Diagnostic statistical analyses to detect the seismicity and geodetic anomalies relative to the normal predictions

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Aftershocks
Omori (1894)

1891 Nobi Earthquake of M8.1

\[ n(t) = K(t + c)^{-1} \]
The Omori-Utsu formula for aftershock decay rate

\[ v(t) = K(t + c)^{-p} \]

- \( t \): Elapsed time from the mainshock
- \( K, c, p \): Constant parameters

Utsu (1961)
Utsu (1961, 1969)

1891 Nobi (M8) Aftershock freq.

Data from Omori (1895) and Utsu (1969)

Fig. 1. Occurrence rate (top) and cumulative number (bottom) of felt earthquakes at Gifu after the Nobi earthquake of 1891. Smooth curves represent the Omori formula fitted to the data.

Observed time interval: \([S, T]\)

Occurrence time data: \(t_1, t_2, \cdots, t_n\)

**Log-likelihood function:**

\[
\ln L(\theta; S, T) = \sum_{S < t_i < T} \ln \lambda_\theta(t_i) - \int_{S}^{T} \lambda_\theta(t) \, dt
\]

\[
l(K, c, p) = \sum_{S < t_i < T} \ln \frac{K}{(t_i + c)^p} - \int_{S}^{T} \frac{K}{(t + c)^p} \, dt
\]

**Maximum Likelihood Estimate (MLE)**
Utsu & Seki (1954)
Utsu (1969)

\[ \log S = 1.02M - 4.01 \]

\[ \log L = 0.5M - 1.8 \]
Standard aftershock activity:
Occurrence rate of aftershock of $M_s$ is

$$n(t) = \frac{10^{0.85(M_0-M_s)-1.83}}{(t + 0.3)^{1.3}}$$

during $1 < t < 100$ days ($M_0 \geq 5.5$), where $p=1.3$, $c=0.3$ and $b=0.85$ are median estimates.

The constant $1.83$ is the best fit to 66 aftershock sequences in Japan during 1926-1968.
Fig. 105. A schematic graph of Type 1-C aftershock sequence. The broken line represents the curve for the modified Omori formula fitting the whole sequence. The shaded area indicates the secondary aftershocks triggered by shock 3.
Omori-Utsu formula:

\[ \nu(t) = \frac{K}{(t + c)^p} \]

ETAS model:

\[ \lambda(t) = \mu + \sum_{\{ j : t_j < t \}} e^{\alpha \{ M_j - M_c \}} \nu(t - t_j) \]

where

- \( t_j \) is occurrence time of \( j \)th event;
- \( M_j \) is magnitude of \( j \)th event;
- \( \mu \) is background rate; and parameters are \((\lambda_0, K, c, \alpha, p)\).
\[ \lambda(t) = \mu + \sum_{t_i < t} c(m_i)g(t - t_i) \]

- Hawkes and Oakes (1974, *JAP*) Epidemic-type model
- Kendall (1949, *JRSSB*) Age-dependent birth & death branching process
Space-Time ETAS model (Ogata, 1998, AISM)

\[ \lambda(t, x, y) = \mu + \sum_{\{j: t_j < t\}} \frac{K}{(t - t_j + c)^p} \left\{ \frac{Q_j(x, y)}{e^{\alpha M_j}} + d \right\}^{-q} \]

where

\[ Q_j(x, y) = (x - x_j, y - y_j) \cdot S_j \left( \begin{pmatrix} x - x_j \\ y - y_j \end{pmatrix} \right) \]

Heterogeneity in Space

\[ \mu \Rightarrow \mu(x, y); \]
\[ K \Rightarrow K(x_j, y_j); \quad \alpha \Rightarrow \alpha(x_j, y_j); \]
\[ p \Rightarrow p(x_j, y_j); \quad q \Rightarrow q(x_j, y_j) \]

Hierarchical Space-Time (HIST) ETAS model

estimated an Objective Bayesian method with smoothness constraints

Ogata (2003, JGR)
Off the East coast of Izu Peninsula Swarm
Off the East coast of Izu Peninsula Swarm

\[ \lambda(t | H_t) = \mu(t) + \sum_{\{j: t_j < t\}} \frac{K_0 e^{\alpha(M_j - M_c)}}{(t - t_j + c)^p} \]

Elapsed days from 16 APR 1998
Diagnostic Analysis
by
Time Transformation
Theoretical cumulative number of the events:

\[ \Lambda(t) = \int_0^t \lambda(s) \, ds \]
Theoretical cumulative number of the events:

\[ \Lambda(t) = \int_0^t \lambda(s) \, ds \]

Graph showing residual point process with parameters:

- \( M = 2.5 \), \( S = 0.01 \), \( T = 5.7 \), \( T_{end} = 5.7 \)
- \( K = 95.93739 \), \( c = 0.05307 \), \( p = 0.94405 \)
M7.0
2003 May
depth 71km
2003 Miyagi-Ken-North foreshocks (M >=1.5) till the mainshock

