Uncertainties in Seismicity Models: Towards Bayesian Earthquake Forecasting

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It is not certain that everything is uncertain.
- Blaise Pascal

Outline

I) **Motivation:**

How do uncertainties impact earthquake forecasts?

How can we characterize magnitude uncertainties?

II) **Magnitude Noise in an Aftershock Model**

Impact on seismic rate estimates, forecasts and tests

III) **Data assimilation**

Framework for uncertainties

Recursive Bayesian forecasting for temporal renewal process

Uncertainties: Pallett Creek

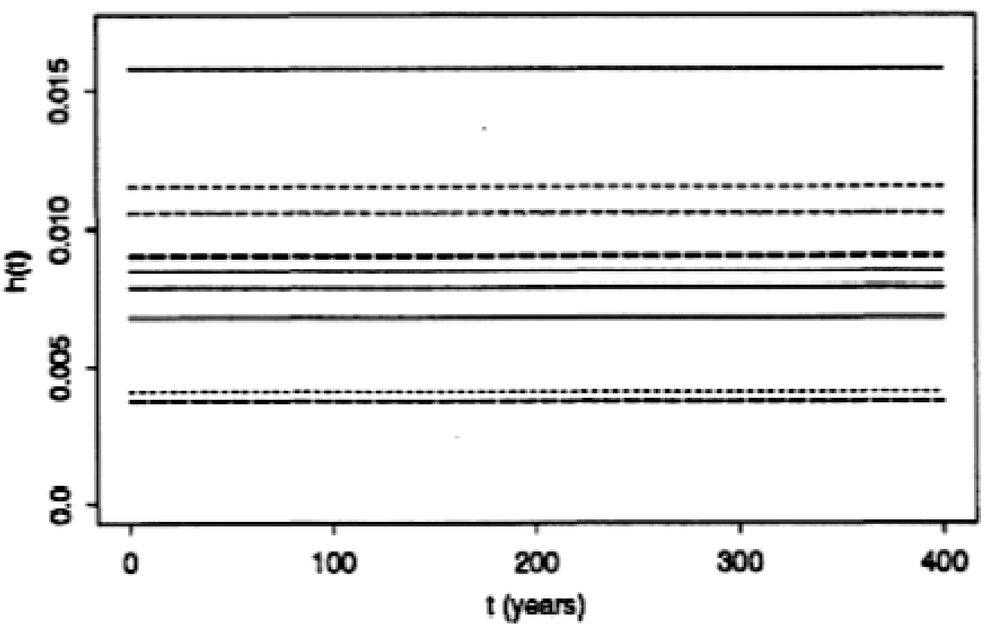


Figure 5. Pallett Creek hazard functions based on the exponential model for 10 randomly sampled parameter values, using central estimates of past rupture times.

Table 1. Estimated Dates of Occurrence for Events at Pallett Creek Based on Table 3 of *Sieh et al.* [1989]

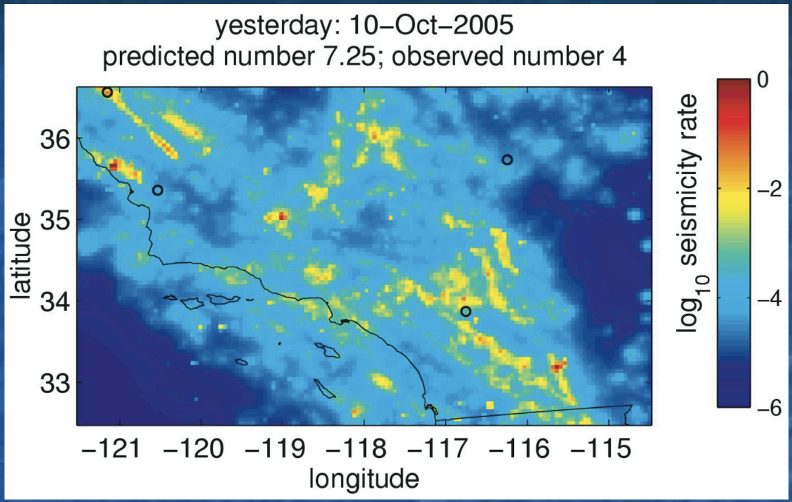
Event	Date	95% Confidence Interval	s.d.
Z	Jan.9, 1857	NA	0
X	Dec.8, 1812	NA	0
V	1480	(1465-1495)	7.5
T	1346	(1329-1363)	8.5
R	1100	(1035-1165)	32.5
N	1048	(1015-1081)	16.5
I	997	(981-1013)	8
F	797	(775-819)	11
D	734	(721-747)	6.5
C	671	(658-684)	6.5
B	before 529	NA	NA

NA, not applicable

Rhoades et al (1994)

Ogata (1999, 2002)

ETAS Forecast



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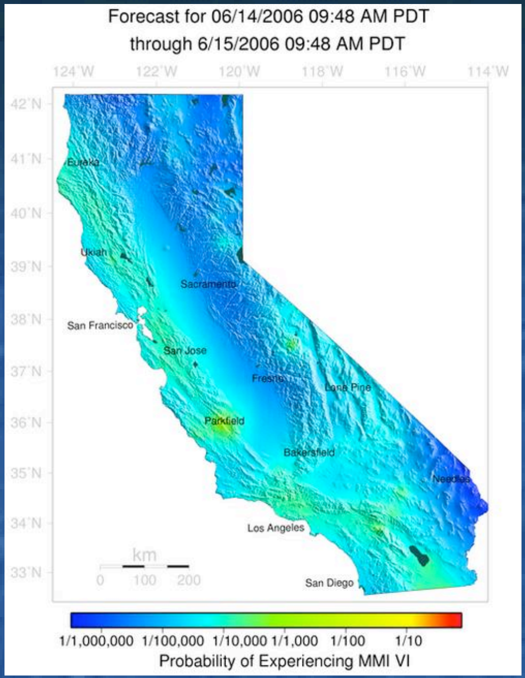
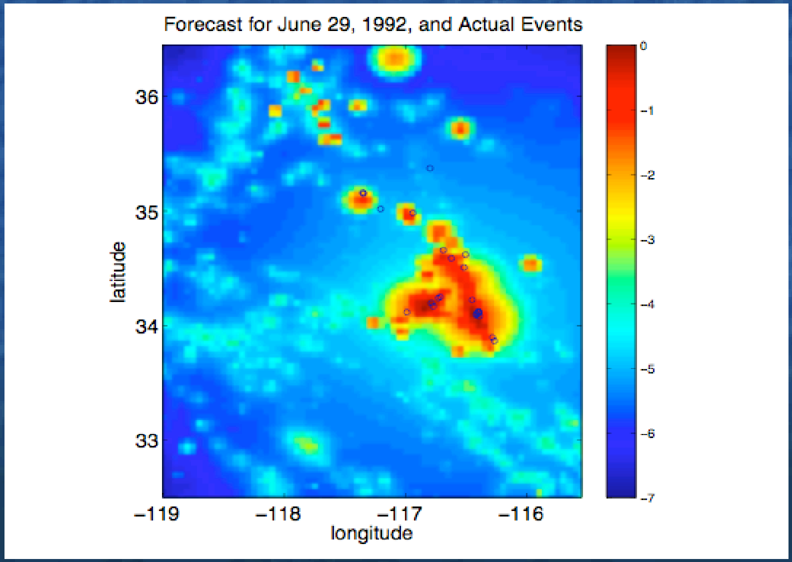


