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Key Points:

- Turbulence has captivated scholars for centuries, yet its complete understanding eludes classical physics
- Understanding turbulence is crucial for weather forecasting, climate modeling, and practical applications, paving the way for progress
- The relentless pursuit of turbulence understanding persists, offering exciting prospects for scientific progress and real-world advancements in diverse fields

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Why (Still) Studying Turbulence in Fluids and Plasmas?

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Abstract Turbulence, a captivating and intricate phenomenon, continues to attract researchers across diverse scientific disciplines. Despite considerable efforts, turbulence remains a fascinating challenge and stands as one of the unsolved enigmas in classical physics. Researchers strive to unravel the underlying physical mechanisms and refine mathematical models to unlock a comprehensive understanding of this complex phenomenon. This paper delves into the reasons why the study of turbulence still persists for a long time, highlighting its history and fundamentals, wide-ranging applications, significance in environmental and climate sciences, and outstanding open challenges. Through these endeavors, the quest for unraveling the mysteries of turbulence promises to yield profound scientific insights and practical applications in the years to come.

Plain Language Summary The study of turbulence has a rich historical backdrop, marked by pioneering contributions from renowned scientists. From Leonardo da Vinci's observations of swirling water to Osborne Reynolds' early experiments, turbulence has fascinated scholars for centuries. Despite sustained efforts, turbulence remains an enigmatic challenge, defying complete understanding within classical physics. Turbulence finds its relevance in a wide array of applications as aircraft design, automotive engineering, oil pipelines, and power generation systems. In environmental and climate sciences, turbulence plays a pivotal role in the transport of heat and pollutants within the atmosphere and oceans. Understanding turbulent processes is essential for accurate weather forecasting, climate modeling, and assessing environmental impacts. This text briefly explores the enduring pursuit of turbulence study, shedding light on its historical foundations, fundamental aspects, diverse applications, and persisting unresolved questions. By delving into these aspects, this paper aims to highlight the ongoing quest for unraveling the mysteries of turbulence and the promising avenues it presents for scientific advancements and practical applications.

1. Leonardo da Vinci, Reynolds' Experiment and Navier-Stokes Equations

Turbulence is a fascinating and complex phenomenon that pervades various aspects of our everyday lives, from the weather patterns that dictate our climate to the flow of fluids in engineering (Dubrulle, 2019; Pope, 2000). Defined as the chaotic, irregular, and unpredictable motion of fluids or particles, turbulence has long captivated the minds of scientists, mathematicians, and engineers since ancient times as testified by beautiful paintings by Leonardo da Vinci (Pedretti, 1982). Unlike laminar flows, where the motion occurs in well-defined and organized patterns, turbulence introduces a complex interplay of multiple flow patterns occurring simultaneously, making difficult to model and predict accurately (Pope, 2000).

The distinction between laminar (regular) and turbulent (irregular) flows can be made by means of the so-called Reynolds number Re, a dimensionless parameter measuring the strength of the nonlinear forces over the viscous ones, and depending on some parameters of the flow (Reynolds, 1883). The Reynolds number has been experimentally found to be

$$\mathrm{Re} = \frac{U\,L}{\nu}, \tag{1}$$

where U is the flow speed, L is a characteristic length (in the original Reynolds' experiment it was the length of the pipe), and ν is the kinematic viscosity of the fluid (Reynolds, 1883). He discovered that below a certain threshold, known as the critical Reynolds number Re^{crit}, the flow remained laminar, with distinct streamlines and minimal mixing, while when crossing it turbulence emerged, marked by the formation of vortices, eddies, and rapid fluctuations in the flow properties. Interestingly, Re can be also directly derived from the incompressible Navier-Stokes equations (NSE), previously introduced by Navier (1823) to describe the motion of fluids,

Figure 1. The formation of numerous small-scale eddies in a river when energy is injected via a waterfall which makes clear the idea of the Richardson cascade.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \nabla^2 \mathbf{u} = -\nabla p + \mathbf{f} \tag{2}$$

with $\mathbf{u}$ being the velocity field, p the fluid pressure, and $\mathbf{f}$ an external force. The Reynolds number represents the ratio between the inertial and viscosity contributions, that is,

$$\mathrm{Re} = \frac{|(\mathbf{u} \cdot \nabla)\mathbf{u}|}{|\nu \nabla^2 \mathbf{u}|}. \tag{3}$$

namely the larger the nonlinear term compared to the linear dissipative one, the larger the Reynolds number will be.

