Effect of faults and fractures on oilfield flow rate data: long-range correlations in a complex system

Ian Main, Lun Li, Thomas Leonard & Orestis Papasouliotis
University of Edinburgh

Kes Heffer
Reservoir Dynamics Ltd.

Xing Zhang & Nick Koutsabeloulis
Schlumberger

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References


*In press*


Preamble: triggering phenomena

- Earthquake-earthquake
  (Marsan et al, Huc & Main, Helmstetter & Sornette, Felzer & Brodsky)
  - static or dynamic?
- Pore pressure-earthquake
  (Segall, Kohl & Megel)
  - poroelasticity and induced seismicity
- Earthquake-pore pressure
  (Doan & Cornet)
  - borehole response
- Pore pressure-pore pressure?
  (Main et al, Heffer et al)
  - flow rate-flow rate

=> Reservoir modelling
Outline

- Background concepts: statistical forecasting, parsimony in model fitting, principal component analysis, geo-mechanics and permeability
- The statistical reservoir model
- Application to the Gullfaks field, North Sea
- Application of a geo-mechanical model to the same field
- Conclusions
Statistical forecasting
Example: Buying a house

££ = ??
Buying a house: A statistical model

\[ ££ = w_1 x_1 + \ldots \]
Buying a house: A statistical model

Local School?

££ = w_1 x_1 + w_2 x_2 + ...
Buying a house: A statistical model

Local School?  South Facing Garden?

\[ \text{££} = w_1 x_1 + w_2 x_2 + \ldots + w_i x_i + w_{i+1} x_{i+1} + \ldots \]
Buying a house: A statistical model

Local School?  South Facing Garden?

\[ \begin{align*}
\£\£ &= w_1 x_1 + w_2 x_2 + \ldots + w_i x_i + w_{i+1} x_{i+1} + \ldots w_n x_n \\
\end{align*} \]

Period features?
Buying a house: A statistical model

Define a set of weights $w_i, i = 1, n$

$££ = w_1 x_1 + w_2 x_2 + \ldots + w_i x_i + w_{i+1} x_{i+1} + \ldots w_n x_n$

Local School? South Facing Garden? Period features? Kitchen Sink

SIGNAL

NOISE
A History of Parsimony

- **William of Ockham**: “That which is accomplished by fewer (assumptions) is accomplished in vain with more”

- **Carl Friedrich Gauss**
  \[
  \sigma^2 = \sum_{i=1}^{n} \frac{[y_i - \hat{y}(x_i)]^2}{n - p}
  \]

- **Hirotugu Akaike**
  \[
  AIC = n \ln \left( \sum_{i=1}^{n} \frac{[y_i - \hat{y}(x_i)]^2}{n} \right) - p
  \]

  \(n = \text{no. of data points}\)
  \(p = \text{no. of parameters}\)
Information vs. goodness of fit

\[ y = 1 + x - x^2 : \sigma^2 = 1 \]

\[ BIC = n \ln \left\{ \sum_{i=1}^{n} \left[ y_i - \hat{y}(x_i) \right]^2 / n \right\} - p \ln(n / 2\pi) \]

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<th>5</th>
<th>6</th>
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Figure 1.28. Fitting a quadratic regression curve.

Figure 1.29. Log-likelihood plot for polynomial regression.
Principal component analysis
Example: stress rotation

\[
\begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3 \\
\end{bmatrix}
= R^T(\theta, \phi)
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} \\
\end{bmatrix} R(\theta, \phi)
\]

\[\sigma_1 \geq \sigma_2 \geq \sigma_3\]
Second Example: mineral prospecting

### Table 1.1. The raw data matrix for the lead–zinc prospecting problem

<table>
<thead>
<tr>
<th>Local-</th>
<th>T</th>
<th>D</th>
<th>P</th>
<th>Causes</th>
<th>Geological properties</th>
<th>Data matrix</th>
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</table>
Map of first three principal components

Figure 1.2. Distribution of controls imposed at the locations of the samples.
The statistical Reservoir Model

Input (past record) \[ C_{ij} \] \[ \times \] Output (future production) \[ P_{i}^{(t+1)} \]

Input Data
- \( I_{1}^{(t)} \)
- \( I_{N}^{(t)} \)
- \( P_{1}^{(t)} \)
- \( P_{M}^{(t)} \)

Predicted Response
At Producers
- \( P_{1}^{(t+1)} \)
- \( P_{M}^{(t+1)} \)

How the reservoir responds
Separating signal from noise: a parsimonious model

Real Array($N \times N \times L$)  \times  Binary Array($N \times N \times L$)  \Rightarrow  \text{Statistical Reservoir Model}

$W_{ij}$  
Optimal Regression Model

$N_{ij}$  
Significance Test For Multiple Regression

Feedback

Now

-1 Month

-2 Month

UK patent application 0524134.4 filed 26/11/2005
Geo-mechanical model: long-range poro-elasticity

Deformation and faulting associated with fluid extraction (Segall, 1989)
Geo-mechanical model: Permeability response to deformation

Linear

Non-linear, near critical
Test case: The Gullfaks field (after Arild Hesjedal)

- Complex, faulted reservoir
- In block 34/10 in the northern part of the Norwegian North Sea.
- Total of 133 months data 1986-1997
- 106 platform wells (79 producers +27 injectors >24 months) used
- Data provided free for academic use
Time series for flow rate
Cross-correlation function

‘Direct’ and time-delayed effects seen over a few months
Significantly correlated wells

Fault map of the Gullfaks Field (Fossen et al., 1998).
Correlation function for significantly correlated wells
Correlation function for significantly correlated wells

Flow rate correlations

Earthquake aftershocks
(Felzer & Brodsky, 2006)
Anomalous (slow) diffusion

\[ <x> \sim t^{0.3} \] for significantly correlated well pairs

- \log_{10}(\text{time in months})
- \log_{10}(\text{mean distance in m})
Anomalous (slow) diffusion

\[ \langle x \rangle \sim t^{0.3} \] for significantly correlated well pairs

\[ \langle x \rangle \sim t^{0.07} \] for earthquakes (Mw\(\geq 5.0\))

(Huc & Main, 2003)

Erice seminar

30 Sep 2007
Principal component analysis
Predictive trial for a single well

- **raw flow rate**
- **forecasted**
- **95% C.I. up**
- **95% C.I. low**

**Flow rate (m³/day)**

**Time (in month)**

- History Match
- Forecasted
Predictive trial for a group of wells

Note good statistical averaging
Geo-mechanical model: Reservoir architecture

Cross-section through Gullfaks

Fossen & Hesthammer (1998) 2D cross section
Geo-mechanical simulation
(2D model in cross section)

Pressure change and volumetric strain: critical case
Geo-mechanical simulation

Shear strain changes

Permeability changes

Erice seminar

30 Sep 2007
Geo-mechanical simulation
(2D model in plan view for a synthetic regular grid of wells)

c.f. data from Gullfaks
Conclusion

- Oilfield flow rate correlations behave very similarly to earthquake-earthquake triggering
- The results agree with deterministic geo-mechanical simulations \textit{iff} the system is critical
- The first principal component agrees with fault architecture
- Good short-term predictability
- Many potential applications
Next steps...

- Further field trials (DTI ‘RESURGE’ project)
  - more data welcome

- Calibration and interpretation of principal components

- Compare with induced seismicity response (independent validation of geo-mechanical simulations)

- Apply to earthquake-earthquake triggering (EU ‘TRIGS’ network)