A REALISTIC 3-D FAULT MODEL INCLUDING THERMAL PRESSURIZATION OF PORE FLUIDS

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Goals

- Understanding the traction evolution and the weakening mechanism during earthquakes;
- Quantify the importance of one of the so-called second order effects in the formulation of a realistic governing law;
- Modeling the dynamic fault weakening through thermal pressurization;
- Quantify the frictional heat developed during dynamic fault slip;
- Emphasizing the implications on the inferred values of the characteristic slip–weakening distance $d_0$ and the fracture energy $E_G$;
- Explore how a time variable porosity can modify the breakdown processes;
- Infer some scaling relations between relevant physical quantities and thermal pressurization parameters.
We solve fully dynamic, spontaneous problem (the fundamental elasto–dynamic equation), without body forces $f$.

We consider truly 3-D (not mixed – mode) problem, for which solutions (for instance the fault slip, i. e. the particle displacement discontinuity) are in the form: $u = (u_1(x_1,x_3,t), 0, u_3(x_1,x_3,t))$

Numerical experiments refer to a vertical fault.
The spatial computational domain is discretized using cubic building blocks in a conventional grid. The medium is linearly elastic except that in the fault plane …

… that obeys to the Fault Boundary Condition, i.e. to the governing law, that relates the fault friction $\tau$ to some physical observables. In general is:

$$
\tau = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e) \sigma_n^{\text{eff}}(\sigma_n, p_{\text{fluid}})
$$

where $\mu$ is the friction coefficient and $\sigma_n^{\text{eff}}$ the effective normal stress that can change during time.

An explicit displacement discontinuity is assumed between the two sides of the fault: Traction–at–Split–Nodes scheme (Day, 1982a, 1982b; Andrews, 1999).

Mathematical background

1 – D Fourier’s heat conduction equation:

\[ \frac{\partial}{\partial t} T = \chi \frac{\partial^2}{\partial \zeta^2} T + \frac{1}{c} q \]

Coupling of temperature \( T \) with pore fluid pressure \( p_{\text{fluid}} \):

\[ \frac{\partial}{\partial t} p_{\text{fluid}} = \frac{\alpha_{\text{fluid}}}{\beta_{\text{fluid}}} \frac{\partial}{\partial t} T - \frac{1}{\beta_{\text{fluid}} \Phi} \frac{\partial}{\partial t} \Phi + \omega \frac{\partial^2}{\partial \zeta^2} p_{\text{fluid}} \]

where \( \chi \) is the thermal diffusivity, \( c \) the heat capacity for unit volume, \( \alpha_{\text{fluid}} \) the coefficient of thermal expansion, \( \beta_{\text{fluid}} \) the compressibility coefficient, \( \Phi \) the porosity and \( \omega = k/\eta_{\text{fluid}} \beta_{\text{fluid}} \) the hydraulic diffusivity (being \( k \) the permeability of the medium and \( \eta_{\text{fluid}} \) the dynamic fluid viscosity). Analytical solutions at \( \zeta = 0 \) are:

\[
T^{\omega f}(\xi_1, \xi_3, t) = T_0 f + \frac{1}{2c \omega(\xi_1, \xi_3)} \int_0^{t-\tau} dt' \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2\sqrt{\chi(t-t')}} \right) \tau(\xi_1, \xi_3, t') v(\xi_1, \xi_3, t')
\]

\[
\bar{p}_{\text{fluid}}^{\omega f}(\xi_1, \xi_3, t) = p_{\text{fluid}0 f} + \frac{\gamma}{2c \omega(\xi_1, \xi_3)} \int_0^{t-\tau} dt' \left\{ \frac{\chi}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2\sqrt{\omega(t-t')}} \right) + \frac{\omega}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2\sqrt{\omega(t-t')}} \right) \right\} \tau(\xi_1, \xi_3, t') v(\xi_1, \xi_3, t') - \frac{2w(\xi_1, \xi_3)}{\gamma} \frac{1}{\beta_{\text{fluid}} \Phi(t')} \frac{\partial}{\partial t'} \Phi(\xi_1, 0, \xi_3, t')
\]

