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Key Points:
- We find an analytical expression of volcanic earthquakes occurrence rate
- We analyze some volcanic seismic catalogs
- We found some differences respect the traditional Omori law

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An Analytic Expression for the Volcanic Seismic Swarms Occurrence Rate. A Case Study of Some Volcanoes in the World

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Abstract Seismic swarms are defined as a group of earthquakes occurring very close in time and space but without any distinctively large event triggering their occurrence. Up to now no simple law has been found to describe the swarm occurrence rate. Here we find an expression able to fit the average occurrence rate on some volcanic areas. This expression exhibits some differences in respect to the classical Omori law. Namely the c parameter of the Omori law is equal to zero and the power law decay of the average occurrence rate of the earthquakes is followed by an exponential decaying regime. Both the results can be interpreted in term of fluid injection and/or movements. Indeed this is a more impulsive phenomenon compared to the occurrence of a large earthquake, with a duration compatible with a $c = 0$. The exponential decay following the power law one could be explained by a viscoelastic relaxation of the stress induced by the injection and/or movements of fluids in the earth crust.

1. Introduction

When an earthquake occurs the stress released is redistributed to the surrounding rocks causing a number of aftershocks which depends on the magnitude of the triggering event (Helmstetter, 2003). The rate of occurrence is governed by the Omori law (Omori, 1894)

$$n = \frac{k}{(t + c)^p} \tag{1}$$

where n is the number of aftershock, t is the time elapsed from the mainshock occurrence and k, c, and p are experimentally estimated parameters. k depends exponentially on the mainshock magnitude (Helmstetter, 2003), c makes the Omori law normalizable, whereas p controls the velocity of aftershocks rate decay.

The Omori law is one of the principal components for the Epidemic Type Aftershock Sequences (ETAS) model (Ogata, 1985, 1998) which views the earthquake occurrence as the superposition of a constant rate of occurrence, μ, and the aftershocks occurrence rate. Aftershocks occur following a cascade process: a parent earthquake can generate some offspring who can, in turn, generate their own offspring. This is a very general characteristic of aftershocks occurrence.

Differently from mainshock—aftershock sequences, earthquake swarms are defined as earthquakes clustered in space and time without a triggering mainshock (Hainzl et al., 2012). Swarm activity has been associated to stress changes induced by aseismic processes such as pore pressure changes (Miller et al., 2004) or fluid intrusion (Toda et al., 2002) or seismic ones as observed at the Corinth Gulf (Mesimeri et al., 2016). Mogi (1963) first suggested that swarms occur in regions characterized by high heterogeneity in terms of material properties and stress concentration. Swarms are recorded in volcanic, geothermal or tectonic environments (Hainzl & Fischer, 2002; Tramelli et al., 2021; White & McCausland, 2015) and their triggering mechanism is interpreted as due to processes of injection and/or movement of fluids (Chouet, 1996; Glazner & McNutt, 2021; Hainzl, 2003; Tramelli et al., 2021). Volcanic swarms are usually the main reported seismic precursor for volcanic eruptions especially for volcanoes that have been silent for decades or more (White & McCausland, 2015).

A mechanical model to simulate the swarm occurrence was introduced by Hill (1977) and Hainzl (2003), modifying the Burridge and Knopoff (1967) model. They were able to explain some of the characteristics of earthquake swarms occurring in seismogenic structures driven by fluid injection and/or movements.

Some statistical models for swarm occurrence modified the original ETAS one (Ogata, 1985, 1998) introducing a non stationary background seismic occurrence. The simplest approach was introduced by Lombardi et al. (2006) who used the stationary ETAS model in moving time windows explaining the fluctuations of the ETAS model parameters. Marsan et al. (2013); Reverso et al. (2015) took into account of seismic transients like fault interactions, fluid and dike injections being able to recover, both in duration and in intensity, the changes in fault loading rates. Kumazawa and Ogata (2014) expressed the non stationary background, $\mu(t)$, as a piece wise linear function, whereas Kattamanchi et al. (2017) used a spline function which allowed them to identify slow slip earthquakes occurring on subduction zones. The authors enlighten as their approach could model earthquake sequences triggered by fluid/magma injections.

Even if many of these models can reproduce some statistical feature of the seismic swarms, it was not possible the fitting of the occurrence rate and neither the Omori law nor a simple relationship can well describe the temporal evolution of the volcanic earthquake swarms. The reason for such a difficulty rely on the duration of the swarms which are often very short and not provide a sufficient number of events for a reliable statistical analysis.

In the following we will show that, stacking many swarms in an average rate of occurrence, an analytic expression for the earthquake swarms time evolution can fit the experimental observations.

2. The Data

Here we analyze five earthquake catalogs of corresponding volcanic areas (Figure 1): Campi Flegrei (CF) (1982–1984), CF (2000–2019), Etna (Alparone et al., 2015), Hawaii archipelago, volcanic cordillera of Guacanaste in Costa Rica.

It is worth to note that CF and Etna are single volcanoes whereas Hawaii and Guacanaste cordillera are volcanic complex. Even if a comparison of the two kind of earthquake catalogs could appear unjustified, the reason of our choice is the homogeneous tectonic regime of Hawaiian volcanoes and Costa Rica ones. Conversely CF and Etna volcanoes cannot be treated as a single volcanic complex because they volcanism is different for genesis and magmatic properties. Moreover the seismic activity of Hawaii is mostly concentrated on the Big Island. The advantage of considering the volcanic complex catalogs is, of course, the increase of the number of earthquakes and, as a consequence, a more stable statistical analysis.

The Websites where the catalogs can be downloaded are reported in Table 1, whereas Table 2 reports the time periods of the catalogs, the earthquake number in each one of them and the adopted completeness magnitude. This quantity has been estimated by using the goodness of fit method (Wiemer & Wyss, 2000) and the method introduced by Godano (2017). In most of cases the two methods estimate the same value and, when there are differences, we adopted the larger value between the two estimations.

3. Individuating the Seismic Swarms and Defining the Rate of Occurrence

We used the clustering properties of earthquake occurrence for separating earthquakes from background seismicity. In order to characterize these properties we use the distribution of the time interval between two successive events. In the following we refers at this quantity as the inter-event time Δt. Let us, first, recall the fundamental results obtained on this distribution.

