Anisotropic modeling of elastic wave velocities evolution in deformed shales

Joël Sarout and Yves Guéguen

Ecole Normale Supérieure – Department of Earth-Atmosphere-Ocean Sciences
CNRS – Geology Laboratory – UMR 8538
• **Motivation:** Radioactive waste disposal in geologic layer (Callovo-Oxfordian shale)

• **Objectives and methodology**

• **Laboratory Experiments:** Elastic wave velocity measurements (ultrasonic frequency) under triaxial stresses on dry rocks

• **Effective Medium Modelling:** Intrinsic + microstructural anisotropies and prediction of fluid-saturated rock elastic properties

• **Conclusions and perspectives**
Radioactive waste disposal in shale layer
Callovo-oxfordian shale

**Microstructural Qualitative Analysis**

**Parall. bedding**

Clay Particules

\[ L = 10^{-7} - 10^{-6} \text{ m} \]

**Perpend. Bedding**

Clay Platelets

\[ L = 10^{-8} - 10^{-4} \text{ m} \]

**SCALE 2**

\[ L = 10^{-5} - 10^{-4} \text{ m} \]

\( \pm \) Ordered Clay Particules

MICROSCOPIC SCALE

**SCALE 3**

\[ L = 10^{-6} - 10^{-3} \text{ m} \]

Shale: Composite material

MACROSCOPIC SCALE

**Ambiant Conditions Data**

**Transverse Isotropy**

5 elastic parameters

P-wave anisotropy ~ 20-40%

Zamora et al. (2003), David et al. (2004)

Elastic wave velocity data:

S-wave ~ 1.3 à 2 km.s\(^{-1}\)
P-wave ~ 2.4 à 3.7 km.s\(^{-1}\)

Gasc et al. (1999), Zamora et al. (2003)

**Clay matrix ~50% vol. (grey)**

**Porosity ~10-15% vol. (blue)**

**Tectosilicates ~15% vol. (red)**

**Carbonates ~25% vol. (green)**

ANDRA Research Outcomes - 2005
Objectives

Explore micristructural changes under varying triaxial stresses

Methodology

Experimental tool:
Ultrasonic elastic wave velocity measurements

Theoretical tool:
Micromechanical modelling
Eshelby’s inhomogeneity

Possible Causes of Anisotropy ??

Intrinsic

Microstructural

Solide matrix:  
- clay matrix  
- Clastic inclusions

Porosity :
- Microcracks
- Equant pores

Porosity : Geometry and Consequences

Permeability...

→ Sensitive to anisotropic porosity (crack-like pores, microcracks, interfacial porosity)
→ Stress-sensitive (microcracks closure)

Callovo-Oxfordian Shale
Experiments: ambiant conditions

**Shale Specific Properties**
- Very low permeability ($10^{-20}$-$10^{-22}$ m$^2$)
- Sensitivity to aqueous fluids (hydration and osmotic swelling)

**Samples Preparation**
- Dry cutting
- Preserved samples (isolated cells) or
- Equilibrated in given air humidity conditions

![Sample Preparation Images]

**Ambiant Conditions Data**

![Graphs and Oscilloscope Image: Université Cergy Pontoise]
Experiments: Undrained triaxial deformation

<table>
<thead>
<tr>
<th>Exp. #</th>
<th>Sample</th>
<th>Loading</th>
<th>I.C.</th>
<th>Cycle $P_c$ [MPa]</th>
<th>Cycle $\sigma_{ax}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>MSE 101</td>
<td>Isotropic</td>
<td>Preserved</td>
<td>0-180-0</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>MSE 101</td>
<td>Isotropic</td>
<td>HR = 98%</td>
<td>0-200-0</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>MSE 101</td>
<td>Triaxial</td>
<td>105°C</td>
<td>0-55-15</td>
<td>0-91-0-103</td>
</tr>
<tr>
<td>IV</td>
<td>REP 2206</td>
<td>Triaxial</td>
<td>HR = 0%</td>
<td>0-55-11</td>
<td>0-88</td>
</tr>
<tr>
<td>V</td>
<td>MSE 101</td>
<td>Triaxial</td>
<td>Preserved</td>
<td>0-20-2-55-15</td>
<td>0-45-0-48</td>
</tr>
<tr>
<td>VI</td>
<td>MSE 101</td>
<td>Triaxial</td>
<td>HR = 98%</td>
<td>0-55-15</td>
<td>0-36-0-37</td>
</tr>
</tbody>
</table>
Experimental setup: triaxial cell

- **Samples:** $\varnothing = 3-4$ cm, $L = 6-8$ cm
- **Isotropic stress:** $P_c = 300$ MPa
- **Axial stress:** $\sigma_{ax} = 800$ MPa
- **Pore pressure:** $P_p = 100$ MPa
- **Direct measurements:** 16 transducers (strain gauges, piezoelectric ceramics), 2 external LVDTs, 10 pressure transducers
- **Automatic control:** in stress or in displacement (Labview™)
- **Data acquisition:** computerized (Labview™)
Experimental results: stress-strain data