2. Richardson Cascade and Kolmogorov Theory

The most striking fact of the Reynolds' finding is that after a proper re-scaling of the NSE they become only dependent on Re. Nevertheless, since the NSE are nonlinear partial differential equations still there is no rigorous proof of the existence and uniqueness of solution (given a smooth initial data, cfr. the Clay prize, https://www.claymath.org/millennium/navier-stokes-equation/), claiming for a phenomenological framework. A pioneer of phenomenological models of turbulence was Lewis Fry Richardson who coined the idea of *energy cascade* which provides a simple cartoon for describing how energy is transferred from large-scale motions to smaller scales in turbulent flows via eddies (see Figure 1). As the flow progresses to smaller scales, eddies become increasingly numerous until they dissipate.

By exploiting the concept of Richardson cascade in 1941 Kolmogorov devised the first theory of turbulence (Kolmogorov, 1941). Focusing on three dimensional turbulence and using he hypothesis that the small-scale structure of the flow is statistically homogeneous, isotropic, and independent of the large-scale structure, Kolmogorov made the assumption that the rate of energy dissipation ϵ is independent of Re (viscous anomaly). Next, energy transfer from large to small scales (usually referred to as *energy cascade*) can be described as a self-similar process across a range of scales between the integral scale L, where energy is injected, and the Kolmogorov scale η, where energy is dissipated via viscous interactions, known as *inertial sub-range*. This description is supported by the well known 4/5 Kolmogorov equation which related third order velocity correlation function with ϵ, ν and the second order correlation function. This hypothesis has a mathematical foundation on the scale-invariant nature of NSE under re-scaling transformations of the form

$$(t, \mathbf{x}, \mathbf{u}) \rightarrow \left(\lambda^{1-h} t, \lambda \mathbf{x}, \lambda^h \mathbf{u}\right) \tag{4}$$

for any constant $\lambda > 0$ and, in principle, for any $h \in R$ (Frisch, 1995). If we define $\delta u(r) = \mathbf{u}(\mathbf{x} + \mathbf{r}) - \mathbf{u}(\mathbf{x})$ the velocity field fluctuations at the scale r, under re-scaling (Equation 4) we have $\delta u(r) \sim r^h$ (Frisch, 1995; Kolmogorov, 1941). Under the assumption that the energy transfer rate ϵ is constant across scales r and scale-invariant

$$\epsilon' \doteq \frac{\delta u'^2(r)}{t'} = \frac{\lambda^{2h} \delta u(r)}{\lambda^{1-h} t} = \lambda^{3h-1} \frac{\delta u(r)}{t} = \lambda^{3h-1} \epsilon, \tag{5}$$

it immediately follows that

$$h = \frac{1}{3} \rightarrow \delta u(r) \sim r^{1/3}. \tag{6}$$

Thus, a generalized law can be derived for any statistical moment p

$$S_p(r) \doteq \langle \delta u^p(r) \rangle = C_p \epsilon^{p/3} r^{p/3}, \tag{7}$$

providing a full and universal statistical characterization of the inertial sub-range (Frisch, 1995; Kolmogorov, 1941). Kolmogorov was able to prove that the above equation is exact for $p = 3$ in the limit $\nu \to 0$, namely

$$S_3(r) = -\frac{4}{5}\,\epsilon\, r. \tag{8}$$

Notice that the above equation is the basic signature of the Kolmogorov theory and it is striking agreement with existing experimental and numerical data (Benzi & Toschi, 2023; Bruno & Carbone, 2016; Dubrulle, 2019, 2022).