3.1. The Inter-Event Time Distribution

The main result obtained on the inter-event time distribution during the last 20 years, is that the Δt distribution $p(\Delta t)$ can be considered universal when the inter-event times Δt are rescaled by the mean occurrence rate, R (Corral, 2003, 2004, 2006). Namely, $p(\Delta t)$ is independent of the geographic zone and the magnitude threshold. This implies that R defines a "local" time scale that characterizes the earthquake occurrence whereas their clustering properties can be considered universal. This result was first obtained for pseudo-stationary periods revealing that earthquakes tend to cluster even if their occurrence is apparently Poissonian (Corral, 2004).

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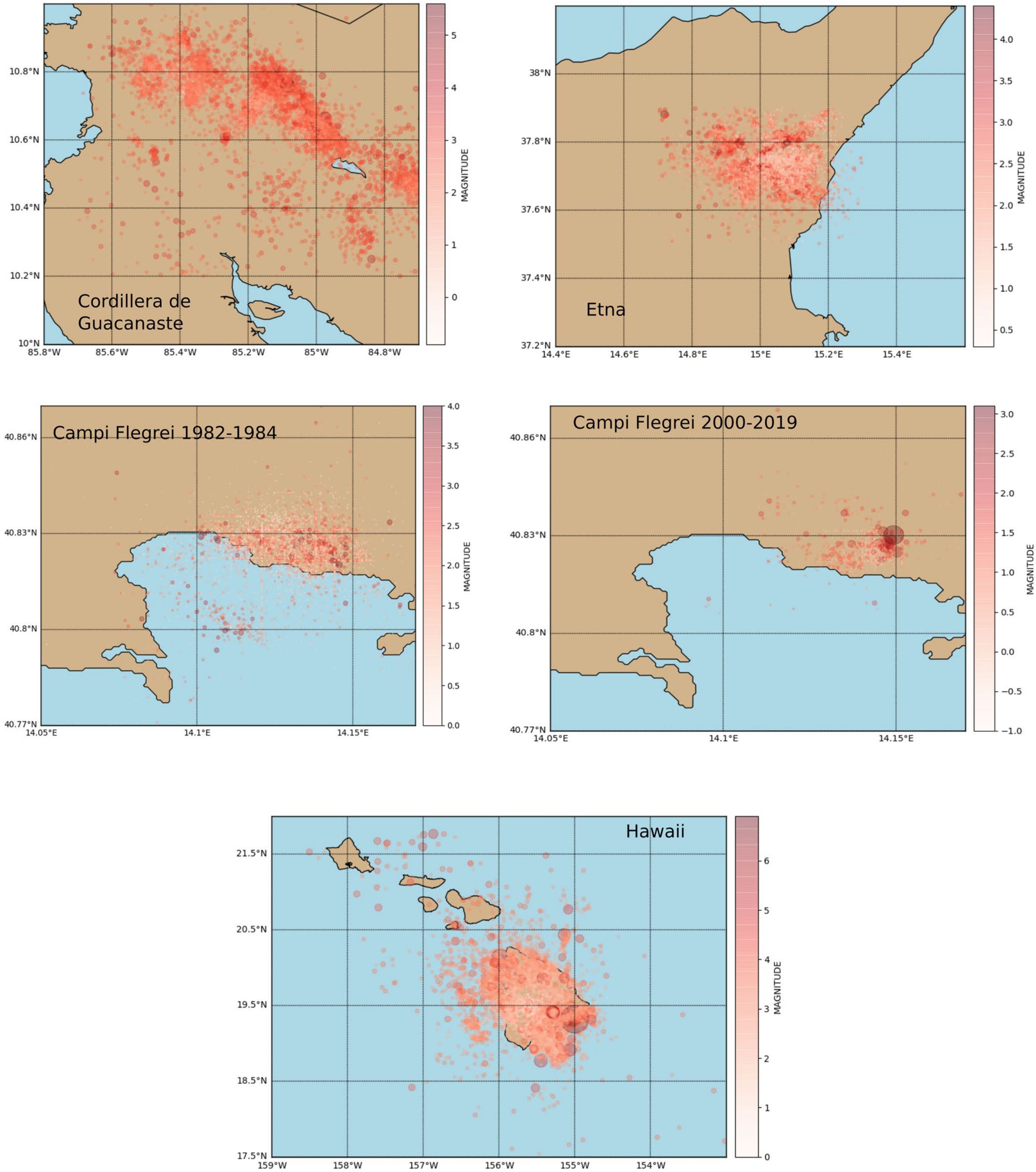

Figure 1. The maps of the seismic activity of the five volcanic areas here analyzed. The red dots indicate the events for the considered volcano. The size of the red dots is proportional to the magnitude of the event.

The universality of $p(\Delta t)$ has been also observed for non-stationary periods (Corral, 2009) and for aftershock sequences (Bottiglieri et al., 2011; Shcherbakov et al., 2005). However the universal behavior of $p(\Delta t)$ has been questioned (Hainzl et al., 2006; Lindman et al., 2005; Molchan, 2005; Saichev & Sornette, 2006, 2007; Sornette et al., 2008; Touati et al., 2009). In particular, deviations from universality at small Δt have been related to

Table 1
The Areas and the Web Sites From Where the Earthquake Catalogs Can Be Downloaded

Catalog	Web site
Cordillera de Guacanaste	https://doi.org/10.5281/zenodo.6383911 (Tramelli, 2022b)
CF 1982–1984	https://doi.org/10.5281/zenodo.6376561 (Tramelli, 2022a)
CF 2000–2019	http://sismolab.ov.ingv.it/sismo/index.php?PAGE=SISMO/last%26area=Flegrei
Etna	https://www.ct.ingv.it/index.php/monitoraggio-e-sorveglianza/banche-dati-terremoti/terremoti
Hawaii	https://zenodo.org/badge/latestdoi/555488289

interplay between correlated earthquakes, which follow a Gamma distribution (see Appendix A for more details), and uncorrelated events, which follow pure exponential decay. The departure from universality has been solved by Bottiglieri, de Arcangelis, Godano, and Lippiello (2010) who showed that four typical time scales are relevant for the interevent time distribution scaling: the inverse rate of independent events, λ, the mean inverse rate of correlated events, the time parameter c defined in the Omori law, and the catalog duration T.