Bizzarri and Cocco (2006a, 2006b, JGR, 111, B05303, B05304)
Temperature change on the fault

Rupture onset

Crack like models

Pulse like models (with healing and finite slip duration)

Numerical experiments

Adiabatic prediction:

\[ \Delta T^{(adiab)} \approx \frac{\tau_f u}{2cw} \]

Bizzarri and Cocco (2006a, 2006b, JGR, 111, B05303, B05304)
Results with SW law

Dry fault ( $\sigma_n^{\text{eff}} = \text{const}$ )

Wet fault ( $\sigma_n^{\text{eff}}$ varies )

Fluid pressure change (Pa)

Slip velocity (m/s)

Along strike direction (m)

Along dip direction (m)

Fluid pressure change (Pa)

Slip velocity (m/s)

Along strike direction (m)

Along dip direction (m)

$\sigma_n^{\text{eff}} = 0$

$\omega \equiv \frac{k}{\eta_{\text{fluid}} \beta_{\text{fluid}} \Phi}$
Results with DR law

\[ \tau = \left[ \mu_\ast + a \ln \left( \frac{v}{v_\ast} \right) + b \ln \left( \frac{\Psi v_\ast}{L} \right) \right] \sigma_n \]

Bizzarri and Cocco (2006a, 2006b, JGR, 111, B05303, B05304)

Dry fault
Dependence on friction parameters

- Temperature change vs. Time
- Traction vs. Slip

Graphs showing the effect of different friction parameters on temperature change and traction.
The breakdown zone

- **Traction (Pa)**
  - Time (s)
  - Slip (m)

- **Temperature change (°C)**
  - Time (s)

- **State variable (s)**
  - Time (s)
Scaling laws #1

\[
\Delta \tau_b = \Delta \tau_b^{(\text{dry})} \left( 1 + a_2 e^{-\frac{w}{\bar{d}}} \right) \left( a_3 + a_4 \sqrt{\frac{\omega_*}{\omega}} \right)
\]

where \( \omega_* = 0.02 \text{ m}^2/\text{s} \) and

for SW: \( a_2 = 0.25, a_3 = 0.86, a_4 = 0.14, \bar{d} = d_0^{(\text{dry})} \)

for RS: \( a_2 = 5.5, a_3 = 0.1, a_4 = 0.9, \bar{d} = d_0^{eq(\text{dry})} \)

Bizzarri and Cocco (2006a, 2006b, JGR, 111, B05303, B05304)
Scaling laws #2

\[ d_{0}^{eq} = d_{0}^{eq(dry)} \left( 1 + b_{2} e^{-\frac{w}{d}} \right) b_{3} + b_{4} \sqrt{\frac{\omega_{*}}{\omega}} \]

where \( \omega_{*} = 0.02 \text{ m}^2/\text{s} \) and

\[ b_{2} = 5, \quad b_{3} = 0.2, \quad b_{4} = 0.8; \]

\[ d = d_{0}^{eq(dry)} \]

**Table:**

<table>
<thead>
<tr>
<th>( w ) (m)</th>
<th>( d_{0}^{eq(dry)}/L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>9.33</td>
</tr>
<tr>
<td>0.5</td>
<td>10.7</td>
</tr>
<tr>
<td>0.1</td>
<td>22.2</td>
</tr>
<tr>
<td>0.035</td>
<td>44.5</td>
</tr>
<tr>
<td>0.01</td>
<td>53.2</td>
</tr>
<tr>
<td>0.001</td>
<td>53.5</td>
</tr>
</tbody>
</table>

**Bizzarri and Cocco (2006a, 2006b, JGR, 111, B05303, B05304)**
Effects of porosity evolution #1

If porosity $\Phi$ evolves through time the heat source function for the elementary solution is:

$$
\tilde{q}(\xi, \zeta, \xi_3, t) = \begin{cases} 
\frac{\tau(\xi, \zeta, t)v(\xi, \zeta, t)}{2w(\xi_3, \zeta_3)} & t > 0, |\zeta| \leq w(\xi_1, \xi_3) \\
0 & |\zeta| > w(\xi_1, \xi_3)
\end{cases}
$$