A change-point

$T_s$ 141.0E 141.2E 141.4E

38.6N 38.5N 38.4N 38.3N

$M \geq 1.5$ $S = 0.01$ $T_c = 0.1621$ $T_e = 0.291$

$\mu = 0$ $K_0 = 137.22$ $c = 2.7662\times 10^{-19}$

$\alpha = 19.476, p = 0.56366$

ORDINARY TIME (DAYS)
2003年M6.2宮城県北部地震の前震
Foreshocks of the Miyagi-Ken-North
Change-Point Analysis
Change-Point Analysis by the AIC

\[ AIC = (-2) \max \{\text{log-likelihood}\} + 2\{\text{number of parameters}\} \]

\[ S[**-----------------------------****t---**]T \]

\[ AIC_0 = (-2) \max_{\theta} \log_e L(\theta; S, T) + 2k_0, \]
\[ AIC_1(t) = (-2) \max_{\theta} \log_e L(\theta; S, t) + 2k_1, \]
\[ AIC_2(t) = (-2) \max_{\theta} \log_e L(\theta; t, T) + 2k_2; \]

\[ AIC_0 \] is compared with \[ AIC_{12}(t) + 2k(N) \] to choose the case of smaller value, where

\[ AIC_{12}(t) = AIC_1(t) + AIC_2(t), \]

and \( k(N) \) is the bias correction in search of \( t \) minimizing \( AIC_{12}(t) \) (change-point problem) and is a certain function of the number of events \( N \) in the interval \([S, T]\).
$AIC_{12} = AIC_1 + AIC_2 + 2k(N)$

$AIC_0$ is compared with $AIC_{12}(t) + 2k(N)$ to choose the case of smaller value, where

$$AIC_{12}(t) = AIC_1(t) + AIC_2(t),$$

and $k(N)$ is the bias correction in search of $t$ minimizing $AIC_{12}(t)$ (change-point problem) and is a certain function of the number of events $N$ in the interval $[S, T]$. 
1997 Kagoshima NW Earthquake

[Graph showing cumulative number of earthquakes and magnitude over time with annotations indicating relative quiescence.]

KAGOSHIMA 1997.3/26 - 5/13 M>2.7

N=522  T=47.880  T0=0.030  T1=5.370

ETAS

Relative Quiescence
The mainshocks location of investigated Aftershock activity
The mainshocks location of investigated Aftershock activity

The subsequent large earthquakes
Mainshocks of investigated aftershocks

Sea M >= 7.0 Land M >= 6.0 H <= 100 km

(a) No change-point

Lapse Time After the Mainshock (years)

Distance from the Mainshock (deg)
(a) first 6 years span

(b) next 14 years span

Expected Number of Events / (deg²/year)

Distance from the mainshock (deg)
Proposed possible mechanisms of precursory seismic quiescence

- Dilatancy hardening [Kelleher and J. Savino, 1975]
- Bi-modal asperity model [Kanamori, 1981]
- Stress weakening due to a creep [Wyss et al., 1981]
- Slip-weakening [Cao and Aki, 1985]

→ Mechanism within the source
Seismic Quiescence

• **Reports**: Inoue (1965), Utsu (1968), Mogi (1969), Ohtake et al. (1977), Wyss (1986), Kisslinger (1986), etc., about 200 papers

• **Significantly wider area than the source**: Inoue (1965), Motoya (1987), Ogata (1988, 1992, 1999)

• **Simulation based on rate- and state-dependent friction law**: Kato, Ohtake and Hirasawa (1997)
The November 2006 and January 2007 Kuril Earthquakes

Collaborative with Shinji Toda)
Background seismicity off Kuril Island and aftershocks of the November 2006 great earthquake of M8.3

USGS/NEIC 1973-2006.11.14
Background seismicity off Kuril Island and aftershocks of the November 2006 great earthquake of M8.3

Background seismicity off Kuril Island and aftershocks of the November 2006 great earthquake of M8.3

USGS/NEIC 1973-2006.11.14

2006.11.15-2007.1.13
Before Thrust

After Thrust

coupled

Lay, Astiz, Kanamori and Christensen, 1989, PEPI
2006年11月15日千島列島東方海溝型地震による周辺正断層へのクーロン応力変化

Coulomb 3.0 (Toda, Stein, Lin & Sevilgen) による計算結果

Outer rise
Normal faulting events

Before Thrust

After Thrust

Coupled
Aftershock activity of 15Nov2006 event (M8.3)
\( M \geq 4.6 \)
M >= 4.6
All detected events
All detected events
A Receiver – Normal faulting

B Receiver – Thrust faulting
The 2004 Chuetsu Earthquake
1997/01/01 - 2004/10/30  Number of events = 4060
Depth <= 25.0 km, M >= 2.0

Coulomb Stress change (mbar)
1997/01/01 - 2004/10/30  Number of events = 4060
Depth <= 25.0 km, M >= 2.0

(a)

(b)
No-Change

\[ \mu = 0.062894 \quad K_0 = 2.3616 \quad c = 0.00046861 \]

\[ \alpha = 1.0081 \quad p = 0.97466 \]
A Change

No-Change

\[
\begin{align*}
\mu &= 0.062894 \\
K_0 &= 2.3616 \\
\alpha &= 1.0081 \\
p &= 0.97466
\end{align*}
\]
The March 2007 Noto Peninsula Earthquake
The 2005 Off coast of Western Fukuoka–Ken Earthquake
Summary with the ETAS model

- The ETAS model summarizes the seismicity due to triggering effect within the contiguous hierarchical complex faults.
- Deviations of the seismic activities from the predicted rate by the ETAS model are useful to detect regional stress changes.
- The respective deviations are explained by the changes in Coulomb failure stresses that are caused by seismic or aseismic slips.

Software and manuals are available at
http://www.ism.ac.jp/~ogata/Ssg/softwaresE.html
Thank you very much for listening

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