3.2. The Role of the Space

Earthquake swarms can be characterized also by their spatial clustering. In order to take into account the role of the space we evaluate the conditioned probability density of Δt given a $\Delta r < \delta$, where Δr is the epicentral distance between the same successive events with an inter-event time Δt and δ is a fixed value. More precisely, if Δt is the inter-event time between the ith earthquake and the $1 + i$th one, it is counted in the distribution only if Δr (the inter-distance between the ith earthquake and the $1 + i$th one) assumes a value $\leq \delta$. δ represents the characteristic spatial scale of the swarms and it has been fixed in different ranges depending on the size of the investigated areas. In particular the minimum δ values have been selected so that each distribution is performed over at least 150 events. Whereas the maximum δ values are fixed at approximately one half of the whole extension of the volcanic area. The δ values can be found in the legends of Figure 2 showing the conditioned probability density $p(\Delta t | \Delta r \leq \delta)$.

As can be seen $p(\Delta t | \Delta r \leq \delta)$ does not depend on δ with the exception of Etna and Hawaii for the smallest values of δ. This implies that, for Etna and Hawaii, no background activity is included in the analysis when δ assumes very small values. Nevertheless, here we need to include some background activity in order to individuate the swarms as an increase of the seismic activity as compared to the background occurrence rate.

The $p(\Delta t | \Delta r \leq \delta)$ follows a Gamma distribution characterized by two parameters: α controlling the power law decay of the distribution and Θ representing the Δt value after which the exponential decay becomes dominant. The two parameters have been estimated using a maximum likelihood method (see Appendix). In Table 3 we report only the values of Θ because it is useful in the swarm individuation (see next section).

3.3. The Earthquakes Swarms and the Omori Law

As stated before we use the value of Θ for discriminating between the background activity and the swarms occurrence. More precisely the swarm starts when $\Delta t \leq \Theta | (\Delta r \leq \delta)$ and ends when $\Delta t > \Theta | (\Delta r \leq \delta)$. Indeed Θ^{-1} can be viewed as the largest observable occurrence rate of the background earthquakes (where observable means not hidden by the swarm events occurrence rate). Conversely the $\Delta t \leq \Theta$ intertime values characterize the occurrence of clustered events in the earthquake swarms. In order to have a qualitative feedback of our choice, Figure 3 shows the cumulative number of events for the whole catalog (black squares) and for the events selected as swarms (red circles). The rate of occurrence increases significantly in correspondence of the individuated swarms revealing that the method recognize the seismic swarms efficiently. In Figure 3 we use the largest δ value reported in the Figure 2 labels. However the δ value does not influence the

Table 2
The Temporal Periods, the Number of Events in the Catalogs and the Completeness Magnitude of the Catalogs Here Analyzed

Catalog	Initial date	Final date	N	m_c
Cordillera de Guacanaste	2005/01/03	2021/12/31	19,372	2.4
CF 1982–1984	1982/02/04	1984/12/31	5,775	1.0
CF 2000–2019	2000/08/22	2019/12/31	1,489	0.4
Etna	2000/01/01	2016/12/31	8,983	2.6
Hawaii	2000/01/01	2018/05/31	64,076	1.8