The solution for the pore fluid pressure is (erf(.) is the error function):

$$
\tilde{P}_{\text{fluid}}^{w}(\xi, \zeta, \xi_3, t) = P_{\text{fluid}}^{st} + \frac{\gamma}{4w(\xi_1, \xi_3)} \int_0^{t-\varepsilon} dw \left\{ -\frac{\varepsilon}{\omega - \chi} \left[ \text{erf} \left( \frac{\zeta + w(\xi_1, \xi_3)}{2\sqrt{\chi}(t-t')} \right) - \text{erf} \left( \frac{\zeta - w(\xi_1, \xi_3)}{2\sqrt{\chi}(t-t')} \right) \right] + \\
+ \frac{\omega}{\omega - \chi} \left[ \text{erf} \left( \frac{\zeta + w(\xi_1, \xi_3)}{2\sqrt{\omega}(t-t')} \right) - \text{erf} \left( \frac{\zeta - w(\xi_1, \xi_3)}{2\sqrt{\omega}(t-t')} \right) \right] \right\} \right\} \tau(\xi_1, \xi_3, t')v(\xi, \zeta, \xi_3, t') + \\
$$

On the fault plane (i.e. in the limit $\zeta \to 0$) the pore fluid pressure change is:

$$
\tilde{P}_{\text{fluid}}^{w}(\xi, \zeta_3, t) = P_{\text{fluid}}^{st} + \frac{\gamma}{2w(\xi_1, \xi_3)} \int_0^{t-\varepsilon} dw \left\{ -\frac{\varepsilon}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2\sqrt{\chi}(t-t')} \right) + \\
+ \frac{\omega}{\omega - \chi} \text{erf} \left( \frac{w(\xi_1, \xi_3)}{2\sqrt{\omega}(t-t')} \right) \right\} \right\} \tau(\xi_1, \xi_3, t')v(\xi, \zeta_3, t') + \\
$$
Effect of porosity evolution #2

Assuming that porosity evolves accordingly to the law proposed by Segall and Rice (1995), and assuming $L_{SR} = L$

$$\Phi(\xi_1, \zeta, \xi_3, t) = \Phi_* - \varepsilon_{SR} \ln \left( \frac{\Psi \, v_*}{L_{SR}} \right)$$

Bizzarri and Cocco (2006a, 2006b, JGR, 111, B05303, B05304)
Importance of the evolution law

DR (ageing evolution law)

\[ \tau = \left[ \mu_* + a \ln \left( \frac{v}{v_*} \right) + b \ln \left( \frac{\Psi v_*}{L} \right) \right] \sigma_n \]

\[ \frac{d\Psi}{dt} = 1 - \frac{\Psi v}{L} \]  

Dry fault

RD (slip evolution law)

\[ \tau = \left[ \mu_* + a \ln \left( \frac{v}{v_*} \right) + b \ln \left( \frac{\Psi v_*}{L} \right) \right] \sigma_n \]

\[ \frac{d\Psi}{dt} = -\frac{\Psi v}{L} \ln \left( \frac{\Psi v}{L} \right) \]  

Dry fault
Melting caused by thermal pressurization should be considered as a *macroscopic phenomenon* involving the whole slipping zone thickness.

Flash heating of contact asperities of fault (*Rice, 1999, 2006*) is due to the highly localized heating created by a fast slip episode in the region near the moving asperity contacts on the slipping surface (⇒ *scale of the process* ≅ 1 – 10 μm).

In other words, it is generated on a *relatively small contact area* that sustains high shear strength. Under these conditions, the sudden and local increase of temperature causes a diminution of the contact’s shear strength and a friction drop with slip rate.

Thermal pressurization *strongly depends* on the effective normal stress changes, while flash heating *does not strongly change* with normal stress.

