ANNALI DI GEOFISICA, VOL. XXXVII, N. 5, September 1994

TEC and f_0F_2 comparison

Paolo Spalla and Luigi Ciraolo
Istituto di Ricerche sulle Onde Elettromagnetiche, C.N.R., Firenze, Italy

Abstract
Following an investigation on the calibration of Faraday Rotation measurements by the slab thickness method, several interesting results have been obtained concerning the limits of application of TEC data in f_0F_2 modelling at middle latitudes.

Key words *ionosphere – total electron content – f_0F_2 modelling*

1. Introduction

Great effort is spent in producing models of the ionosphere allowing to provide either a better understanding of the involved physical phenomena or in radiopropagation forecasting. Several categories of such models can be classified according to the way they are actually organized, or to their limits of validity for what concerns the range in latitude, height or geophysical conditions, and finally according to the type of parameters they provide as an output.

Two categories are actually the most important to be considered:

a) theoretical models starting from the classical physical equations of continuity, of conservation of energy and momentum for electrons, ions ad neutral molecules, try to derive the distribution of ionization, and subsequently all the parameters needed for theoretical investigations and practical applications. A typical example in this category is the Time Dependent Ionospheric Model (TDIM) (Schunk *et al.*, 1986). These models are obviously very interesting in the study of the physics of the ionosphere and of the interactions either among its various constituents or with the solar wind and the geomagnetic and solar fields. On the other side, the models belonging to this category offer generally a very modest practical fall out at the expense of heavy times of computation on large computing systems (several hours on CRAY);

b) empirical and semiempirical models: these assume some simple theoretical statements and predetermined functions providing the shape of the ionization profile at different heights (Chapman, Parabolic, Epstein, etc.) which are subsequently tuned using historical, long term and wide geographical coverage, data bases trying to extract the information about the dependence on particular parameters (such as the spot number, as an example). These parameters will be subsequently used as an input to models attempting to predict the behaviour of the ionosphere. This is the category to which most of the more used and known models belong, such as the IRI (Bilitza *et al.*, 1993).

Most of the data bases used by the models of the category (b) – the most widely known is that of the Consultative Committee of International Radiocommunications, CCIR, 1967 and following – are composed by ionosonde measurements (normal and oblique sounding, from ground and top sounding from satellite), rockets (local composition), incoherent radar (profile of electron densities and velocities of the irregularities, airglow, etc. Because spatial and temporal coverage of such observations is generally partial, some process of interpolation or extrapolation is generally needed. This process is called mapping, and it attempts to fill the gaps of data and, whenever possible, to perform some forecasting. For all these reasons, these maps are very important for the modelling of the ionosphere, and any improvement of the data base by observations or maps whose limits of validity are known, helps the models to provide more reliable results.

Just in this framework the following model can be placed, able to evaluate f_0F_2 (one of the parameters most widely used by semiempirical models) from TEC measurements obtained observing the Faraday Rotation affecting the signals from geostationary satellites, or the differential Doppler effect between two armonically related carriers transmitted by orbiting satellites.

More than proposing the model, widely known and accepted (Titheridge, 1973), the following analysis attempts to provide an estimation of the involved parameters and the error to be expected in its use, at the different times of the day and in the different seasons of the year.

2. The slab thickness: definition, use, and statistical analysis

The total electron content of the ionosphere (TEC) is defined as

$$\int_0^h N(h)dh \tag{2.1}$$

where the integral is evaluated in a region where the electron density $N(h)$ is significatively different from zero. The electron density presents an absolute maximum N_{max}, which determines the maximum frequencies of the waves (Ordinary and Straordinary: O and X, which can be reflected by the ionosphere. Therefore these frequencies play an important role in the down sounding probing by ionosonde: namely the maximum frequency of the ordinary wave, the f_0F_2 which is one of the most measured parameters of the ionosphere.

It has been fruitful finding a relationship between TEC and N_{max} or better f_0F_2 (Davies, 1990)

$$\mathrm{TEC} = 1.24\,10^{-6}\,\tau(f_0F_2)^2 \tag{2.2}$$

where: TEC is measured in TEC units: electrons $m^{-2}\,10^{16}$; f_0F_2 is measured in MHz; τ the slab thickness measured in m, in which τ can be considered as an effective breadth of the ionosphere, depending on the actual shape of the profile.

Equation (2.2) can be used to model TEC from the knowledge of f_0F_2 or vice versa, provided that behaviour and variability of τ are known. As it will be shown in the following the assumption of the constancy of slab thickness during nighttime over long periods of time provides the tool to solve the phase ambiguity inherent to the measurements of the rotation of the polarization of a wave crossing the ionosphere (the Faraday effect).