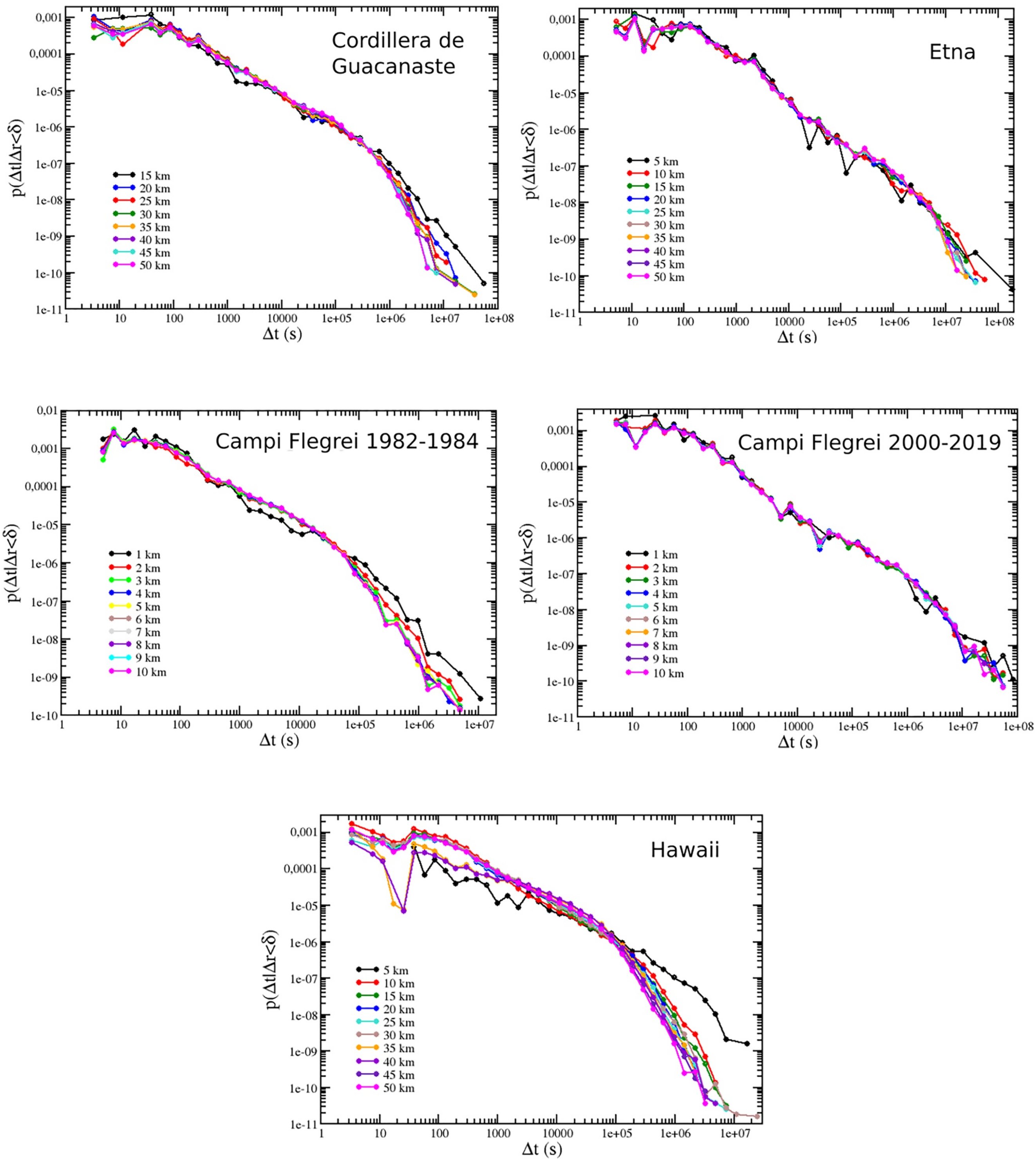

Figure 2. The conditioned probability density $p(\Delta t | \Delta r \le \delta)$ for the five volcanic catalogs here analyzed. The minimum δ value has been obtained under the request of at least 150 events satisfying the condition $\Delta t | \Delta r \le \delta$. Different colors of the curves represent the same distribution obtained at different values od δ, as indicated in the legend.

Table 3
The Values of θ for the Earthquake Catalogs Here Analyzed

Catalog	Θ (s)
Cordillera de Guacanaste	450,000
CF 1982–1984	55,000
CF 2000–2019	500,000
Etna	200,000
Hawaii	80,000

selection of the swarms, indeed, as shown in Figure 2, Θ is δ independent and assumes the same value for any δ. As a consequence the use of different values of δ does not influence our results.

As a counterproof of the goodness of our choice for the earthquake swarms, in Figure 4 we show the intertime distribution during the periods outside the swarms. As expected the distribution appear to be exponential for all the catalogs here analyzed revealing their Poissonian occurrence (Cox & Lewis, 1966) and confirming the goodness of our choice.

The sample size of many identified swarms does not allow the investigation of their time behavior. As a consequence we have stacked all of them in a unique average Omori law for each catalog. Namely we count the number of events occurred at the time t elapsed from the beginning of the swarm and for each class t the different $n(t)$ are summed and divided by the number of classes with $n(t) \neq 0$ building an average rate of occurrence $\nu(t|\Delta r \leq \delta)$ for each catalog here analyzed. Figure 5 shows $\nu(t|\Delta r \leq \delta)$ opportunely rescaled in order to have a collapse on a unique master curve and to better evidence their independence of the δ value.

For all the catalogs we obtain a $\nu(t|\Delta r \leq \delta)$ that can be described as a power law tapered by an exponential decrease after a given value of t. However the CF catalogs after the power law decay presents a bump in the average occurrence rate eventually followed by an exponential decay whereas the Hawaii catalog exhibits a more flat regime at $t > 10^6$(s) for $\delta = 30$, 35 and 40 km. An interpretation and the fit of these behavior will be provided in the next section.

3.4. The Productivity Law

We have verified that the volcanic swarms occurrence rate can be assimilated to an Omori law. Let us verify if the productivity law holds also for volcanic earthquakes. As suggested by Helmstetter (2003), the number of events in a seismic sequence, grows exponentially with the magnitude of the mainshock. However, for the earthquake swarms, it is not possible to speak of a mainshock. As a consequence, we evaluate the number of events in a swarm as a function of the largest event magnitude, m_L, in the swarm. Figure 6 shows that the productivity laws are independent of the δ values with the exception of the Hawaii catalog.

Following Shebalin et al. (2020) we also investigated the productivity through the distribution $p(n)$ of the number of earthquakes per swarm. Figure 7 confirms the results of Shebalin et al. (2020): $p(n)$ follows a power law distribution.

Figure 8 confirms the expected result that the distribution of occurrence time t_L of the largest earthquake in the swarm in respect to the beginning time t_b of the swarm reveals that in all the cases $t_L > t_b$ confirming that the mechanism of triggering the earthquake swarms is different by the one of the tectonic sequences which are triggered by the mainshock.

This result confirms the idea that earthquake swarms are not triggered by the occurrence of a large events and excludes that we are including in our analysis any tectonic sequence occurred on the active faults of the Guacanaste Cordillera (indeed for tectonic sequences the first event is generally the largest one triggering the whole sequence).

4. Discussion

Fluid injections are considered as the responsible for triggering earthquake swarms in many volcanic areas of the world (Chiodini et al., 2017; Massin et al., 2013; Padrón et al., 2021; Saccorotti et al., 2002) and intraplate geothermal environments (Hainzl & Fischer, 2002; Parotidis et al., 2003). This hypothesis has been confirmed by experimental results at laboratory scale (Cebry & McLaskey, 2021) and inspired models for swarm triggering (Hill, 1977). As a consequence, we can hypothesize that it can be related to our occurrence rate expression.

In the previous section we have shown that many of the results obtained for the earthquake sequences can be extended to the volcanic earthquakes with some substantial differences.

