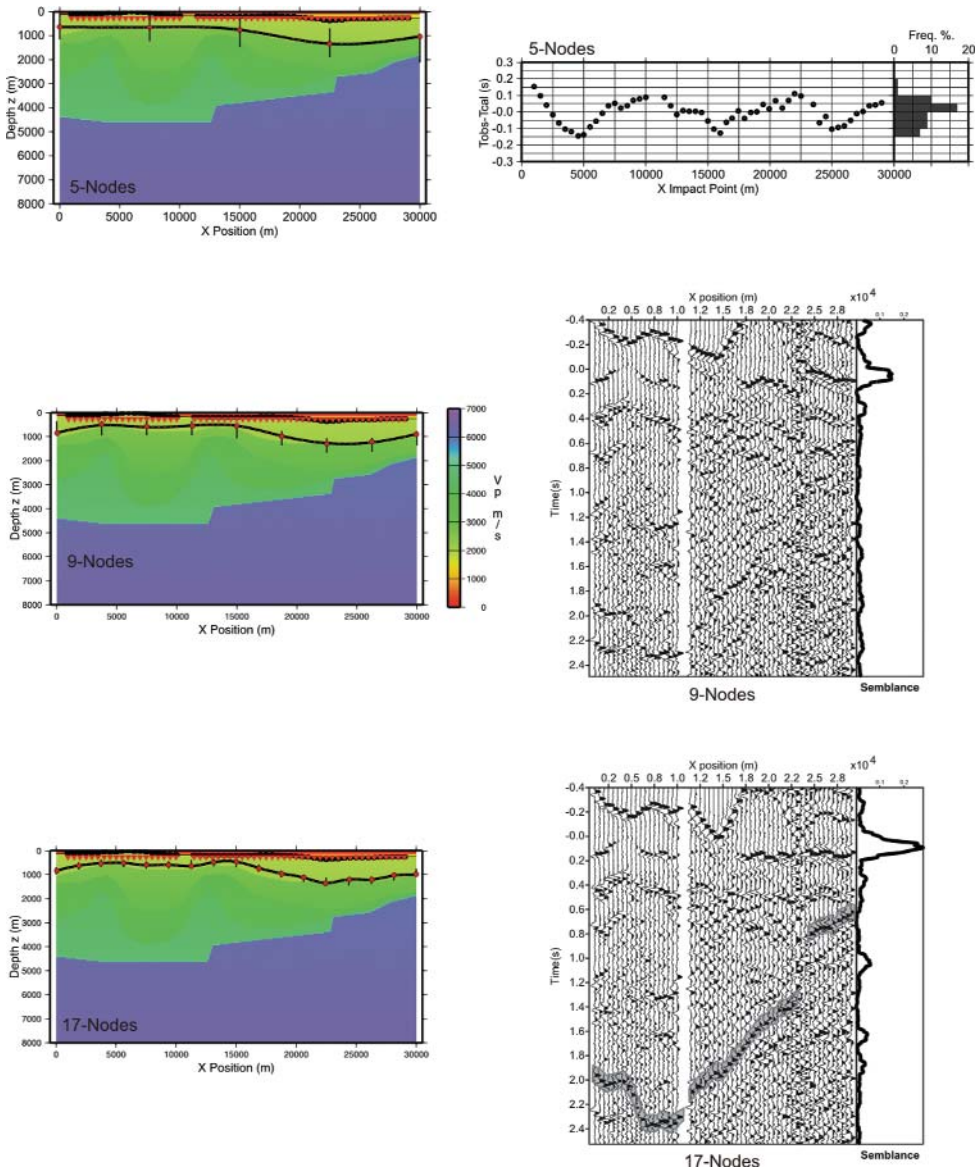

Fig. 2. Results of the inversion method for the reflected phase shown in the section of Figure 1b. The three images on the lefthand side show the interface models used to calculate the distribution of the residuals and the ZTMO panels on the right. The three interface models were obtained by performing a succession of inversion runs with an increasing number of interface nodes (large dots). The five-node model (top) was obtained by performing the inversions based on the optimization of reflected traveltimes. The nine-node and 17-node models (middle and bottom) were obtained using the semblance optimization starting with the five-node model. The vertical bars are the search intervals of each of the spline nodes. The semblance trace shown on the right of the ZTMO panels as a heavy dark line allows us to quantify the coherence of the aligned phases. A semblance maximum trace is seen at zero-time, confirming the phase alignment modelled.

nine and 17 nodes are shown in (Figure 3), which also shows the trend of the residuals and the ZTMO sections relative to the models with five, nine and 17 nodes. The final semblance obtained for the model defined by nine nodes is 0.13, while that for the model defined by 17 nodes is 0.19.