Cumulative Faraday Rotation measurements can provide TEC evaluation according to:

$$\Omega_{\mathrm{meas}} + \Omega_{\mathrm{amb}} = \Omega_0 + k\,\mathrm{TEC} \tag{2.3}$$

where: Ω_{meas} is the measured rotation of the polarization; Ω_{amb} is the integer number of the cycles of the rotation at the beginning of the measurements. This parameter is unknown; Ω_0 is the initial polarization at the satellite. If it is unknown it will be included in Ω_{amb}, which is no more an integer number (Spalla, 1989).

Rewriting eq. (2.3) in terms of eq. (2.2), it will be obtained

$$\Omega_{\mathrm{meas}} = k\,1.24\,10^{-6}\,\tau(f_0F_2)^2 - \Omega_{\mathrm{amb}} \tag{2.4}$$

Under the circumstances in which τ can be assumed to be constant, if the relative TEC – *i.e.* the TEC accounted before that ambiguity has been solved – is plotted vs. $f_0F_2^2$ one should get a line whose slope accounts for the slab thickness and the intercept for the ambiguity, allowing the calibration of the TEC measurements.

To ensure the constancy of τ eq. (2.4) must be applied only during nighttime hours and for periods long enough to allow sufficient points for the statistics, but sufficiently short to avoid long period effects due to variability of τ. Generally this last condition is fulfilled as it is difficult to have continuous records lasting more than twenty days on the average.

Using this method, Faraday rotation data from 1975 to 1992 have been calibrated, obtaining intercepts and then ambiguities with an error ranging from 1 to 3 TEC units. These measurements made the largest TEC data base in Europe, presenting also the interesting peculiarity of a very dense sampling in time (1 min).

Great care was paid to ascertain the meaning of the results: the regression-correlation coefficient r was greater than .8 for all regression and the error for slope was ± 25 km.

3. Results and discussion

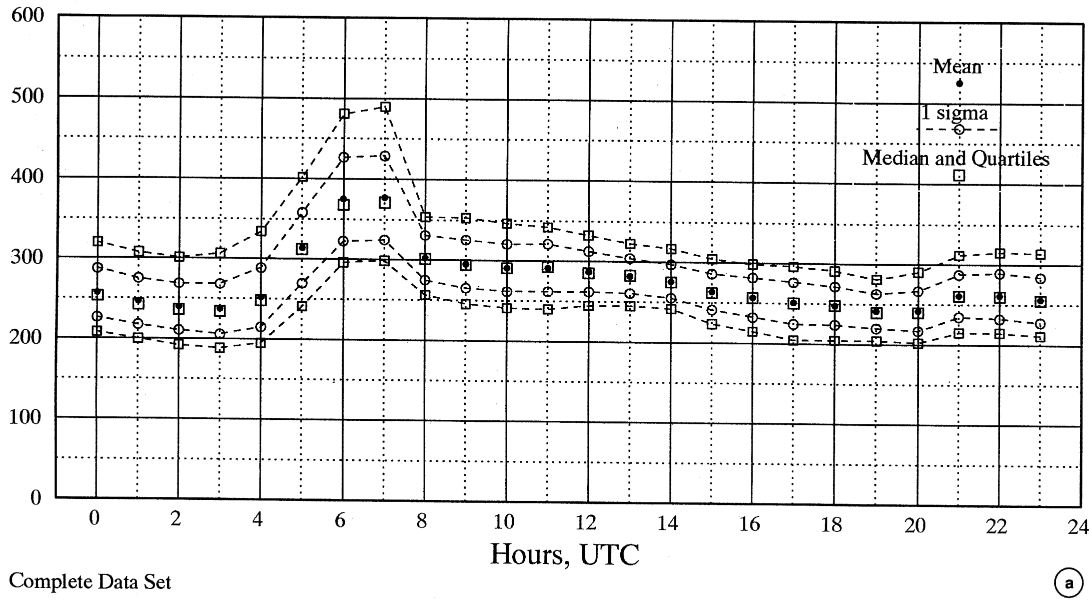
Following the solution of the ambiguity as outlined in the previous section, a lot of estimates of slab thickness values become available (the slopes of the regression lines). It is worth reminding that these values refer to nighttime hours. An interesting behaviour of the slab thickness was noticed: the assumption of its (statistical) constancy – for nighttime hours – turned out to hold for all processed periods. In fact the evaluated τ's range from 120 to 360 km with a mean value of 230 km and a standard deviation sigma = 50 km; however the 70% of the values are in the range of 230 ± 50 km; the median is 230 km and lower and upper quartiles are respectively 190 and 260 km (figs. 1a-f). At present no investigation has been made on 30% out of these limits.

After the solution of the ambiguity of eq. (2.4), calibrated TEC values are available. Using these TEC values and simultaneous f_0F_2 measurements of a near ionosonde, it becomes possible to evaluate slab thickness τ according to eq. (2.2) and during any time in the day. A statistical study of the slab thickness during the period 1975-1992 was carried out. The data have been grouped according to periods of three months, centered about solstices ad equinoxes. For each period, median, mean ad their corresponding confidence levels (quartiles and $\pm\ 1\sigma$ values) were computed, and plotted in figs. 1a-f. The following points can be noticed:

1) mean and median take very close values, and corresponding confidence levels have the same behaviour: this ensures that the statistical distribution of the values is well behaved as it does not present significant outliers;

2) the values of τ lie around an average value of about 230 km not only during the night but also during sunrise.