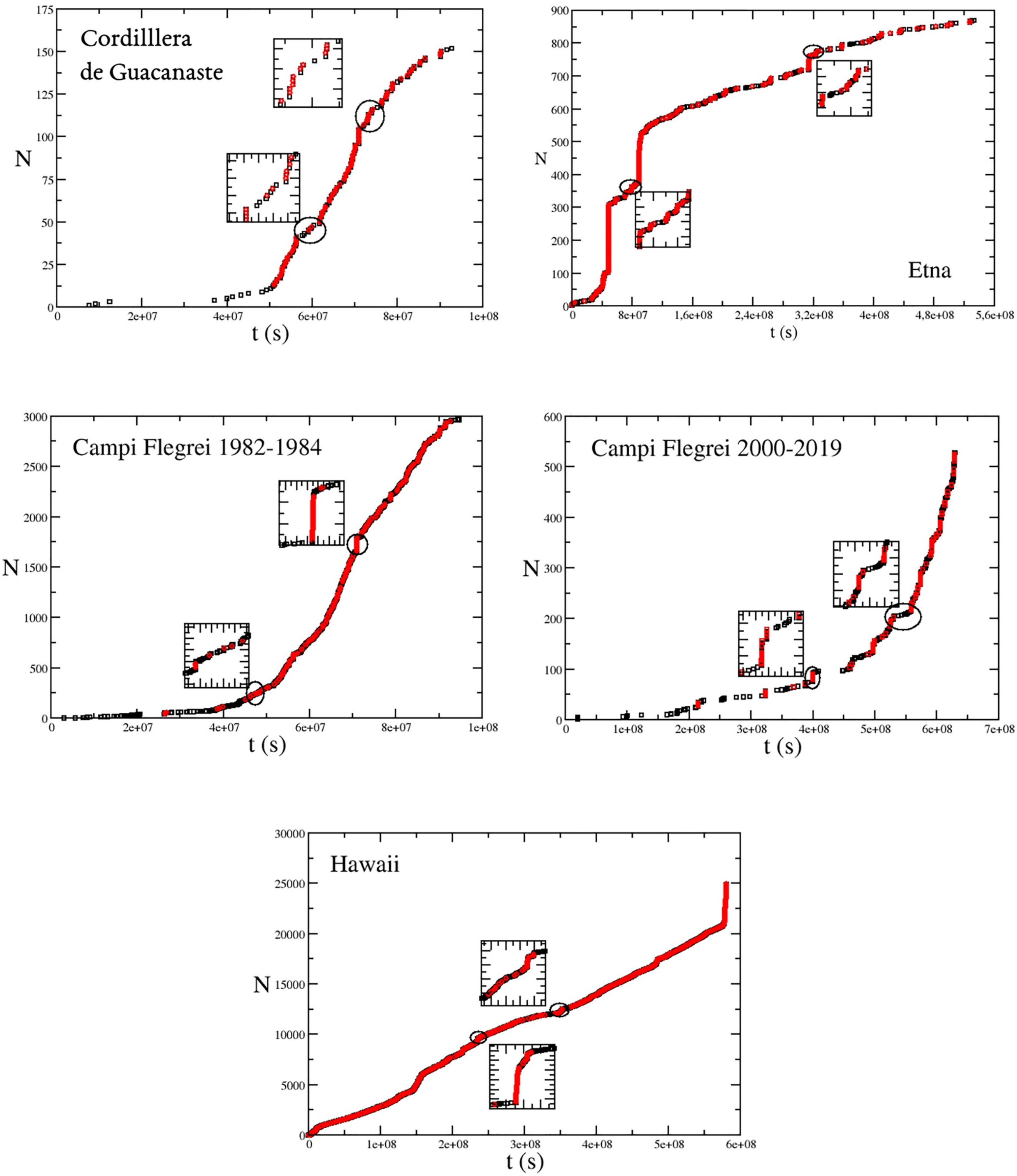

Figure 3. The cumulative number of earthquakes as a function of time for the whole catalogs (black squares) and for the individuated swarms (red circles). The insets represent a zoom of the curve in the ellipses.

The intertime distribution follows, as for the earthquake sequences, a Gamma one. As well known this distribution is characterized by a power law decreases with Δt followed by an exponential one. The first regime is characteristic of earthquakes clusters, whereas the exponential regime characterizes the occurrence of the background seismic activity (Bottiglieri et al., 2010; Corral, 2003, 2004, 2006; Godano, 2015; Molchan, 2005).

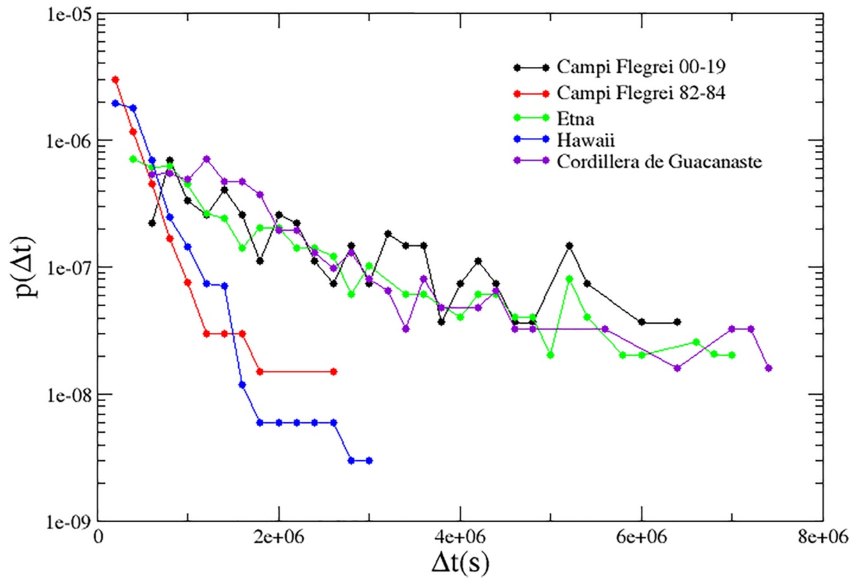

Figure 4. The intertime between two successive events occurring during periods outside the swarms.

The parameter Θ separate the two regimes and has been here used for successfully individuate the seismic swarms. The Δt distribution appears to be independent of the δ value revealing that it is not influenced by the spatial occurrence properties. This result was unexpected because the spatial proximity is part of the definition of earthquake swarms. However it simply reveals that the occurrence probability of simultaneous swarms at different zone of the same volcanic area is very small, even if the area is characterized by the presence of many volcanoes as in Costa Rica or Hawaii.

The analysis of the Δt distribution allows us to individuate the seismic swarms and to build for each catalog an average swarm rate of occurrence. Figure 5 shows as this occurrence rate exhibits some differences in respect to the Omori law. The first one is that the parameter c appears to be equal to zero. However the presence of an exponential tapering at high values of t allows the normalization of the occurrence rate expression:

$$\nu(t) \propto (t^{-p} + \mu)e^{-\frac{t}{\tau}} \tag{2}$$

where p assumes the same meaning of the usual Omori law p value and τ is the elapsed time from the beginning of the swarm at which the exponential decay becomes dominant. The parameter μ is the constant rate of background occurrence seismicity here introduced to explain the bump in the rate of occurrence just before the exponential regime (Figure 5) observed for the CF catalogs.