ETAS Hindcast



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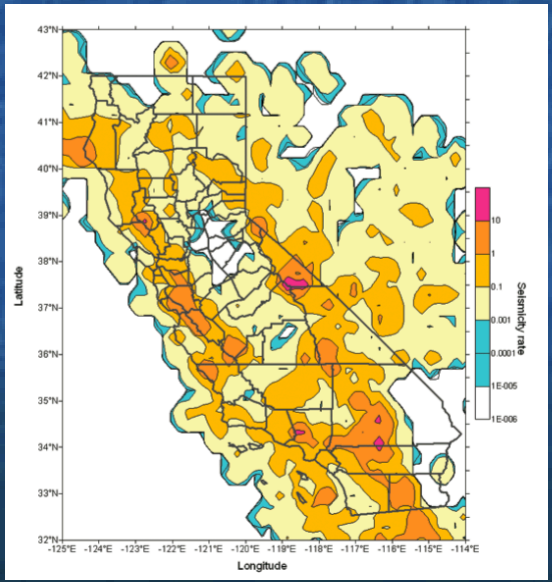


Magnitude Uncertainties

What is the magnitude uncertainty in forecast models?

- Uncertainty in measuring a particular magnitude?
  - Inversion process (e.g. hypoinverse - reports uncertainties)
  - Velocity model, parameters, assumptions, instruments
  - Compare estimate of same type of magnitude from different methods or catalogs (e.g. CMT vs USGS moment magnitude)
  - For ANSS in California, only NCSN reports hypoinverse uncertainties

INTRA

- Uncertainty in the "forecast magnitude" - Mf ?
  - Systematic and random differences between different magnitudes reported in catalogs (e.g. CMT's Mw vs PDE mb and Ms)
  - CSEP's authorized data ANSS reports many different magnitudes
  - E.g. NCSN reports two magnitudes (Ms and Md)
  - Which is the forecast magnitude?

INTER

Moment Magnitude Uncertainty

Y.Y. Kagan / Physics of the Earth and Planetary Interiors 135 (2003) 173–209

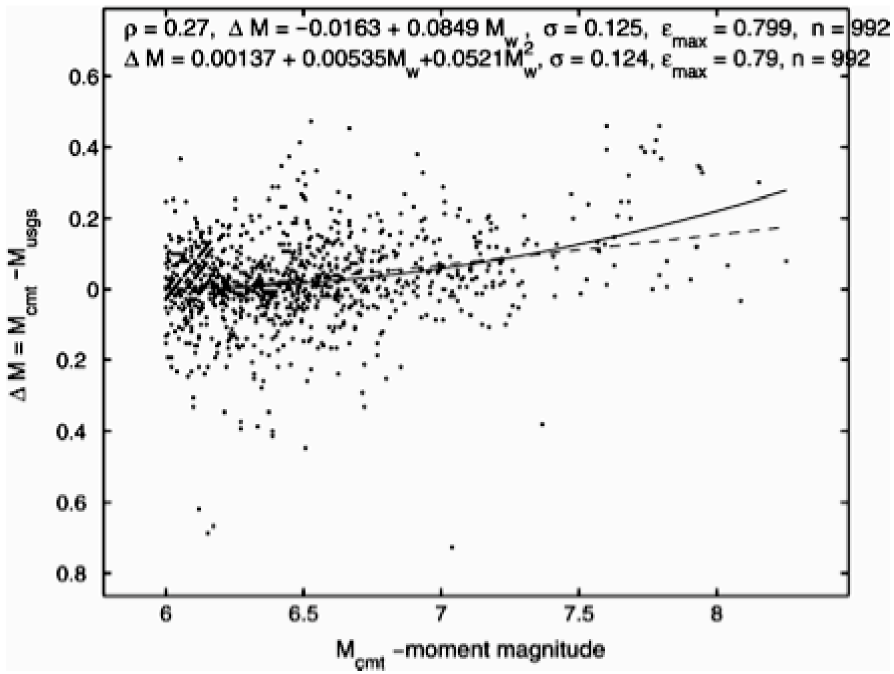
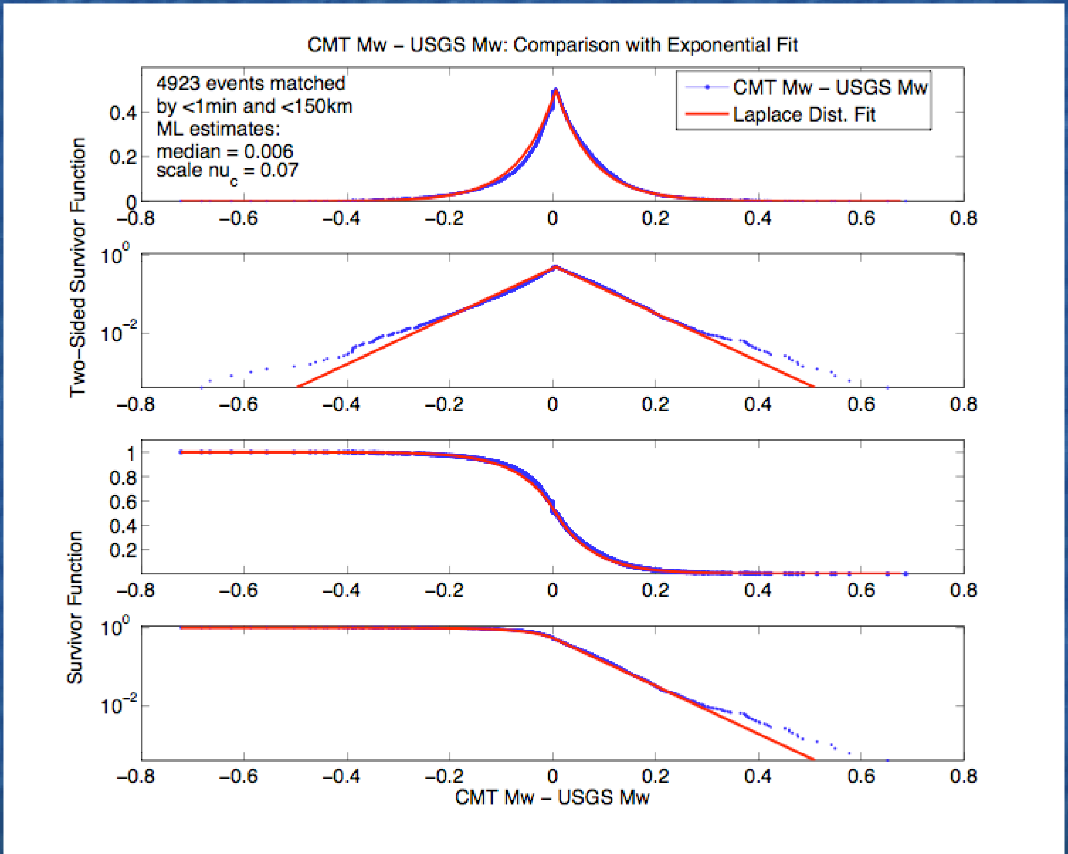
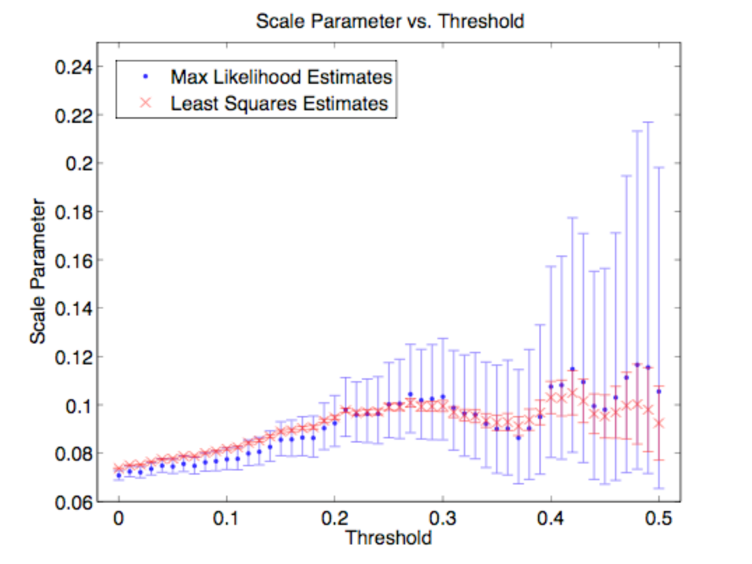