3. Landau's Criticism and Parisi-Frisch Multifractal Theory

While Kolmogorov theory has provided valuable insights into the statistical properties of turbulence, it has soon attracted some objections in terms of its applicability. Among them, in 1944 Landau raised significant criticism, questioning the concept of universality of the constants C_p (this is the content of a famous footnote on page 126 of the original version of the 1959 book by Landau and Lifshitz (2013)). According to Landau C_p depends on the geometry of the large-scale forcing and on the intermittent stochastic nature of the energy redistribution across scales. This implies that the statistical properties of velocity fluctuations may not be universal. Then one can also question about the Kolmogorov conjecture on the scaling $p/3$ in Equation 7 for $p \neq 3$. This point, together with the experimental findings by Anselmet et al. (1984) who found a nonlinear behavior of the scaling exponents $\zeta_p \neq p/3$, motivated, in 1980s, the birth of a new theory of turbulence based on the *multifractal* paradigm (Parisi & Frisch, 1985). Taking into account that NSE are formally scale-invariant for any choice of h, Parisi and Frisch (1985) generalized the concept of self-similarity in a non-homogeneous environment by searching for local (not global) self-similar probabilistic solutions of the NSE (Dubrulle, 2022; Frisch, 1991, 1995; Parisi & Frisch, 1985). The assumption is of having a probability $P_r(h) \sim r^{3-D(h)}$ of observing a re-scaling exponent h for the velocity increment $\delta u(r)$ at the scale r such that the $p-$ order statistical moment $S_p(r)$ must be written as

$$S_p(r) = \int dh\, r^{hp}\, P_r(h) \sim r^{\zeta_p} \tag{9}$$

with being $\zeta_p = p/3 + \tau_p$ a nonlinear convex function of p (Benzi et al., 1984; Frisch, 1995; Parisi & Frisch, 1985), with τ_p taking into account for intermittency correction of the energy transfer rate at different scales (Frisch, 1991). Equation 9 is referred to as the multifractal theory of turbulence and it can be generalized to predict the scaling properties of velocity fluctuations experienced by a Lagrangian particle (tracer) flowing in a turbulent environment. Although the function $D(h)$ is not know, the multifractal theory provides well defined predictions on the statistical properties, for instance, of velocity gradients and the velocity fluctuation of Lagrangian turbulence which are in agreement with existing numerical and experimental data (see Benzi and Toschi (2023) for details).

The key of Equation 9 is the existence of a scale-invariant nature of the field fluctuations, that is, calling for the seminal idea by Kolmogorov that small-scale structures within a turbulent flow exhibit statistical similarity to larger-scale structures. In 1993 Benzi et al. (1993) extended the concept of self-similarity to a form of extended self-similarity (ESS) where statistical correlations extend over a range of scales rather than being limited to just a range of scales. The core idea here revolves around seeking a form of scale-invariance between two structure functions, denoted as $S_q(\tau)$ and $S_p(\tau)$, expressed as:

$$S_q(\tau) \sim S_p(\tau)^{\xi(q,p)}. \tag{10}$$

where $\xi(q, 3) = \zeta(q)$. ESS offers several advantages compared to the conventional scaling against τ since it provides an accurate means of determining scaling exponents, while remaining valid even within the dissipative regime and across a wide range of Reynolds numbers, encompassing both high and mid-to-low values (Benzi & Toschi, 2023).

4. Turbulence in Plasmas

After 20 years of the first theory of turbulence proposed by Kolmogorov, using the data from Mariner 2, one of the first spacecraft orbiting around the Earth, measured a well defined Kolmogorov spectrum in space plasmas (Coleman & Paul, 1968), even for magnetic fluctuations. This opened a strong increasing interest in making similar approximations to describe the turbulent behavior of plasmas. A plasma is a highly ionized gas composed of charged particles, mainly electrons and ions, that coexist in a nearly equal number. Plasmas are abundant in natural (e.g., stars, interstellar/interplanetary medium, near-Earth environment) and man-made systems (e.g., fluorescent lights, fusion reactors). The collective behavior of a plasma, at large scales, is governed by electromagnetic forces and, under the assumption that ions are inertial and electrons are considered at rest, can be described by the incompressible magnetohydrodynamic (MHD) equations (Biskamp, 2003)