This circumstance suggests to devise a simple model able to provide TEC given f_0F_2 by straight application of eq. (2.2) using a slab thickness of 230 km and f_0F_2, data measured in a near ionosonde (Rome). This model was applied during all the periods when Faraday TEC evaluations were available. The Faraday TEC was compared with TEC modelled day by day, for all the hours of the day following this procedure: the differences between measured TEC and modelled TEC have been grouped for seasonal periods (Spring, Summer, Autumn, Winter, Equinox), then mean, standard deviation, median ad quartiles have been computed. This process has been carried also using, for each set of data, the specific slope provided by the regressions: no important differences are shown by the results, and the model of constant τ presents even better statistical behaviour, so that only figures of the model using this assumption are shown (figs. 2a-f).



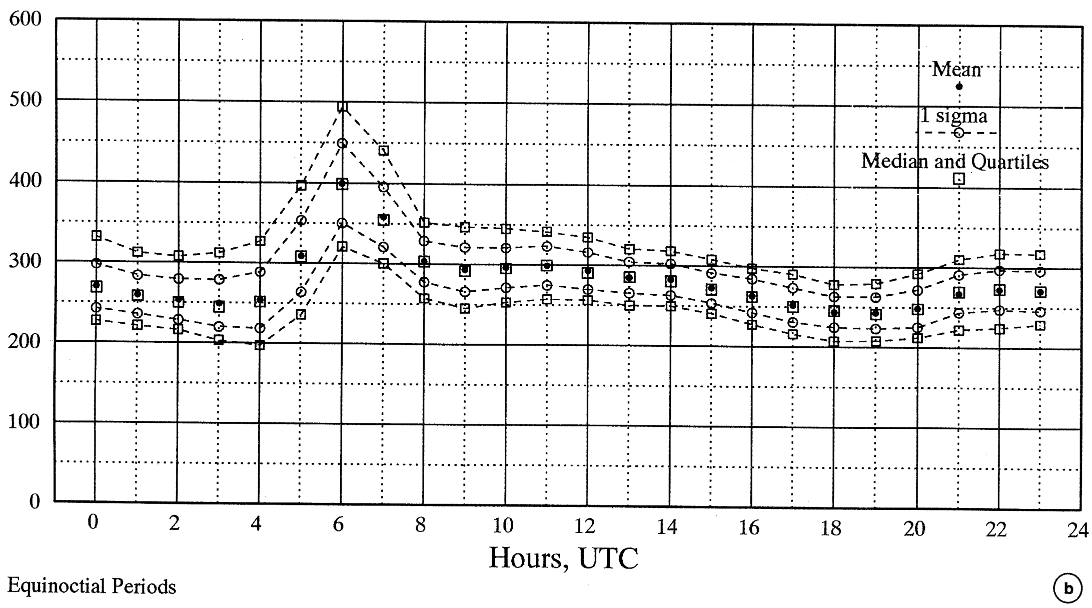
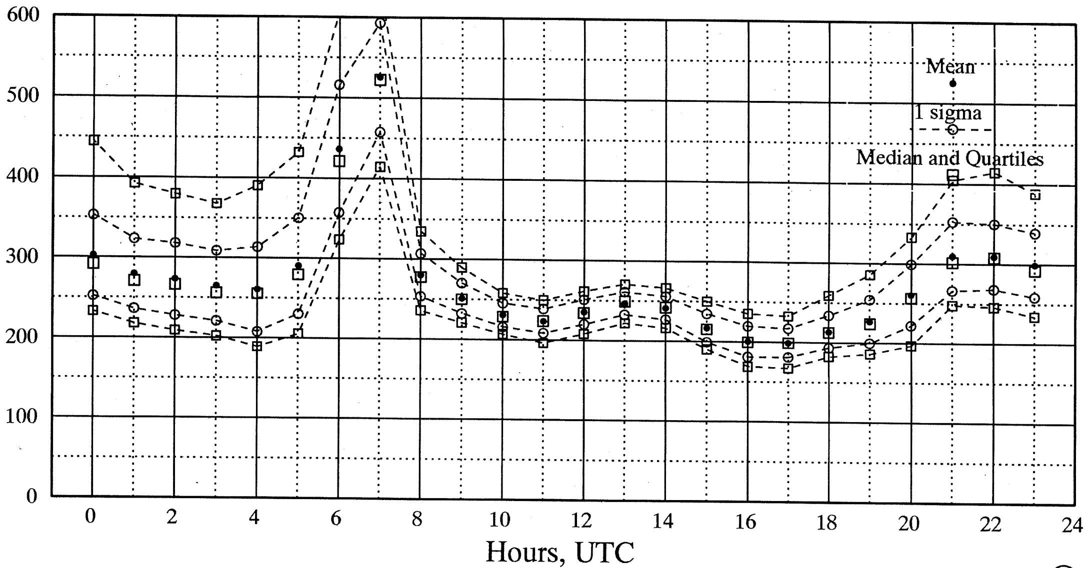
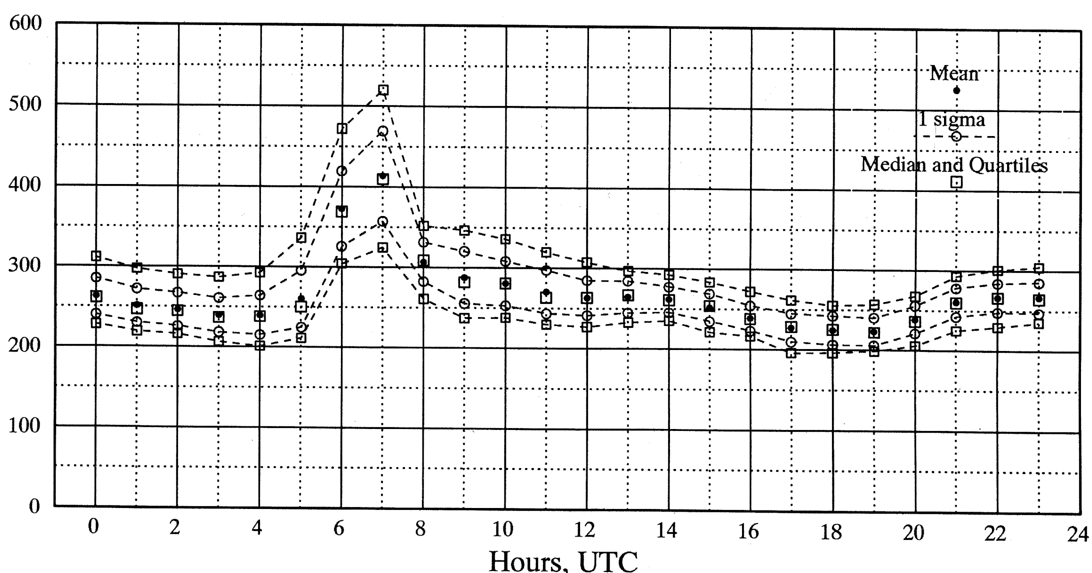