Fig. 12. Dependence of the moment magnitude difference Δm between two catalogs (Harvard and GS-MT) on the Harvard moment magnitude ($m \geq 6.0$). Dashed line—linear approximation (Eq. (8)); solid line—quadratic approximation (Eq. (9)). Results of both regressions are written at the top of the plot: ρ is the coefficient of correlation, σ the S.E., ε_{max} the maximum difference and n the number of pairs.

Kagan (2003)

Moment Magnitude Uncertainties CMT vs USGS

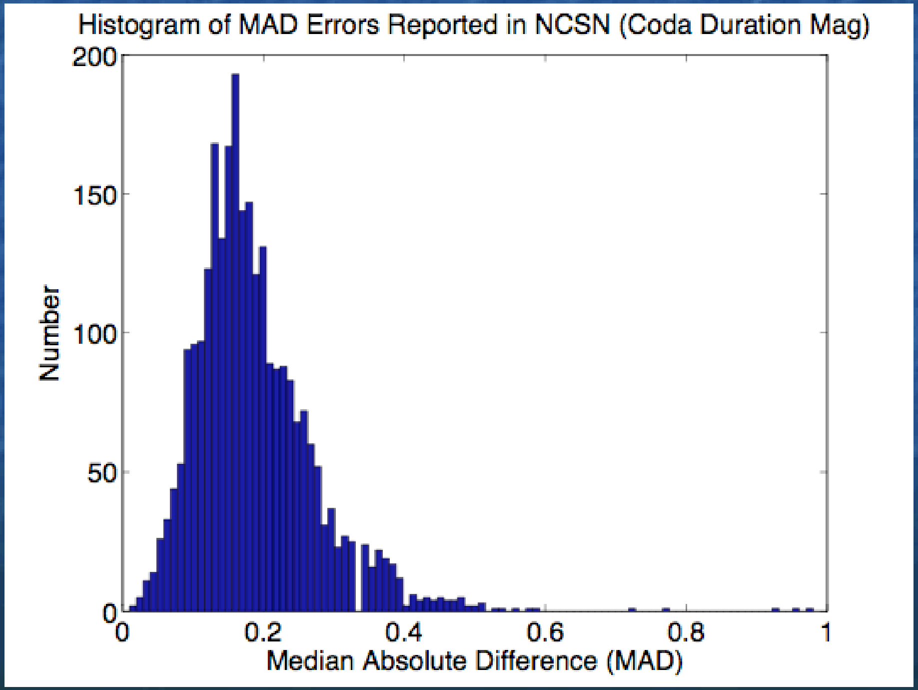


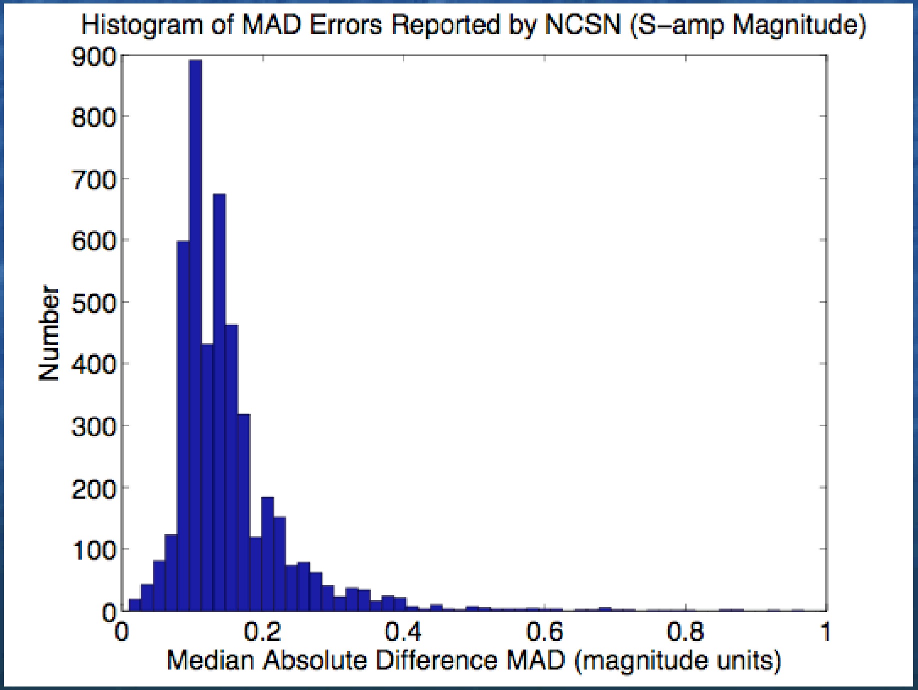


$$p_\nu(\nu) = \frac{1}{2\nu_c}\exp(-\frac{|\nu|}{\nu_c})$$

ANSS: Magnitude Uncertainty

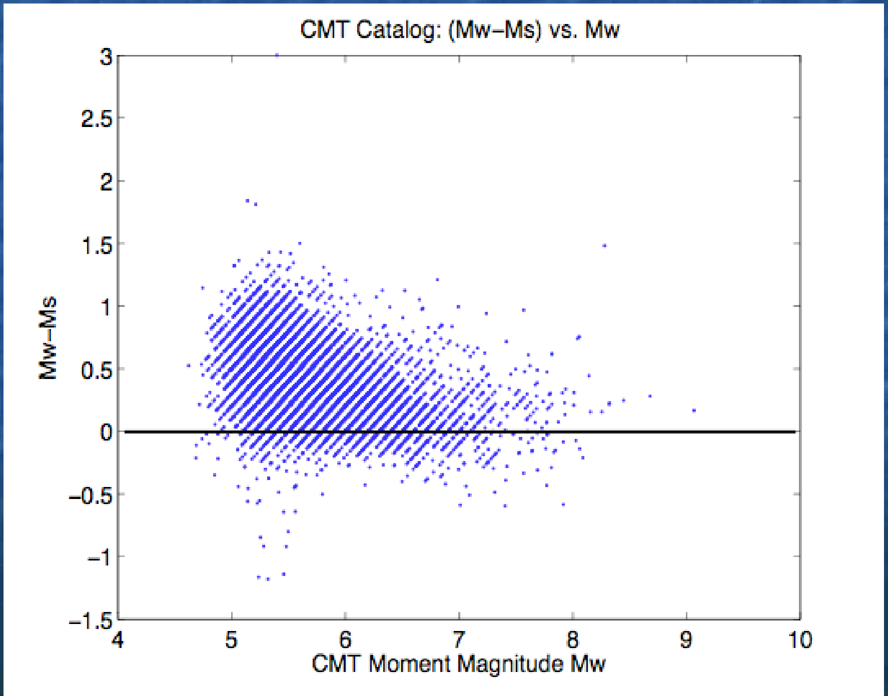
Within California, the only (?) network in the ANSS that routinely reports uncertainties in magnitude is the NCSN using the hypoinverse format: Median Absolute Difference (MAD)

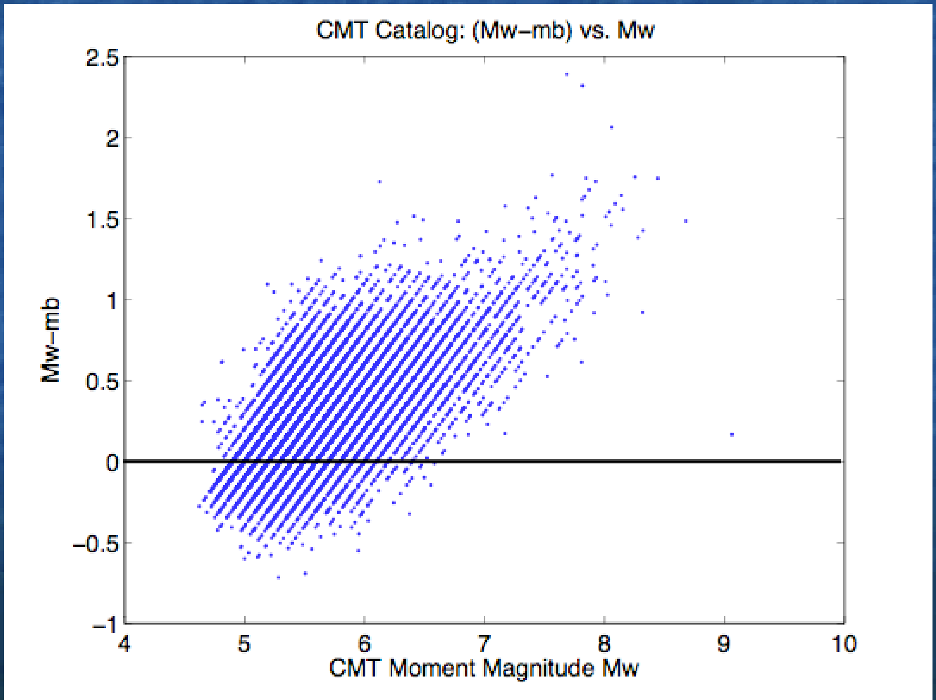




“Forecast Magnitude” Uncertainty

- ANSS in CA reports many magnitudes:
  - Richter, moment, coda duration, coda amplitude, S amplitude ...
  - NCSN reports amplitude magnitude and coda duration magnitude for the same event (in the hypoinverse format):
  - differences up to 0.4 usually 0.1 to 0.2
- CMT reports PDE mb and/or Ms





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I) **Motivation:**

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II) **Magnitude Noise in an Aftershock Model**

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Effects of Magnitude Noise in a Aftershock Cluster Model

Poisson Cluster Model:
(mainshock + aftershocks)

$$\lambda^o = \sum_i^n \frac{k e^{a(m_i^o - m_d)}}{(t - t_i + c)^p} = \sum_i^n w_i \epsilon_i$$