This because in the latter case the affected zone is very small and extremely thin, thus the capacity to support normal stress and the net area of contact might not be affected (*Rice and Cocco, 2006*). The *scale of process is the thickness of the thermal boundary layer* ( \( \sqrt{2 \kappa t} \)) ≅ few cms.
The inclusion of thermal pressurization of pore fluid changes the shape of the rupture front and the traction evolution within cohesive zone;

In particular, thermal pressurization can enhance the transition for sub– to super–shear rupture velocity;

The $\Delta \tau_b$ and $d_0^{eq}$ strongly depend on the physical processes occurring within the cohesive zone $\Rightarrow$ importance of the formulation of the governing law;

In our models they are inversely proportional to fault thickness and hydraulic diffusivity;

Thermal pressurization increases the SW distance up to half of a meter (reference $d_0^{eq(dry)} \cong 10$ mm);

For time variable porosity or for the slip evolution law $d_0^{eq}$ may become meaningless.
1) Results with thermal pressurization will predict a nearly complete stress drop and therefore we should find a signature of these high stress drop values in the radiated seismograms. Seismological estimates of stress drop do not support such an evidence; the estimation of stress drop from seismic waves is biased (for instance by the difficulties in analyzing high frequency radiation)
or the effects of thermal pressurization on the dynamic traction evolution are less pronounced
2) We confirm that thermal pressurization reduces the temperature rise. However, if the thickness of the slipping zone is extremely thin, the increase of temperature is still large: for half meter of slip in 1 mm of slipping zone thickness, the $\Delta T \cong 800^\circ C$ and can still be sufficiently large to generate melting of gouge materials; this might be due to the fact that we consider constant permeability value in the slipping and in the damage zone.

3) …but if melting occurs, the Terzaghi effective stress law is not anymore valid to represent the frictional properties of the fault zone; We need constitutive equation in melting regime!
4) The analytical solution of the frictional heating problem implies the assumption that all the work spent in allowing the crack advance and fault sliding is heat. This raises a further question on the meaning of fracture when thermal pressurization controls the dynamic rupture propagation;

Pittarello et al. (2007) confirms that heat is paramount during coseismic rupture.

5) What is the “true” evolution of the porosity? What about the dependence of permeability from effective normal stress?

We need more observations to constrain the evolution equation for the porosity and permeability.
6) What is the mechanical behavior of the damage zone surrounding the slipping zone? The creation of microcracks can enhance fluid migration.

We need to know the rheological equations!
Thank you!

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Parameters, Notes, etc.

To not be displayed directly. Referenced above.
**Why “truly” 3 – D?**

*Remembering the dimensionality of the problem:*

<table>
<thead>
<tr>
<th>Dimensionality</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 – D Mode II</td>
<td>Pure in–plane</td>
<td>( u = (u_1(x_1,t), 0, 0) )</td>
</tr>
<tr>
<td>2 – D Mode III</td>
<td>Pure anti–plane</td>
<td>( u = (0, u_2(x_1,t), 0) )</td>
</tr>
<tr>
<td>3 – D Mixed mode</td>
<td></td>
<td>( u = (u_1(x_1,t), u_2(x_1,t), 0) )</td>
</tr>
<tr>
<td>3 – D having only one non null component</td>
<td></td>
<td>( u = (u_1(x_1,x_2,t), 0, 0) )</td>
</tr>
<tr>
<td>Truly 3 – D</td>
<td></td>
<td>( u = (u_1(x_1,x_2,t), u_2(x_1,x_2,t), 0) )</td>
</tr>
</tbody>
</table>

**Remark:**

- The terms “pure” and “anti–plane” refer to the nature of the deformation in the plane (Mode II) and normal to the plane (Mode III) respectively.
- The “Mixed mode” indicates a combination of both types of deformation.
- The “having only one non null component” refers to a situation where only one of the components of the displacement vector is non–zero.
- **Truly 3–D** indicates a scenario where all components of the displacement vector are non–null, capturing the full three-dimensional behavior of the problem.
• **Conventional – grid (CG):**