Fig. 1a,b. Seasonal median and mean of the evaluated slab thickness (km) derived from TEC and f_0F_2 measurements (see eq. (2.2)). Mean, mean minus standard deviation σ, mean plus σ are marked by circles. Median, and quartiles are marked by squares. The figures refer respectively to: a) all the measurements (more than 2000 days from 1975 to 1992); b) data of February, March, April, August, September, October, *i.e.* the equinoctial months all together.



Winter Periods

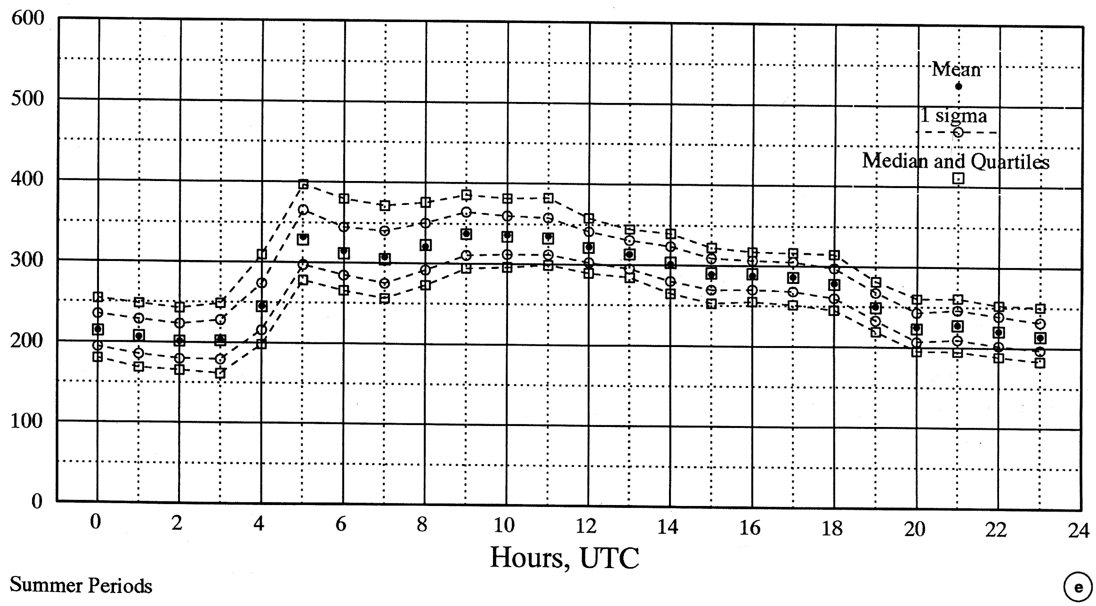
(c)



Spring Periods

(d)

Fig. 1c,d. Seasonal median and mean of the evaluated slab thickness (km) derived from TEC and f_0F_2 measurements (see eq. (2.2)). Mean, mean minus standard deviation σ, mean plus σ are marked by circles. Median, and quartiles are marked by squares. The figures refer respectively to: c) data of November, December, January; d) data of February, March, April.