Noisy magnitudes:

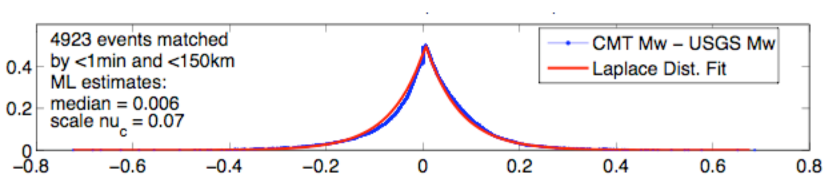
$$m^o = m^t + \nu$$

Quenched weights:

$$w_i = k e^{a(m_i^t - m_d)} (t - t_i + c)^{-p}$$

Random variables:

$$\epsilon_i = e^{a\ \nu_i}, \quad p_\nu(\nu) = \frac{1}{2\nu_c} \exp(-\frac{|\nu|}{\nu_c})$$



What are the fluctuations in the seismic rate estimates?

Magnitude Noise in Cluster Models

What are the fluctuations in the hazard due to noise?

$$\lambda^p(t) - \lambda(t) = \sum_i^n \frac{k e^{a(m_i^o - m_d)}}{(t - t_i + c)^p} (e^{a\nu_i} - 1) = \sum_i^n w_i(\epsilon_i - 1)$$ Sum of random variables

$$p_\epsilon(\epsilon) = \frac{1}{2a\nu_c\ \epsilon^{(1+\alpha)}} \qquad \alpha = \frac{1}{a\nu_c}$$