Discussion and conclusion

We have presented a method for the non-linear inversion of arrival times and waveforms of reflected seismic phases that is aimed at the reconstruction of the depth and morphology of seismic reflectors. With respect to existing non-linear 2-D interface inversion methods [7] [12] we introduce a new approach based on the combined optimization of traveltime (global search) and waveform semblance data (local search). Moreover, we adopt a criterion (the Akaike Information Criteria [1]) based on the the principle of parsimony, for determining when the multi-scale iterative process should be stopped. The method is based on the search for the interface model that optimises at successive steps an objective function defined either through a comparison between the arrival times calculated and those observed, or through the similarities of the waveforms (waveform semblance) around the calculated arrival times. The general principle used here is that reflected travel times provide long wavelength information about the interface morphology while more refined models can be retrieved by the waveform semblance optimization, using as a starting model the one obtained from traveltime misfit. The direct forward problem is solved by a modified version of the method of Podvin and Lecomte (1991) [11], which allows for obtaining accurate and fast estimates of the reflection arrival times even in strongly heterogeneous velocity models. The global search for the optimal model in the parameter multidimensional space is carried out using the Genetic Algorithm (GA) [5] [4], a search technique well adapted for the determination of the solution of non-linear problems. The parameter uncertainty is evaluated by the local exploration of the semblance around the best-fit model along orthogonal directions of the parameter space.

The method has been applied to synthetic data generated in a heterogeneous velocity structure that simulates a complex volcanic structure. The choice of a non-linear approach for the reconstruction of the reflector morphology is motivated by the high level of non-linearity that exists between the reflected arrival times and waveforms particularly in the presence of strong heterogeneity of the propagation medium. The use of an non-linear optimisation scheme coupled to a multiscale inversion strategy results in more stable and robust estimates of the model parameters, and reduces the risk of convergence towards local minima.

The inversion based on the optimization of the semblance is advantageous since it limits the human and manual intervention in the final definition of the