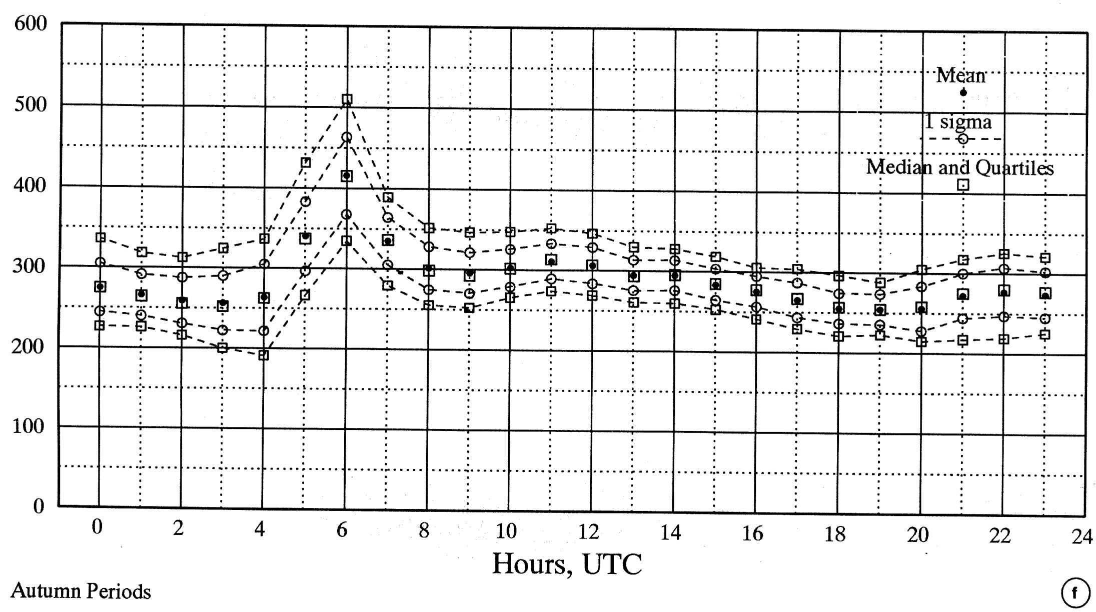
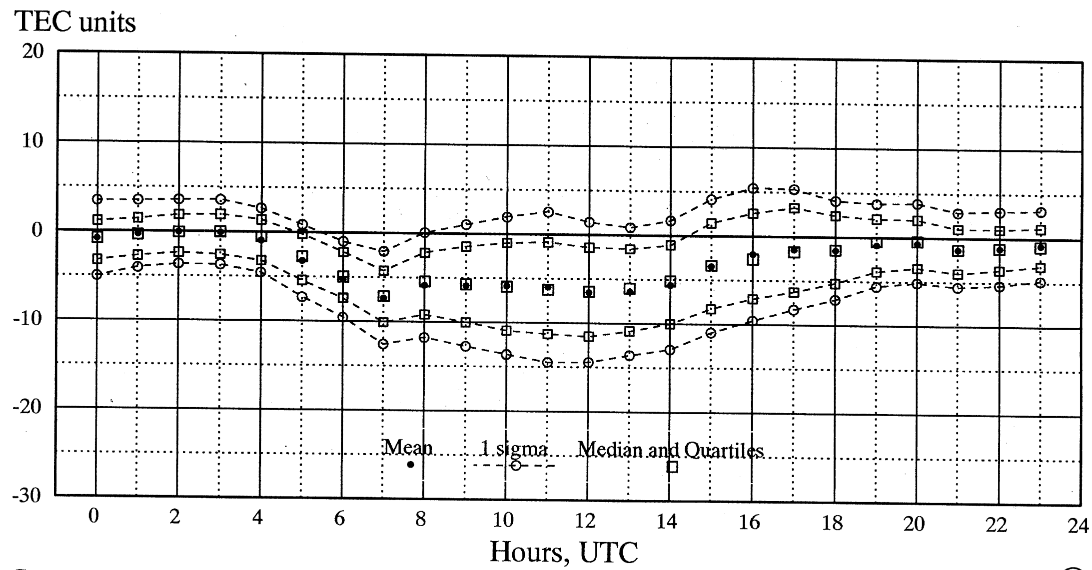
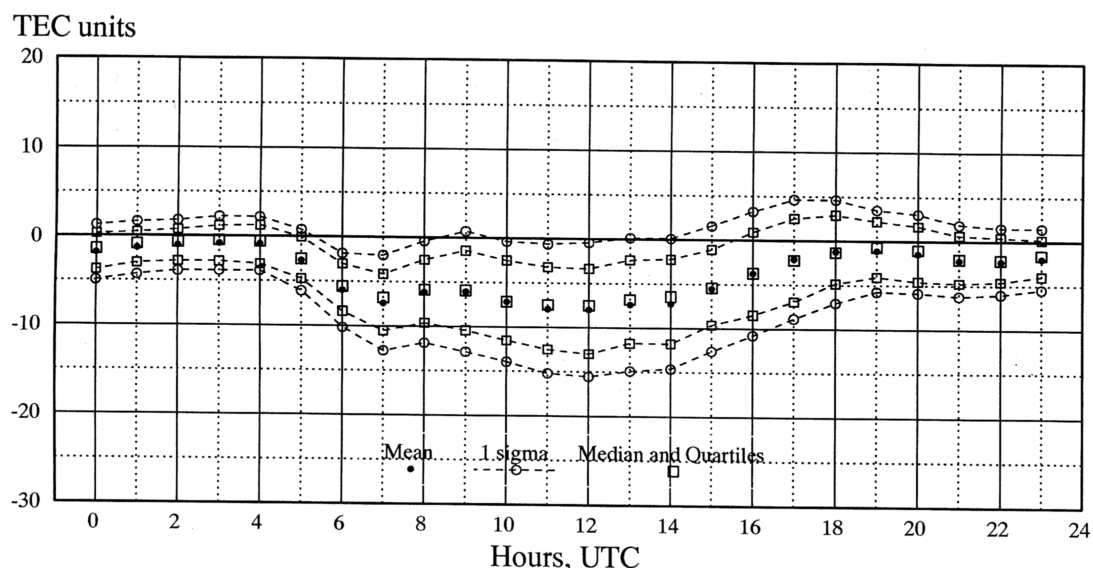