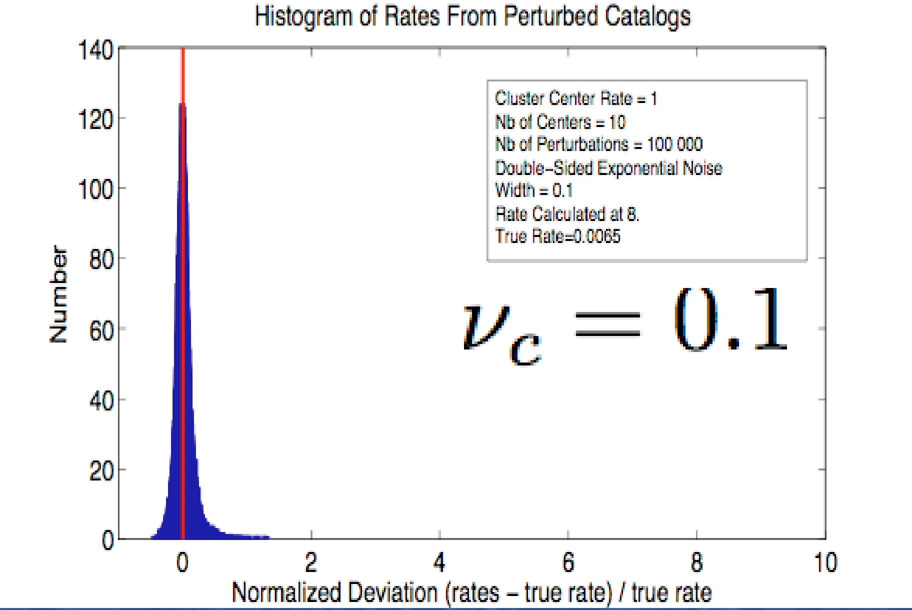
$$a = \alpha_{ETAS} \ln(10) = 2.3$$

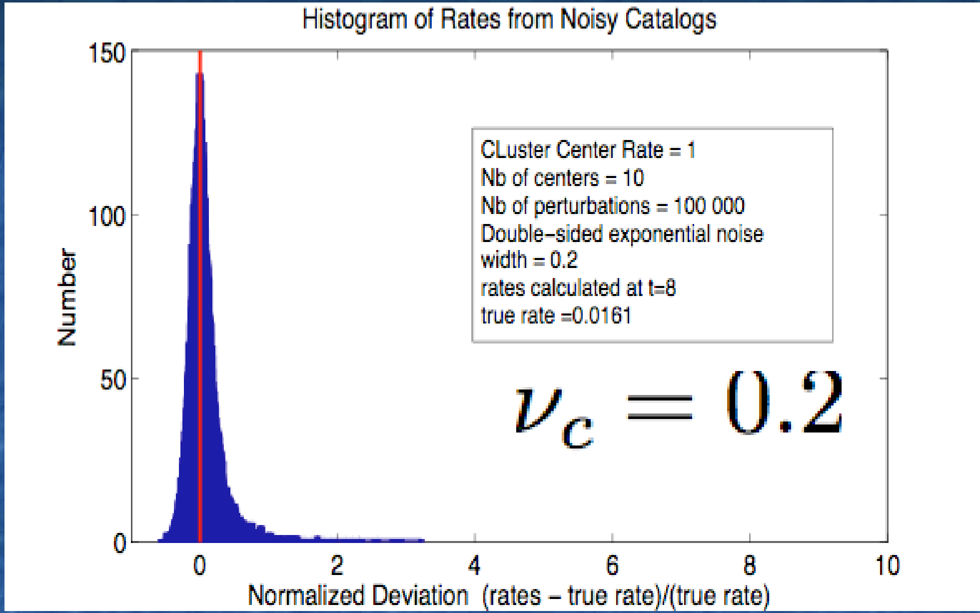
$$\nu_c = 0.2 \qquad \nu_c = 0.3$$

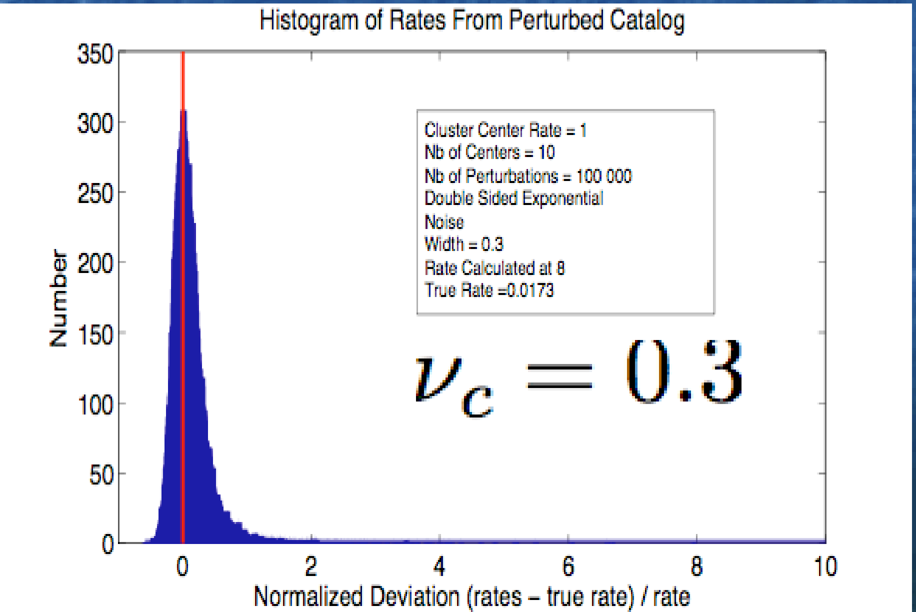
$$\alpha = 2.17 \qquad \alpha = 1.45$$

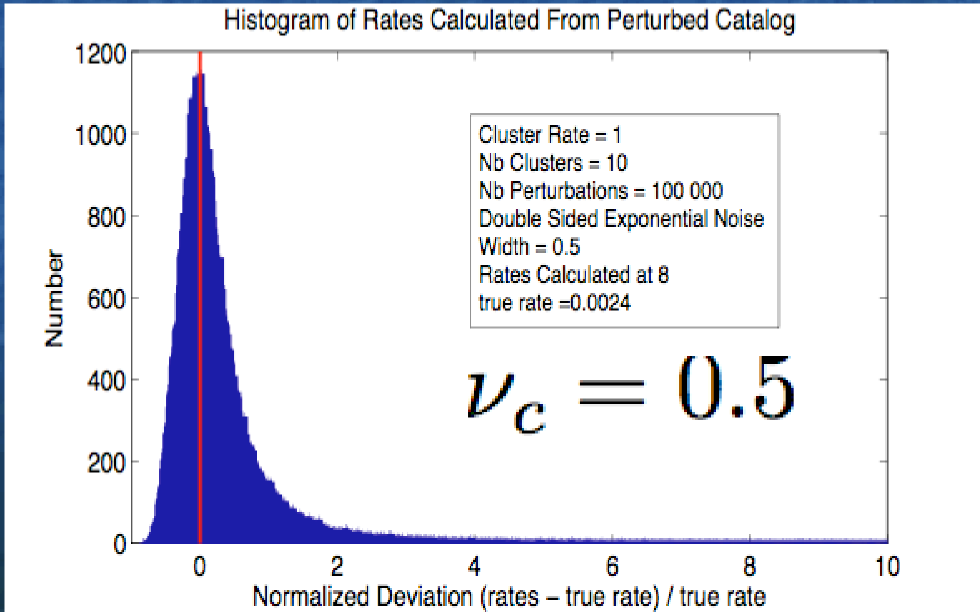
- Strong fluctuations in seismic rate estimates