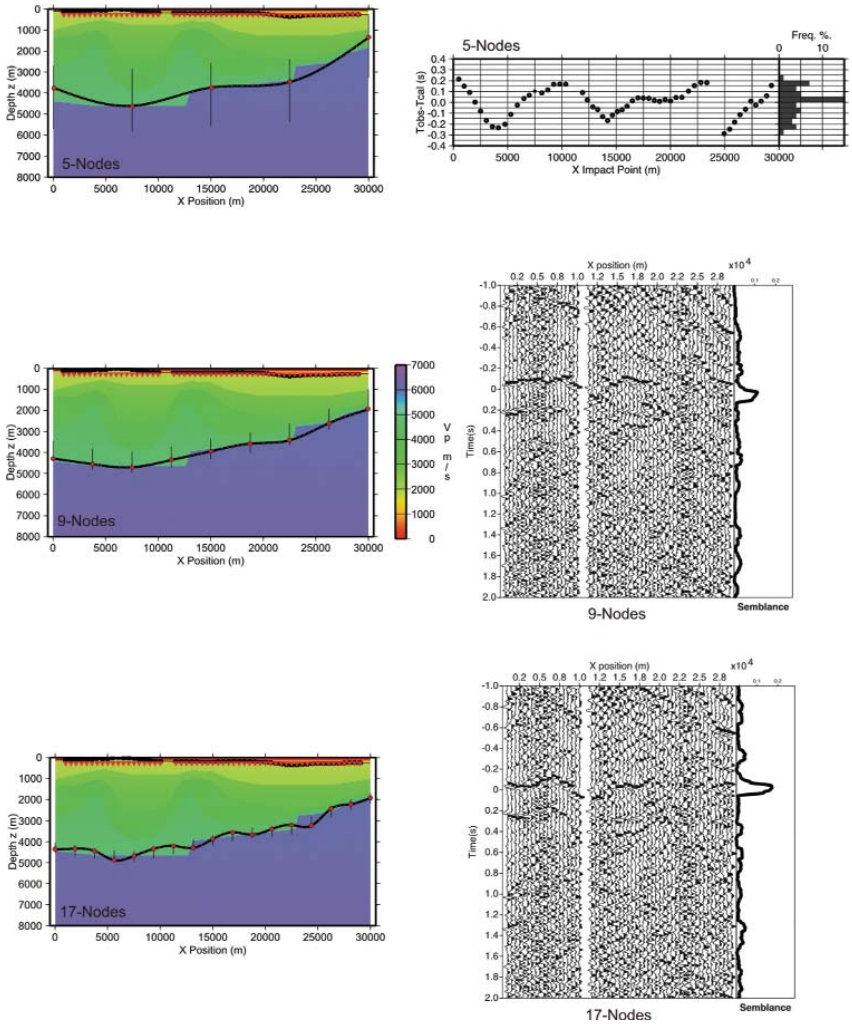

Fig. 3. Results of the inversion method for a reflected phase shown in the 17-nodes ZTMO of Figure 2. The three images on the lefthand side show the interface models used to calculate the distribution of the residuals and the ZTMO panels on the right. The three interface models were obtained by performing a succession of inversion runs with an increasing number of interface nodes (large dots). The five-node model (top) was obtained by performing the inversions based on the optimization of reflected traveltimes. The nine-node and 17-node models (middle and bottom) were obtained using the semblance optimization starting with the five-node model. The vertical bars are the search intervals of each of the spline nodes. The semblance trace shown on the right of the ZTMO panels as a heavy dark line allows us to quantify the coherence of the aligned phases. A semblance maximum trace is seen at zero-time, confirming the phase alignment modelled. We used the Akaike criteria for the choice of the optimal parametrization for both the interfaces (shallow interface and deep interface). Computing the AIC for all the retrieved models we obtained that for both the interfaces the 9-nodes models minimize the AICc values so they were chosen as best model. The uncertainties in depth for the nodes of minimum AIC interfaces are evaluated by the local exploration of the semblance around the best-fit model along orthogonal directions of the parameter space. Figure 4 shows the final interfaces retrieved and the corresponding errors. Geometry and depth of the retrieved interfaces are in good agreement with the interfaces used to compute synthetic data.

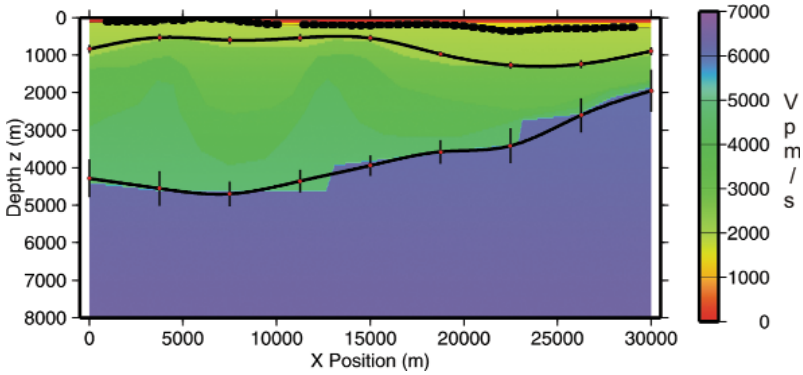

Fig. 4. Final interface models and parameter uncertainty. Uncertainty in depth of the interface nodes are estimated by locally exploring the variation of the cost function around the best-fit interface nodes.

reflector model. Moreover, the use of the zero-time move-out sections (ZTMO) can help in to identify more clearly than on original sections the seismic phases generated by deeper discontinuities. Indeed, particularly for strongly heterogeneous propagation media, the shallow interfaces can destroy the lateral coherence of the reflected seismic phases generated at the deep interfaces. However the inversion procedure is not fully independent of the manual intervention that is needed for the initial arrival time picking of the reflected seismic phases. Indeed, several preliminary tests indicated that this two-step inversion strategy is preferred to the direct inversion of semblance waveforms, which generally leads to unreliable solutions associated with relative minima of the objective function and reflection events not necessarily generated by the same reflector.

The proposed method suffers of some limitations. It allows to reconstruct a single interface at a time, and for each of them, different models are obtained according to the multi-scale strategy. In this way, the computing time can be relatively high when compared with the one needed for the reconstruction of the interfaces with the pre-stack migration techniques that are usually used to solve similar problems [14].

A future development could be represented by the 3D extension of the inversion procedure. This will allows to define seismic discontinuities as surface rather than curves. This is particularly relevant for combining active and passive (earthquakes) dataset. The latter would represent an important advancement allowing through earthquake records to use the complete information carried out by primary P and S reflections in addition to the converted arrivals for constraining the shape and depth of seismic discontinuities.

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