Fig. 1e,f. Seasonal median and mean of the evaluated slab thickness (km) derived from TEC and f_0F_2 measurements (see eq. (2.2)). Mean, mean minus standard deviation σ, mean plus σ are marked by circles. Median, and quartiles are marked by squares. The figures refer respectively to: e) data of May, June, July; f) data of August, September, October.



Complete Data Set, fixed slope

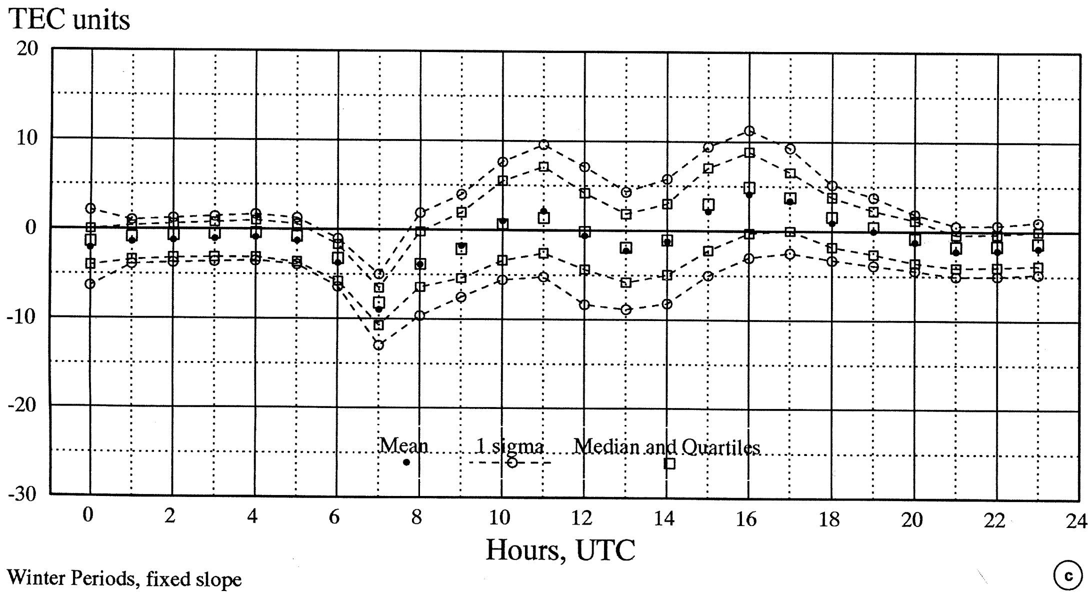
a



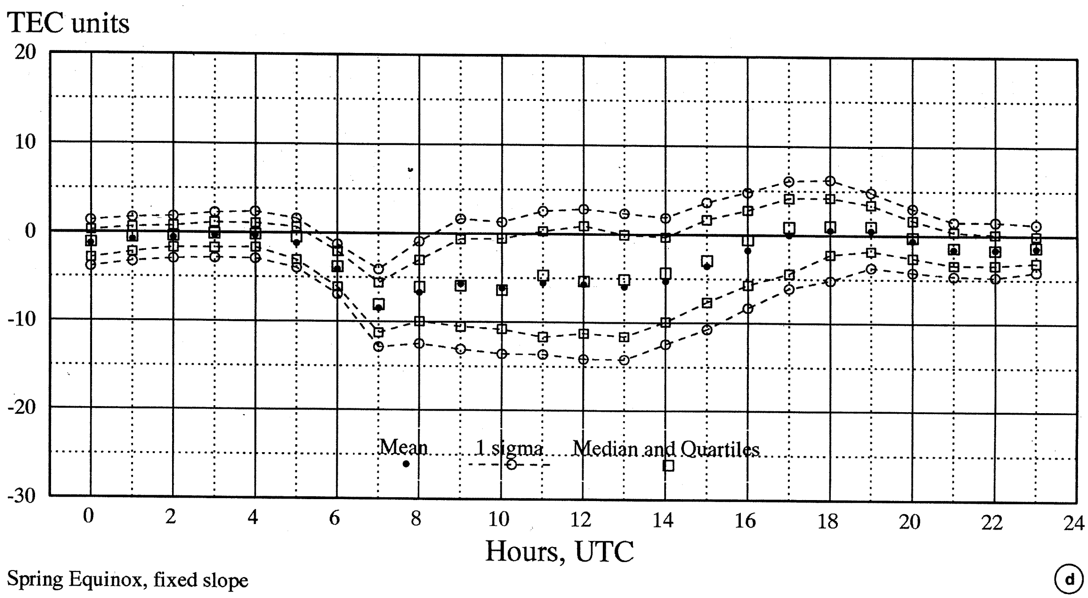
Equinoctial Periods, fixed slope

b

Fig. 2a,b. Seasonal median and mean of the differences between TEC modelled by eq. (4.1) and TEC derived from Faraday measurements. The legenda «fixed slope» means that eq. (4.1) has been used with constant $\tau = 230$ km for all the periods (see text). Mean, mean minus standard deviation σ, mean plus σ are marked by circles. Median, and quartiles are marked by squares. The figures refer respectively to: a) all the measurements (more than 2000 days from 1975 to 1992); b) data of February, March, April, August, September, October, *i.e.* the equinoctial month all together.