- No central limit theorem for noise > 0.2
- Quenched weights strongly depend on catalog realization
- Averaging effects depend on parameter values

Simulations of Perturbed Rates

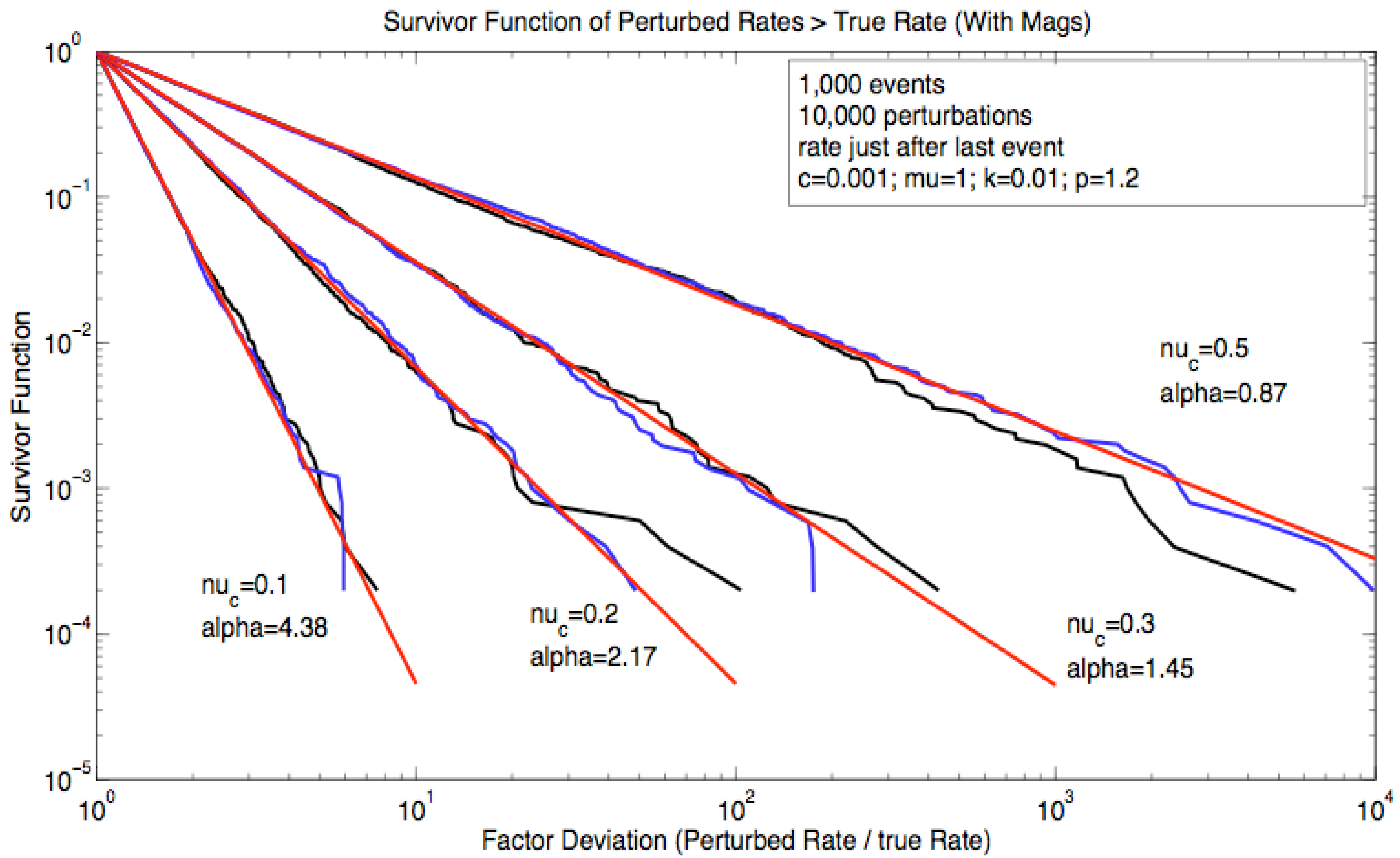








Seismic Rate Estimation: Power Law Tail Behavior



Noisy Forecasts in RELM Likelihood Tests

"Truth / Observations":