We have fitted the parameters of Equation 2 through the minimization of the χ^2 (Hastings & Peacock, 1975). More precisely we explore the parameter space looking for the minimum value of the χ^2. The estimated parameters are reported in Table 4.

Let us to provide an interpretation of the three main differences with the standard Omori law. Namely the c is equal to zero and the power law rate regime is followed by an exponential one.

1. The c value represents a time during which aftershocks do not yet occur or are not recorded. It can be explained in terms of many physical processes, namely the duration of fault fracture, a not perfectly elastic behavior of the rock introducing a delay in the mechanism of stress release, higher resistance of the unbroken patches of the fault delays the occurrence of the aftershocks, etc. (Lippiello et al., 2019, 2021; Petrillo et al., 2022). In the case of the seismic swarms the triggering phenomenon is not the occurrence of a large earthquake, but the intrusion and/or movements of fluids (see, among the others, Chouet, 1996; Glazner and McNutt, 2021; Hainzl, 2003; Hill, 1977; Tramelli et al., 2021). This can be considered a more impulsive phenomenon, moreover the presence of fluids lubricates the existing faults making more rapid the response to the stress impulse. As a consequence the c value becomes negligible as shown in Figure 5.

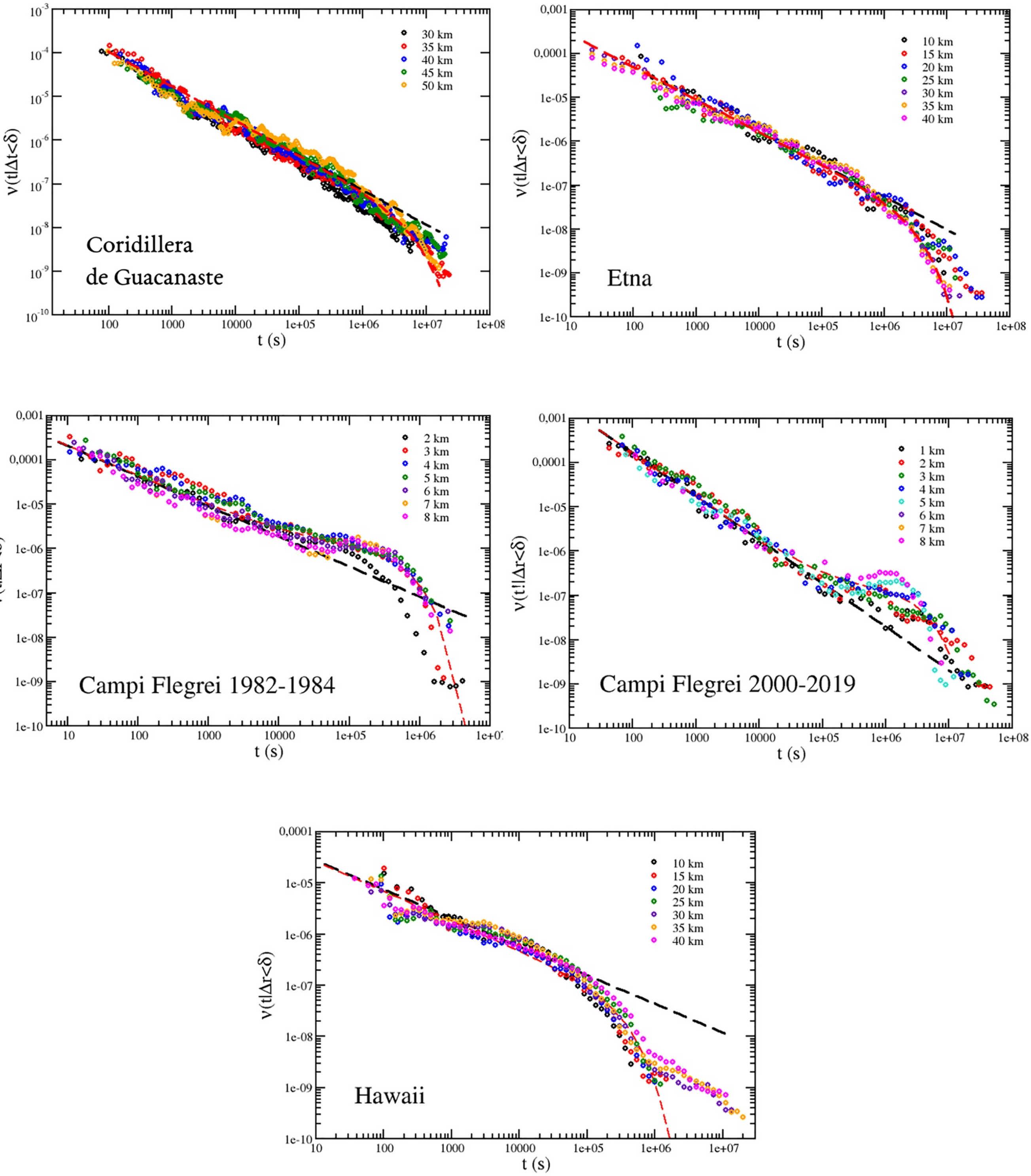

Figure 5. The conditioned rate of occurrence $\nu(t|\Delta r \leq \delta)$ for the five volcanic catalogs here analyzed. The standard deviation is not reported in the figure for reasons of clarity. Its order of magnitude is $\approx 10^{-10}$.