- Anelastic and anisotropic strain under isotropic stress variations
- Static anisotropy increases with isotropic stress
- Irreversible strain under deviatoric stress increase applied \( \perp \) bedding plane
- Dilantancy threshold \( C' \) under deviatoric stress applied \( \perp \) bedding plane
- Shear plane rupture at \( \approx 45^\circ \)
  - \( \varepsilon_{\text{vol}}^{\text{iso}} \approx 0.55 \% \) et \( \varepsilon_{\text{vol}}^{\text{ax}} \approx 0.6 \% \)
Experimental results: elastic wave velocities

- All elastic wave velocities increase with isotropic stress
- $V_p(45^\circ)$ and $V_p(90^\circ)$ increase with deviatoric stress applied $\perp$ bedding plane
- $V_p(0^\circ)$, $V_{SH}(0^\circ)$ et $V_{SV}(0^\circ)$ decrease with deviatoric stress applied $\perp$ bedding plane
- $V_p(0^\circ)$ et $V_{SH}(0^\circ)$ rupture precursors
Elastic interpretation

**Transversely Isotropic Elasticity**

\[ \sigma_{ij} = C_{ijkl} \varepsilon_{kl} \]

\[ C_{ijkl} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \]

\[ C_{12} = C_{11} - 2C_{66} \]

**P-wave**

\[ \varepsilon = \frac{C_{11} - C_{33}}{2C_{33}} \]

**S-wave**

\[ \gamma = \frac{C_{66} - C_{44}}{2C_{44}} \]

**Anellipticity** \[ \eta \propto \varepsilon - \delta \]

\[ \delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} \]

**Measurements Inversion**

\[ C_{11} = \rho V_p^2(0^\circ) \]

\[ C_{33} = \rho V_p^2(90^\circ) \]

\[ C_{44} = \rho V_{Sv}^2(0^\circ) \]

\[ C_{66} = \rho V_{Svh}^2(0^\circ) \]

\[ C_{13} = -C_{44} + \sqrt{(C_{11} + C_{44} - 2\rho V_p^2(45^\circ))(C_{33} + C_{44} - 2\rho V_p^2(45^\circ))} \]

**Thomsen (1986)**
evolution of elastic anisotropy

- Elastic anisotropy decreases with isotropic stress
- Elastic anisotropy decreases with deviatoric stress applied \( \perp \) bedding plane
- Intrinsic elastic anisotropy \( \neq 0 \)

Anisotropy becomes a tool for accurate microstructural properties identification
microcracks evolution under varying stress

Effective medium model =
Transversely isotropic solid (Intrinsic anisotropy) +
Transversely isotropic distribution of pores/microcracks (geometry and orientation)
Micromechanical modelling: Eshelby’s problem

Compliant Porosity ⇔ Source of Strain

\[ \varepsilon = \varepsilon^0 + \Delta \varepsilon = \left( S^o + \Delta S \right) : \sigma \]

\[ \Delta S = \frac{V^*}{V} \left[ \left( S^* - S^o \right)^{-1} + C^o : (J - S) \right]^{-1} \]

Microstructural Parameters

Porosity \( \phi \), Aspect ratio \( \xi \), Fluid Incompressibility \( K_f \) ... and \( C^o \)
Theoretical predictions

Co = High Pressure Moduli
C11o = 34.5 GPa
C33o = 19.9 GPa
C44o = 8 GPa
C66o = 11.5 GPa
C13o = 5.6 GPa

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Ref. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>1/100</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.5 %</td>
</tr>
</tbody>
</table>
Experiments vs theory (1)

Good Fit for:
\[ \xi \approx \frac{1}{100} \]
\[ K_f \approx 0.5 \text{ GPa} \]
Experiments vs theory (2)

- Stanley & Christensen (2001)
- Hornby (1998)
- Vernik & Liu (1997)
- Lo, Coyner & Toksöz (1986)
Conclusions

**Callovo-Oxfordian shale background rock is transversely isotropic**

**Crack-like pores/microcracks are more or less aligned with the rock bedding plane**

Mean aspect ratio ≈ 1/100

**Transversely isotropic effective properties**

**Elastic data:**
- Wet shale anisotropy < Dry shale anisotropy at ambient conditions
- Wet shale elastically stiffer than dry shale at ambient conditions

Elastic anisotropy $\varepsilon$, $\gamma$ and $\delta$ decrease with increasing isotropic stress in dry shale

**Modeling:**
- Elastic shear anisotropy $\gamma$ should be insensitive to fluid incompressibility $K_f$
Perspectives...

**Experimental Perspectives**

- Perform higher confining pressure experiments on dry shale samples
  ⇒ assess intrinsic elastic properties
- Perform experiments on shale samples cored in different directions w.r.t. bedding plane

**Theoretical Perspectives**

- Micromechanical model limited to horizontal oblate spheroids
- Approximate shale intrinsic elastic properties by means of an energetically equivalent isotropic solid
- Extrapolate ultrasonic laboratory data to low frequency range using Brown & Korringa poroelastic relations
Annexe I : ambient conditions data

![Graph showing velocity vs. angle for MSE 101 (depth 613 m)]
appendix II: components of Eshelby's tensor
Appendix III: Water and clays

Pore Water

1 μm

100 nm

10 nm