$$\partial_t \mathbf{z}^{\pm} + \left(\mathbf{z}^{\mp} \cdot \nabla\right)\mathbf{z}^{\pm} - \nu^{\pm}\nabla^2\mathbf{z}^{\pm} = -\nabla p + \mathbf{f}, \tag{11}$$

which look like the NSE (see Equation 2). The key variables are the Elssäser fields $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$ (Elsasser, 1950), which represent the Alfvénic fluctuations propagating in opposite direction with respect to a large scale magnetic field. Here v and b are the velocity and magnetic (in units of Alfvén speed (Alfvén, 1942)) fields, respectively, p is the plasma pressure, and $\nu^{\pm}$ is the sum/difference between the viscosity ν and the magnetic diffusivity η. As for NSE they possess a scale-invariant nature

$$\left(t, \mathbf{x}, \mathbf{z}^{\pm}\right) \rightarrow \left(\lambda^{1-h}t, \lambda\mathbf{x}, \lambda^{h}\mathbf{z}^{\pm}\right) \tag{12}$$

for any $\lambda > 0$ and $h \in R$ (Carbone, 1993). By exploiting similar arguments as made by Kolmogorov, Iroshnikov (1964) and Kraichnan (1965) developed the first theory of plasma turbulence. Assuming that the small-scale statistics, now described in terms of Elsässer field increments $\delta z_r^{\pm} = z^{\pm}(\mathbf{x} + \mathbf{r}) - z^{\pm}(\mathbf{x})$, is statistically invariant under re-scaling transformations (Equation 12) and assuming that the energy transfer rate ϵ is preserved, then

$$S_p(r) \doteq \langle \delta z^{\pm p}(r) \rangle = C_p \epsilon^{p/4}\, r^{p/4}. \tag{13}$$

This scaling, different from the Kolmogorov scaling, is obtained by assuming that the strength of nonlinear interactions can be lower in MHD with respect to usual fluid flows, due to the fact that they only take place between Alfvénic fluctuations propagating in opposite directions (cfr. Equation 11). This sweeping effect lowers the energy cascade which in fact is realized on a time different from the eddy-turnover time. As for fluids, experimental measurements demonstrated the failure of the global self-similar picture of plasma turbulence (Carbone et al., 1995, 1996), claiming for a revised multifractal formalism in a similar manner of that developed by Parisi and Frisch (1985). Thus, the intermittent character of the cascade mechanism (Benzi et al., 1984; Carbone, 1993), forming topological structures of different size and with different geometries (Dubrulle, 2019), is traduced in observing a wide spectrum of re-scaling exponents h (i.e., symmetries of the MHD equations) at different scales, moving from a global to a local concept of scale-invariance (Frisch, 1991), due to the fact that the energy transfer rate is not preserved along scales but it fluctuates both in space and time (Frisch, 1995; Landau & Lifshitz, 2013; Mandelbrot, 1982).

Using MHD equations it is also possible to derive an exact law for the third-order mixed moment, similar to the 4/5-Kolmogorov law (Carbone et al., 2009; Marino & Sorriso-Valvo, 2023; Politano & Pouquet, 1998)

$$\langle \delta z_{\parallel}^{\pm}(r) |\delta z^{\pm}|^2 \rangle = -\frac{4}{3}\epsilon^{\pm} r \tag{14}$$

where $\epsilon^{\pm}$ represents the energy dissipation rates for the two kind of fluctuations. The relation 14 has been carefully verified in space plasma using data from the Ulysses spacecraft (Sorriso-Valvo et al., 2007).

5. Open Challenges for Future Directions

Despite almost a century of scientific progresses in the theoretical, experimental, and numerical aspects of fluid and plasma turbulence, with increased accuracy of laboratory facilities and computational power, some features of turbulence remain elusive, and many aspects of this phenomenon continue to be the subject of active investigation for everyday applications in geophysics, astrophysics and space physics, aviation, engineering, etc.