2. We interpret the exponential decay after the power law regime as due to a viscoelastic effect in the hypothesis that the occurrence rate is proportional to the stress rate. Indeed the higher temperature of the volcanic rocks makes their rheological behavior more viscous than the rocks of the tectonic areas. As a consequence, when the fluid is not injected or moved anymore, the induced stress on the surrounding rocks

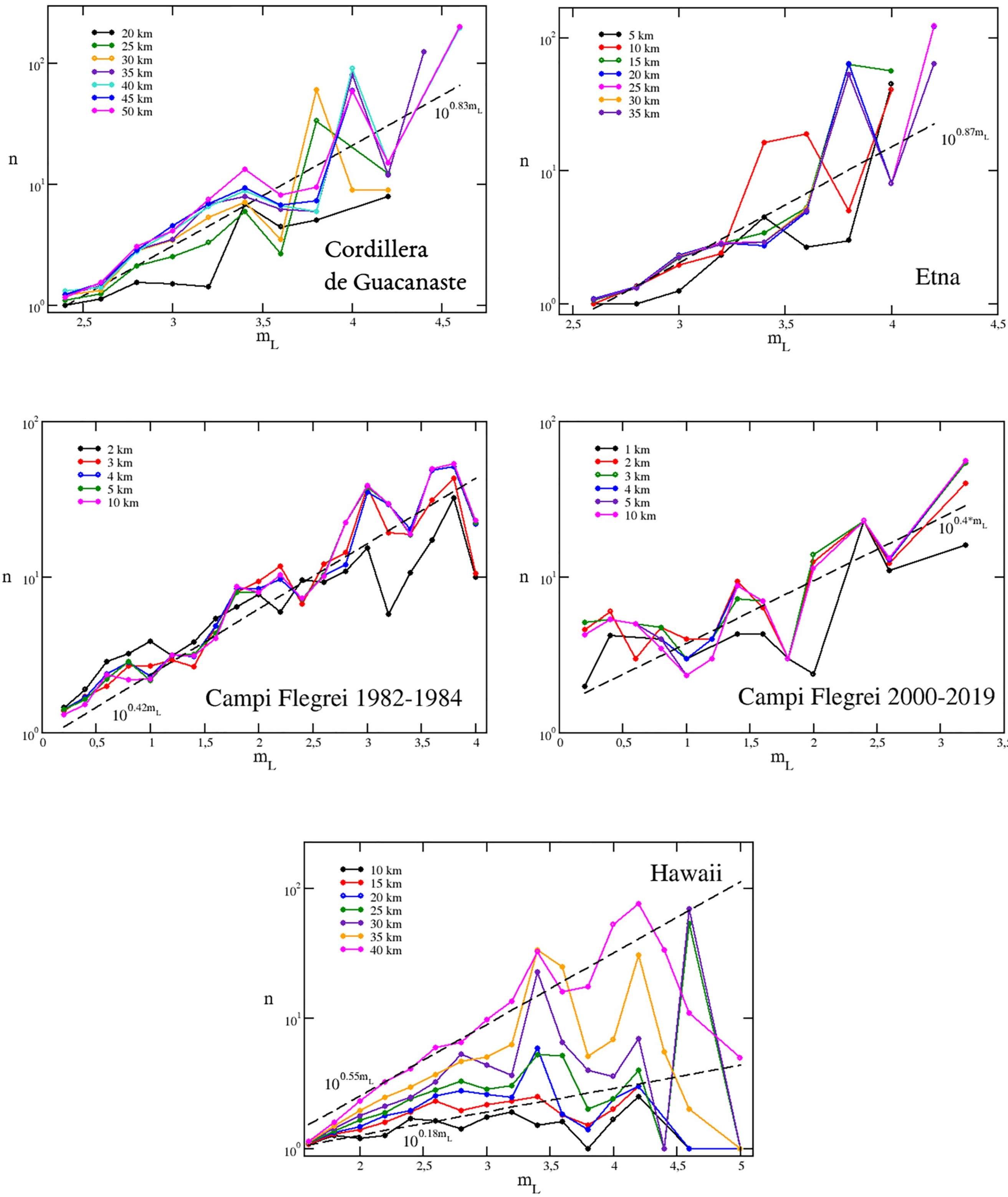

Figure 6. The conditioned productivity laws for the five volcanic catalogs here analyzed. *y* axis and *x* axis represent the number of events and the maximum magnitude in the swarm, respectively.

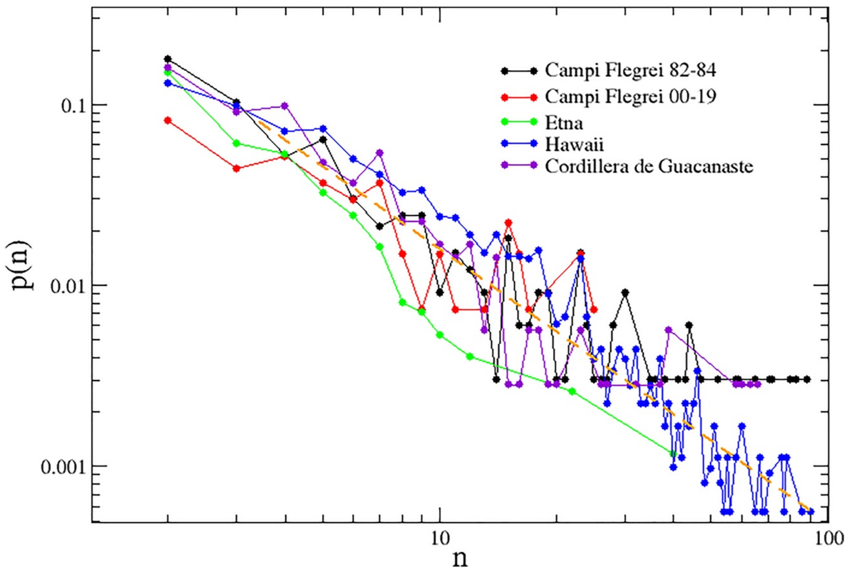

Figure 7. The distribution of the number of events per swarm n for the five volcanic catalogs here analyzed.

is released following a viscous relaxation causing the exponential decay of the occurrence rate (Jagla et al., 2014).