- simulate a synthetic catalog with the Poisson cluster model and GR law
- calculate (true) rate at time steps using known parameters
- generate 1000s of "modified" observations consistent with the rate assuming Poisson process (as in RELM likelihood tests)

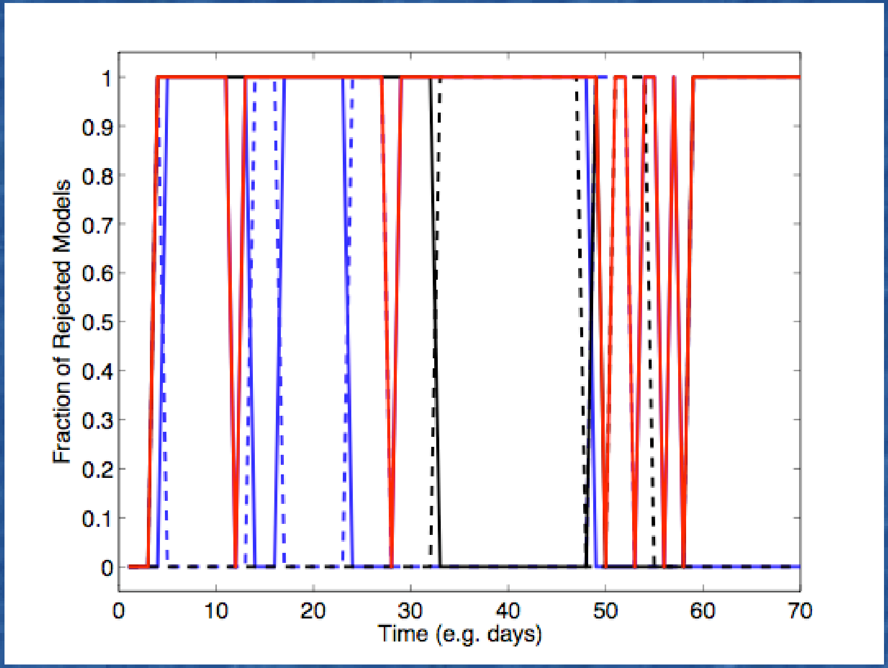
"Models / Forecasts":

- perturb magnitudes of catalog 1000s of times
- calculate seismic rate for each perturbed catalog based on Poisson cluster model equation with known parameters
- for each such "model", generate 1000s of simulated observations consistent with its calculated rate from perturbed catalog

"RELM Likelihood test / Evaluation":

- perform N and L tests on "models" against "observations"

Fraction of Models Rejected by N, L Tests vs Time



Because of magnitude noise, a good model can be rejected.
What are the implications for long term testing?
Which parameter regime is adequate?

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III) **Data assimilation**

Framework for uncertainties

Mathematical set up for point processes

Recursive Bayesian forecasting for temporal renewal process

Data Assimilation

- Talagrand (1997): “The purpose of data assimilation is to determine as accurately as possible the state of the atmospheric (or oceanic) flow, using all available information”

- Statistical combination of observations and short-range forecasts produce initial conditions used in model to forecast. (Bayes theorem)

- Advantages:
  - General framework for uncertainties
  - Unknown initial condition estimation
  - Account for observational noise, system noise, parameter uncertainties

Data Assimilation

$$\text{To obtain analysis}: x_k^{\mathrm{a}} = (1 - K_k) x_k^{\mathrm{f}} + K_k x_k^{\mathrm{o}}$$

$$\text{as estimate of true state}: x_k^{\mathrm{t}} = M_{k,k-1} x_{k-1}^{\mathrm{t}} + \eta_{k-1}$$

$$\text{using model forecast}: x_k^{\mathrm{f}} = M_{k,k-1} x_{k-1}^{\mathrm{a}}$$

$$\text{and observation}: x_k^{\mathrm{o}} = H_k x_k^{\mathrm{t}} + \varepsilon_k$$

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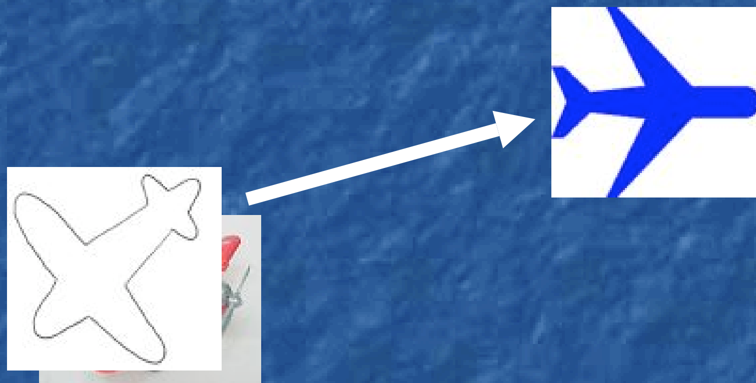

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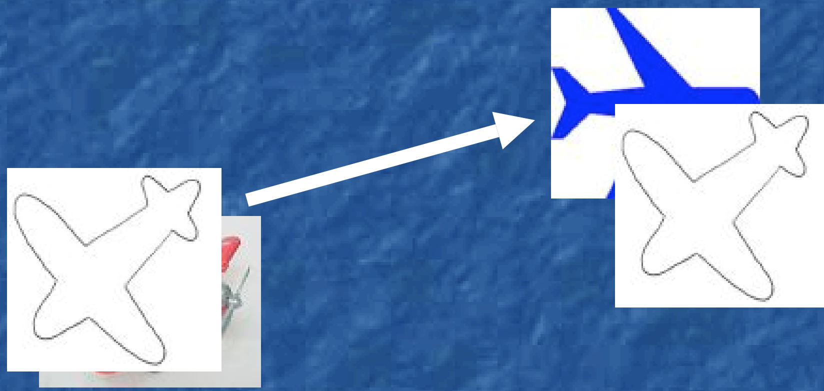