5.1. Solving NSE or MHD Equations

From a purely mathematical point of view the main open challenge is to rigorously prove the existence of viscous anomaly, that is, that energy dissipation remain constant in the limit of large Re number. Recent advances on a simpler but highly non trivial problem of the passive scalar provide some hint on this problem (Falkovich et al., 2001). Based on Reynolds' findings we can provide a different view/definition of turbulence as the dynamical behavior of a neutral or magnetized fluid for $\mathrm{Re} \to \infty$ (analytically) or for high-Reynolds number, exceeding the critical one $\mathrm{Re} \gg \mathrm{Re}^{crit}$ (practically). When Re is small, then the flow is smooth and we can reasonably neglect the contribution coming from the nonlinear term $(\mathbf{u} \cdot \nabla)\mathbf{u}$. Thus, the NSE become linear and they can be analytically solved almost exactly. For moderate Re the flow can be described as a superposition of a large number of periodic or quasi-periodic patterns of time lifting the velocity field to appear *chaotic* as shown by Ruelle and Takens (1971). It means that turbulent flow for moderate Re can be predicted up to an intrinsic time $t_p = \lambda^{-1}$, where λ is the largest positive Lyapunov exponents which quantifies the exponential departure of two close initial conditions, while for times longer than t_p the flow behavior can only be predicted probabilistically. When $\mathrm{Re} \to \infty$ then the non-linear term of the NSE cannot neither be neglected nor be parameterized lifting the velocity field to be *rough* over space with strong variations over relatively small spatial scales. This is a consequence of the *singularity* of the NSE or MHD equations, that is, the velocity gradients become infinite at small scales close to the viscous/dissipative ones without the action of any external small-scale perturbations. This means that we know the large-scale dynamics of a neutral (i.e., the NSE) or magnetized (i.e., the MHD equations) fluid but we are not able to provide a rigorous proof of the viscous anomaly, that is, that the energy dissipation is constant for large Re, although experimentally observed in two-dimensional passive scalar turbulence for the rate of enstrophy dissipation (Benzi & Toschi, 2023; Falkovich et al., 2001).

This is strictly connected to the existence of smooth solutions of the Navier-Stokes existence which is still unresolved for the full set of three-dimensional incompressible equations (Fefferman, 2000). While substantial progress had been made in proving local existence and regularity results (Robinson & Sadowski, 2007), the question of global existence and smoothness for all possible initial data remained an open challenge (Robinson, 2020). Notable results included theorems that established local well-posedness for smooth initial data and provided criteria for the existence of solutions for a limited time (Pooley & Robinson, 2016; Tao, 2016). However, the behavior of solutions at high Reynolds numbers (i.e., flows with high turbulence) remained particularly challenging, and it was not clear whether smooth solutions could persist or if singularities would develop under certain conditions.

5.2. When and How the Excess of Energy Is Dissipated in Fluids and Plasmas

An open issue closely related to that introduced before is related to the extension of the inertial range. Indeed, it extends from the integral scale L, where energy is injected, to the Kolmogorov scale η, where energy is dissipated via viscous effects. The latter is defined as the scale where the Reynolds number of velocity fluctuations $\delta u(\eta)$ is equal to 1, that is, $\nu = \delta u(\eta)\,\eta$. Since $\delta u(r) \sim r^h$, then

$$\frac{L}{\eta} \sim \mathrm{Re}^{\frac{1}{1+h}}. \qquad (15)$$

This means that there is a variety of *dissipative scales* depending on the value of h and that the extension of the inertial range is dependent on both Re and h. Equation 15 holds in the same way for plasmas, thus providing similar constraints both for neutral and magnetized fluids. Indeed, it has profound consequences both in terms of numerical solutions of NSE or MHD equations and in terms of the physical mechanisms behind dissipation.

From the numerical point of view the last 40 years provided a huge increases in terms of computation power and numerical resources to approach a solution of the NSE or MHD equations (Benzi & Toschi, 2023; Dubrulle, 2019). Nevertheless, this should require a number of grid points, in time and space, of the order of $\mathrm{Re}^{\frac{3}{1+h}}$, largely exceeding the computational capabilities of present computers (Benzi & Toschi, 2023; Dubrulle, 2019).