3. The presence of the μ constant term here introduced for explaining the bump in the occurrence rate for the CF catalogs deserves a short discussion. Indeed it should not be confused with the background activity which is a constant rate of occurrence to be added to the swarms activity and represent a Poissonian process independent of the swarm triggering mechanism. Conversely μ should be considered as a constant activity integrated in the swarm that ceases when the fluid intrusion and/or movements stops. In this sense the triggering mechanism causes an increase of the stress generating the swarm activity and, moreover, causes a Poissonian occurrence of other earthquakes that could be generated by a mechanism of fault lubrication due to the presence of fluids. The observation of μ only at CF is easy to explain noting that the m_c values are significantly higher for the other volcanoes. Indeed the background seismicity is dominated by smaller events and, as a consequence, the

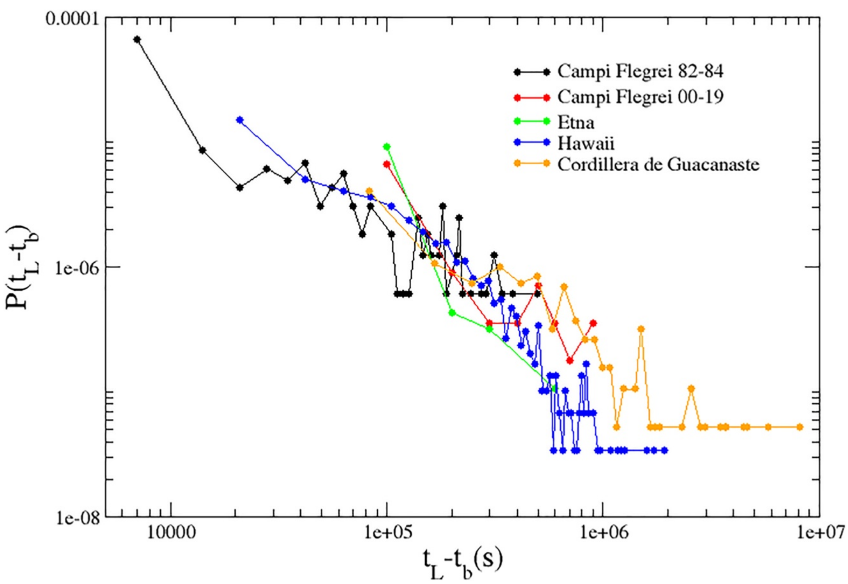

Figure 8. The distribution of the quantity $t_L - t_b$ for the five volcanic catalogs here analyzed.

Table 4
The Estimated Values of the Parameters p, τ, and μ, of Equation 2, for the Five Catalogs Here Analyzed

Catalog	p	τ (s)	μ (s^{-1})
Cordillera de Guacanaste	0.79	$5.8 \cdot 10^6$	0
CF 1982–1984	0.68	$5 \cdot 10^5$	10^{-3}
CF 2000–2019	0.98	$2.8 \cdot 10^6$	10^{-5}
Etna	0.74	$(2.8 \cdot 10^6, 82 \cdot 10^6)$	0
Hawaii	0.58	$3.1 \cdot 10^5$	0

background activity, occurring during the swarms, cannot be observed for the catalogs with a large value of m_c (see Table 2).

Finally the Hawaiian greater productivity for $\delta = 30$, 35, and 40 km can be explained observing that for those δ values the rate of occurrence exhibits an approximately flat regime (for $t > 10^6$ s) (see Figure 5) evidencing that, in these cases, some background activity has been included in the analysis.

5. Conclusions

The general interpretation of earthquake occurrence is that the stress induced by the plate tectonics drives the crustal rocks at a critical state which allows the occurrence of random earthquakes whose magnitude follows the Gutenberg-Richter distribution. Moreover, when an earthquake occurs, it diffuses, in the crust, the accumulated strain generating new stress able to give rise to the occurrence of other earthquakes. Generally the two classes of earthquakes are called mainshocks and aftershocks. The number of aftershocks depends on the magnitude of their mainshock (Helmstetter, 2003) and decreases in time as t^{-p} (Omori, 1894). However such a behavior is not observed for seismic swarms occurring on volcanic, geothermal or tectonic environments where fluids injections and/or movements generate an instant increase of the stress.

We have shown that, stacking the occurrence rate of many swarms it is possible to express the earthquake swarms occurrence rate as a power law decay tapered by an exponential decay allowing its normalization. This kind of approach could be extended to others volcanoes of the world. Obviously this does not implies that the same results are obtained for all the volcanoes of the world and in some cases the seismic activity of the volcano could be more and more complex (see as an example Rodríguez-Cardozo et al., 2021). However we would like to observe that the definition of an analytic expression for the volcanic earthquake occurrence rate could also aim at boosting some debate on the swarms characterization and could be useful in the assessment of the seismic risk where the triggering mechanism is represented by fluids injections and/or movements.

Appendix A: Gamma Distribution and Log-Likelihood

The Gamma distribution can be defined as:

$$p(\Delta t) = \frac{1}{\Gamma(\alpha)} \Delta t^{\alpha-1} e^{-\Delta t/\Theta} \quad \text{(A1)}$$

where, α and Θ are the parameters of the Gamma distribution and $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$ is the Gamma function. The two parameters of this distribution can be easily estimated by the maximum likelihood method. The log-likelihood for the three distribution is:

$$LL = (\alpha - 1) \sum_i \Delta t_i - N \frac{\langle \Delta t \rangle}{\theta} - NK \log\Theta - N \log\Gamma(\alpha) \quad \text{(A2)}$$

A correct estimation of the parameters involves the derivative of $\Gamma(\alpha)$, however an approximated estimation is provided by $\alpha = \frac{3-s+\sqrt{(s-3)^2+24s}}{12s}$ with $s = \log\langle \Delta t \rangle - \langle \log\ \Delta t \rangle$ and $\Theta = \frac{\langle \Delta t \rangle}{\alpha}$.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

Data for Campi Flegrei and Cordillera de Guacanaste can be found at (Tramelli, 2022a, 2022b). Other data here used can be found and downloaded at the web-sites: https://sismolab.ov.ingv.it/sismo/CATALOGO_STATICO/FLEGREI/fle_2000_2019.html, https://www.ct.ingv.it/index.php/monitoraggio-e-sorveglianza/banche-dati-terremoti/terremoti. Indeed this is a more impulsive phenomenon, in respect to the occurrence of a large earthquake, with a duration compatible with a c = 0 (https://zenodo.org/badge/latestdoi/555488289).

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