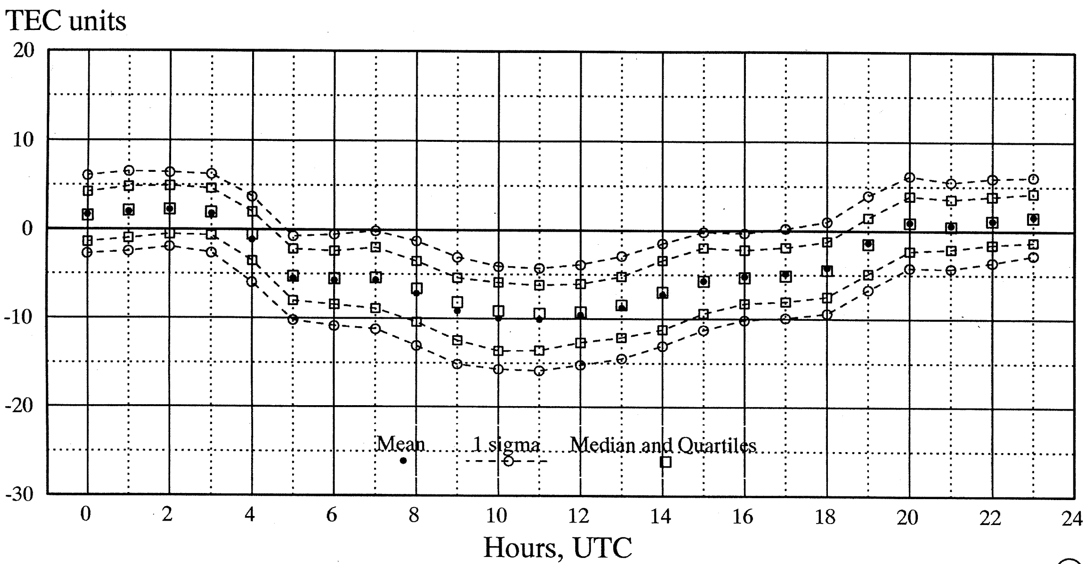


Winter Periods, fixed slope



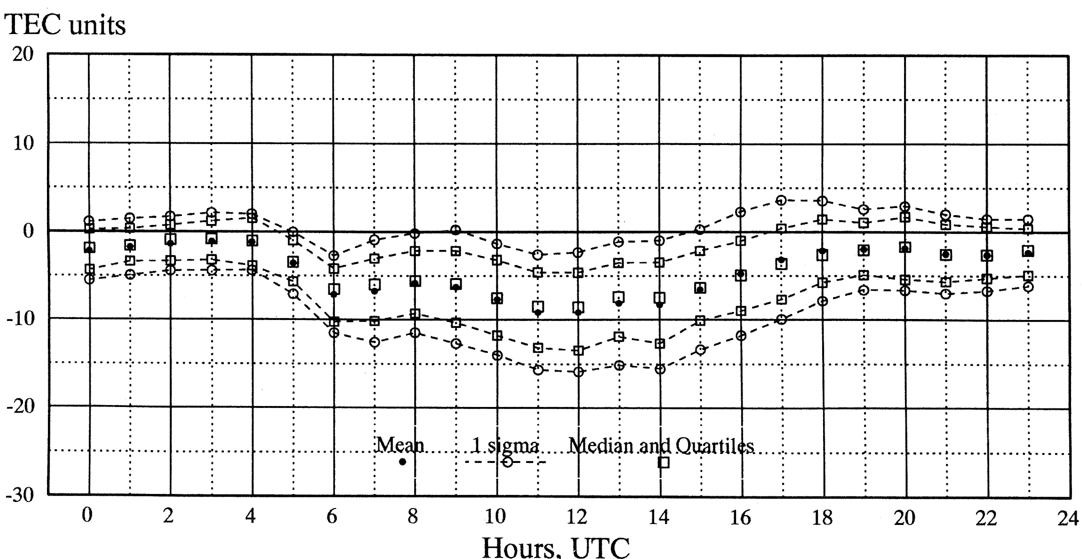
Spring Equinox, fixed slope

Fig. 2c,d. Seasonal median and mean of the differences between TEC modelled by eq. (4.1) and TEC derived from Faraday measurements. The legenda «fixed slope» means that eq. (4.1) has been used with constant $\tau = 230$ km for all the periods (see text). Mean, mean minus standard deviation σ, mean plus σ are marked by circles. Median, and quartiles are marked by squares. The figures refer respectively to: c) data of November, December, January; d) data of February, March, April.



Summer Periods, fixed slope

e



Autumn Equinox, fixed slope

f

Fig. 2e,f. Seasonal median and mean of the differences between TEC modelled by eq. (4.1) and TEC derived from Faraday measurements. The legenda «fixed slope» means that eq. (4.1) has been used with constant $\tau = 230$ km for all the periods (see text). Mean, mean minus standard deviation σ, mean plus σ are marked by circles. Median, and quartiles are marked by squares. The figures refer respectively to: e) data of May, June, July; f) data of August, September, October.

4. Conclusions

In general the differences between measurements and model are zero within 2-3 TEC units, during nighttime hours: if this is obvious from midnight to 4 when the specific slopes were computed, the fact is that this holds also for all the night, and even using the same average slab thickness (230 km) for all the periods. During the day the situation is quite different, however, the presented plots can provide an idea about the validity of the eq. (2.2) and of the involved errors for different seasons and different hours of the day. As general behaviour it can be noticed that the model tends to underevaluate the TEC by about 2 TEC units, and that the error ranges between – 6 and 4 TEC units, rather flatly distributed.