From the physical point of view Equation 15 is consistent with the observation of enhanced intermittent fluctuations around the Kolmogorov scale η with respect to those predicted by the scaling exponent $\zeta(p)$ in Equation 9. This means that the most intermittent structures of a turbulent flow are of the order of η. A closely related question is to identify the sources of these intermittent structures, if they are driven by strong fluctuations in the dissipation range or if they are generated by stochastic fluctuations of the energy transfer across the inertial range. Numerical simulations support the latter scenario (Benzi & Toschi, 2023) suggesting that the small-scale structure of turbulent flows, mainly characterized by vortex filaments or tubes below the Kolmogorov scale, is an intrinsic property of the viscous/dissipative range, being independent on intermittent structure of the inertial

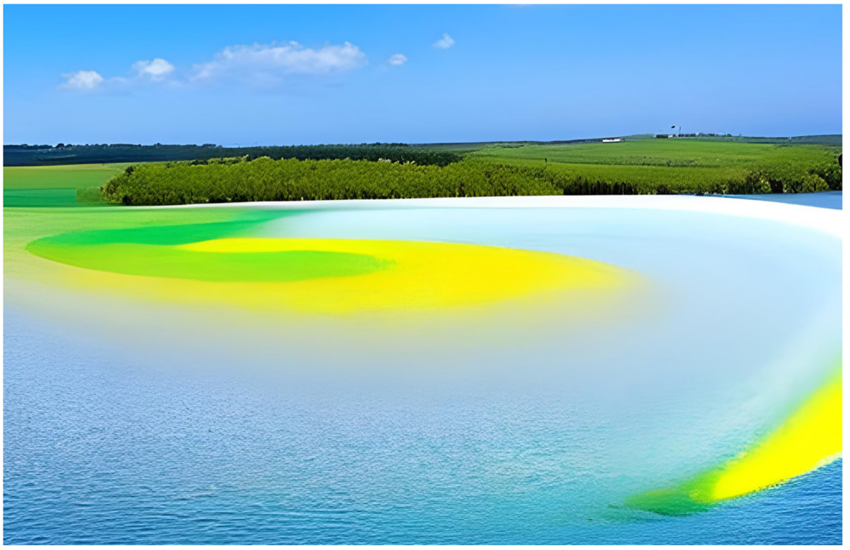

Figure 2. Schematic illustration of the dispersion of pollutants in the ocean driven by turbulent processes. Yellow and green colors represent the level of pollutants (yellow-low, green-high).

range. These independence arises from the fact that the physical mechanisms governing turbulence in these two ranges are fundamentally distinct. In the inertial range, nonlinear interactions among eddies dominate, giving rise to intermittent, high-energy structures. Meanwhile, the dissipative range is characterized by the overwhelming influence of viscosity, leading to the gradual dissipation of energy and the smoothing out of structures (Benzi & Toschi, 2023; Frisch, 1995; Pope, 2000). This has been supported by statistical studies on the Markovian nature of fluctuations within the two regimes based on empirical observations on both fluid and plasma turbulence (Benella et al., 2022; Renner et al., 2001), as well as, by direct numerical simulations (Benzi & Toschi, 2023).

A more complicated scenario needs to be addressed in collisionless plasmas, like the solar wind, where Coulomb collisions between particles are very rare (Bruno & Carbone, 2016). The collision-free nature coupled with electromagnetic fields increases the number of degrees of freedom and provide a different picture of MHD turbulence with respect to the hydrodynamic case. Indeed, the lack or rarity of particle collisions means that the excess of kinetic energy should occur without viscous processes. In this situation, the Kolmogorov scale cannot be appropriately defined (Biskamp, 2003) and the key scale is the so-called ion inertial length d_i (Biskamp, 2003; Bruno & Carbone, 2016). It is the scale at which ions decouple from electrons and the magnetic field becomes frozen into the electron fluid rather than the bulk plasma. Around this scale many physical mechanisms occur as wave-particle interactions, ion cyclotron waves, and resonances (Bruno & Carbone, 2016), that could mediate the turbulent energy dissipation. Nevertheless, the dissipation mechanism that replaces the usual viscosity is still obscure, and this represents a strong challenge for space plasma physics with recent space missions specifically designed to solve this by being equipped with high-resolution instruments to resolve the small-scale physics of the interplanetary medium (Fox & McComas, 2016; Müller et al., 2020).