![Diagram of a conventional grid with variables $U$, $\sigma_{ij}$, and $h$.](image)
• **Traction – at – Split – Nodes ( TSN ):**

\[
\begin{align*}
V(i, j_f^1 + 1,k) \\
V^+(i, j_f^1,k) \quad [\text{vtop (nd)}] \\
V^{-}(i, j_{\text{end}} + 1,k) \quad [\text{vbot (nd)}] \\
\end{align*}
\]

Fault surface \((j_f^1)\)

\(\{\sigma_{ij}\}\)

Andrews (1999), Bizzarri and Cocco (2005)

*Discontinuum medium* (continuum mechanics equations of motion are applied to each half-space individually; the fault is an explicit discontinuity in displacement)
Non - laminar fault model

Fig. 2. Schematic section across the North Branch San Gabriel fault zone illustrating position of the structural zones of the fault. The diagram is not to scale.

Chester, Evans and Biegel, J. Geoph. Res., 1993
Sibson, BSSA, 2003
Chester and Chester, SSA, SCEC meetings 2004
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\lambda = G$</td>
<td>27 GPa</td>
</tr>
<tr>
<td>$v_p$</td>
<td>5196 m/s</td>
</tr>
<tr>
<td>$v_\phi$</td>
<td>3000 m/s</td>
</tr>
<tr>
<td>$\Delta x_1 = \Delta x_1'$</td>
<td>25 m</td>
</tr>
<tr>
<td>$\Delta x_2$</td>
<td>100 m</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>$0.83671 \times 10^{-3}$ s</td>
</tr>
<tr>
<td>$\sigma_n - P_{\text{fluid}}$</td>
<td>30 MPa</td>
</tr>
<tr>
<td>Slip-weakening model parameters</td>
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<tr>
<td>$\tau_0$</td>
<td>20 MPa</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>0.93333</td>
</tr>
<tr>
<td>$\mu'_0$</td>
<td>0.33333</td>
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<tr>
<td>$\delta_0$ (at $t = 0$; reference)</td>
<td>0.8</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.1 m</td>
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<td>Rate- and state-dependent models parameters</td>
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<tr>
<td>$t_{\text{ref}}$</td>
<td>$t (v_{\text{ref}})$</td>
</tr>
<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$b$</td>
<td>0.016</td>
</tr>
<tr>
<td>$L$</td>
<td>0.01 m</td>
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<tr>
<td>$v_{\text{ref}}$</td>
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<tr>
<td>$\mu'$</td>
<td>0.56</td>
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<tr>
<td>$\mu'_{\text{ref}}$</td>
<td>$1 \times 10^{-4}$ s</td>
</tr>
<tr>
<td>$\mu'_{\text{outside the mechanics}}$</td>
<td>$\psi'' (v_{\text{ref}})$</td>
</tr>
<tr>
<td>$\frac{c_{\text{LDM}}}{c_{\text{LDM}}}$ (reference)</td>
<td>0.53</td>
</tr>
<tr>
<td>Thermal pressurization parameters</td>
<td></td>
</tr>
<tr>
<td>$T_0$</td>
<td>100 °C</td>
</tr>
<tr>
<td>$h$ (reference)</td>
<td>$5 \times 10^{-7}$ m$^2/$s</td>
</tr>
<tr>
<td>$\eta_{\text{fluid}}$</td>
<td>$1 \times 10^{-4}$ Pa s</td>
</tr>
<tr>
<td>$c$</td>
<td>$3 \times 10^3$ J/(m$^3$ °C)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>$1 \times 10^{-4}$ m$^2$/s</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\epsilon_{\text{fluid}}$</td>
<td>$1.5 \times 10^{-3}$ °C$^{-1}$</td>
</tr>
<tr>
<td>$\beta_{\text{fluid}}$</td>
<td>$1 \times 10^6$ Pa$^{-1}$</td>
</tr>
<tr>
<td>$w$ (reference)</td>
<td>0.035 m</td>
</tr>
</tbody>
</table>