This suggests that a simple model, based on eq. (2.2) and on its inverse form

$$f_0 F_2^{\,2} = 3.51 \ \text{TEC} \tag{4.1}$$

and on the knowledge of the TEC datum only, can provide a coarse evaluation of the actual f_0F_2 median value, satisfactory within the limits resulting from the statistics reported above, considering that the relative error on f_0F_2 is half of the error shown in the figures relative to TEC (figs. 2a-f).

The use of such model leads certainly to rough estimations, but it results very valuable when no other means are available to get them, and mainly because the reported statistics evidence very clearly the limits of the errors involved.

REFERENCES

BILITZA, D., K. RAWER, L. BOSSY and T. GULYAEVA (1993): International reference ionosphere – past present and future, *Adv. Space Res.,* **13** (3), 3-14.

CCIR (1967, 1971, 1975, 1980, 1984, 1988, 1990, 1992): Atlas of Ionospheric Characteristics (and later supplements), Report 340-1-2-3-4, 894, 434 International Telecommunication Union, Geneva, Switzerland.

DAVIES, K., (1990): *Ionospheric Radio* (Peter Peregrinus Ltd., U.K.), 274-277.

SCHUNK, R.W., J.J. SOJKA and M.D. BOWLINE (1986): Theoretical study of the electron temperature in the high latitude ionosphere for solar maximum and winter conditions, *J. Geophys. Res.*, **91** (A10), 11281-11290.

SPALLA, P. (1989): Faraday rotation measurement and TEC derivation, in *Lectures held at the College on Theoretical and Experimental Radiopropagation Physics, ICTP, Trieste, 13-15 February*, 3-23.

THITHERIDGE, J.M. (1973): The slab thickness of the mid-latitude ionosphere, *Planet Space Sci.*, **21**, 1775-1793.

(received February 10, 1994;
accepted August 30, 1994)