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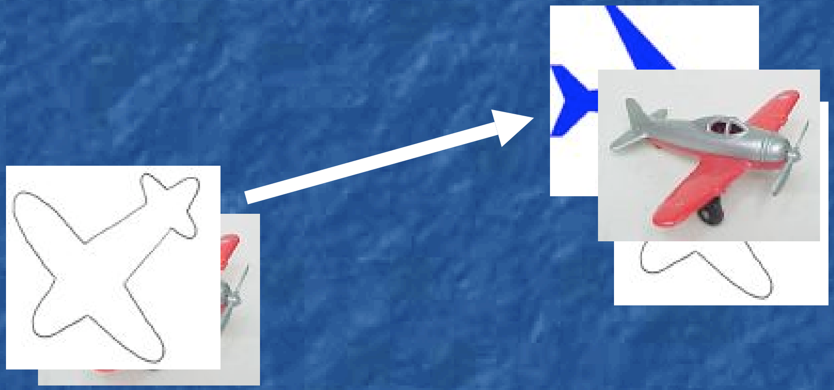

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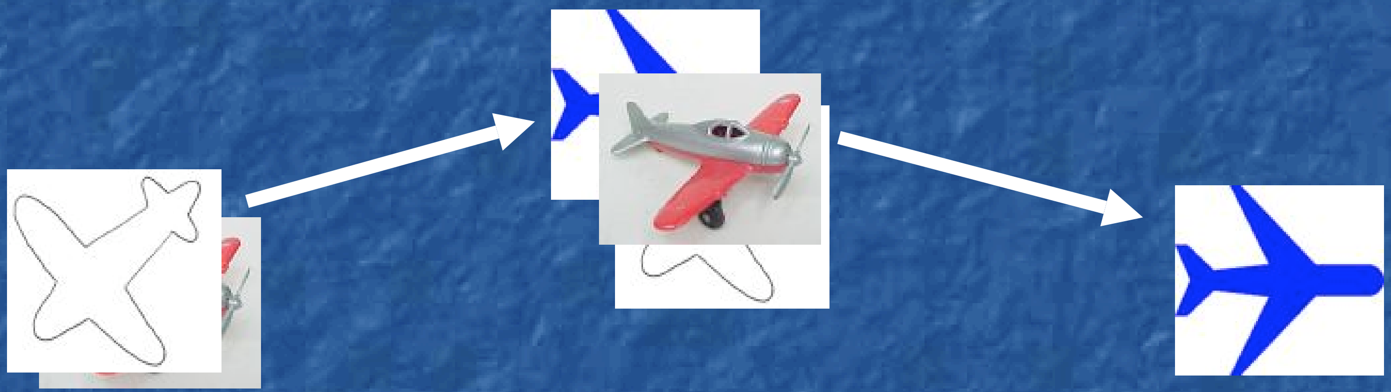

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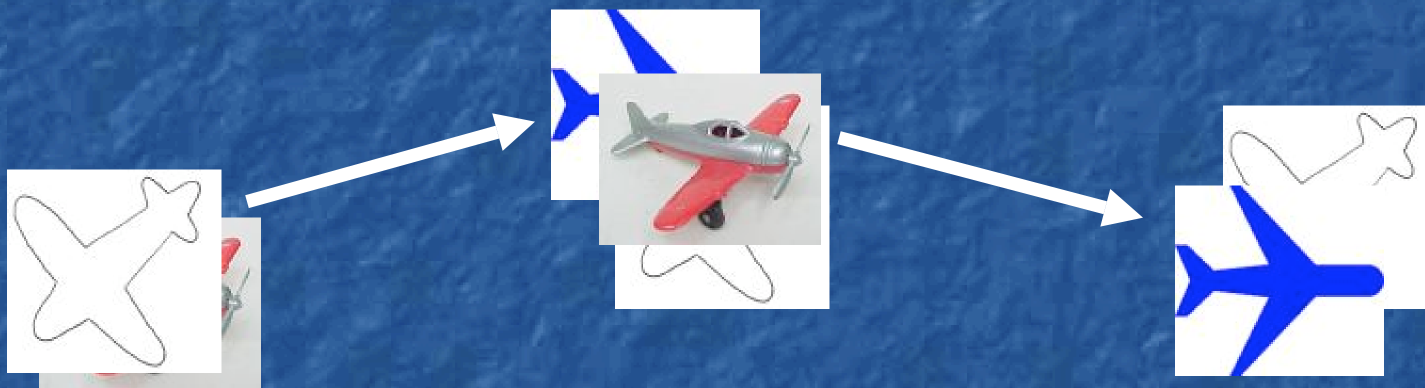

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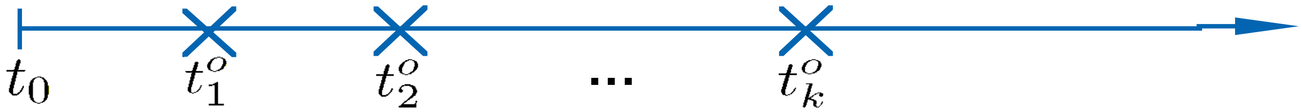
Temporal Renewal Processes



Noise: $t_k^o = t_k^t + \epsilon_k \qquad p_\epsilon(\epsilon_k) = U(-\frac{\Delta}{2}, +\frac{\Delta}{2})$

Renewal process: $p(t_{k+1}|t_k) = p(t_{k+1} - t_k)$

Forecast: $p^f(t_k^f|t_{1:k-1}^o) = \int p^f(t_k^f|t_{k-1}^a)p^a(t_{k-1}^a|t_{1:k-1}^o)dt_{k-1}^a$

Likelihood (observation): $L(t_k^o|t_k^t) = p_\epsilon(t_k^o - t_k^t) = U(-\Delta/2, +\Delta/2)$

Analysis / Posterior: $p^a(t_k^a|t_{1:k}^o) = C^{-1}\ L(t_k^o|t_k^a)\ p^f(t_k^a|t_{1:k-1}^o)$

Werner, Ide and Sornette (2007), in prep

Conclusions

- Uncertainties and noise are important
  - Observational noise can be large
  - Magnitude uncertainties not straight forward.
  - Seismic rate estimates in aftershock model fluctuate
  - Forecasts suffer
  - Testing and evaluating need to be robust

- Data assimilation can help
  - Generic framework for dealing with uncertainties