5.3. Dispersion of Pollutants in Geophysical Flows

An active area of research is on the dispersion of pollutants in geophysical flows, as the atmosphere and the oceans, through turbulence which is a crucial aspect of understanding air/marine pollution and its impact on human health and the environment (Fernández-Pacheco et al., 2023). Dispersion of particulates, molecules, gases, and other form of suspended materials can cause diseases, deaths, and damages to different living organisms and may come from anthropogenic or natural sources (Högström et al., 2002; Wolf et al., 2020). When pollutants are emitted into the atmosphere or the ocean, they undergo turbulent mixing, resulting in their dispersion over a wide area (see Figure 2), causing them to spread out horizontally and vertically (Pasquill, 1971). The key factor affecting the dispersion of pollutants are the stability of the atmosphere, which can suppress or enhance mixing along the vertical direction (Högström et al., 2002), the speed and direction of the wind, mainly determining the distances at which pollutants are transported (as during fires and/or volcanic eruptions), and the height of the atmospheric stable boundary layer, influencing both the vertical and horizontal transport of pollutants (Phalen & Phalen, 2011).

Nowadays, in the increased emission scenarios due to climate change (Masson-Delmotte et al., 2021), there is an increasing interests in understanding turbulent dispersion of pollutant in geophysical flows like the atmosphere and the ocean. Several models, incorporating turbulence and atmospheric dynamics and chemistry, are employed to predict dispersion patterns (e.g., Gaussian plume models, Lagrangian stochastic dispersion models, computational fluid dynamics (CFD) models). These models simulate the movement of pollutants based on inputs such as meteorological data, emission characteristics, and atmospheric stability, helping in estimating pollutant concentrations at different locations and understanding the dispersion patterns under various conditions (Fernández-Pacheco et al., 2023).

5.4. Drag Reduction in Pipe Flows

An ongoing research with many practical applications in fluid transport is the control or reduction of drag and other frictional resistances via drag reducing agents (DRA), that is, materials that are able to reduce frictional pressure loss by reducing the level of turbulence (Lester, 1985). Indeed, airplanes, wind turbine, and pipelines typically encounter turbulent flows that generate skin-friction drag, constraining both speed and fuel efficiency, with significant economic and environmental effects (Marusic et al., 2021). DRA are usually long-chain polymers that are added to the fluid and that, through the so-called *visco-elastic effect*, are able to alter the flow characteristics creating a more streamlined flow profile and reducing the energy dissipation caused by turbulence along the walls and the boundaries (L'vov et al., 2004). By reducing the energy required to overcome friction, DRA can enhance the efficiency of fluid transport and potentially lead to cost savings (Lee et al., 2023). The use of DRA has been successful in various industries, including oil and gas, where the transport of crude oil and petroleum products over long distances is crucial. Nevertheless, future studies are needed to improve the effectiveness of DRA which depends on various factors as the fluid properties and flow conditions, as well as, to determine the appropriate dosage and application method for a given pipeline system (Alsurakji et al., 2023; Benzi & Ching, 2018; Marusic et al., 2021).

5.5. Turbulence and Climate Change

Turbulence in the context of climate change plays a significant role in various aspects of ocean and atmospheric dynamics, including its influence on El Niño events and jet stream dynamics. El Niño, a climate phenomenon characterized by the periodic warming of sea surface temperatures in the central and eastern equatorial Pacific Ocean (Philander, 1983), is amplified by turbulence. As an example, during an El Niño event, the warm water layer in the central and eastern Pacific deepens due to increased turbulence, enhancing the warming associated with El Niño (Lien et al., 1995). Earth's jet streams and weather systems (Tuck, 2021) are also affected by the potential increase in atmospheric turbulence (Lovejoy, 2019), particularly as a result of climate change. Turbulence indeed can occur at various scales in the atmosphere, from small-scale eddies and gusts to large-scale weather systems (Sutton, 2020). It is influenced by factors such as temperature gradients, wind shear, and atmospheric stability. Climate change, on the other hand, is primarily driven by the increased concentration of greenhouse gases in the Earth's atmosphere, mainly carbon dioxide (CO_2) and methane (CH_4), resulting from human activities (Masson-Delmotte et al., 2021) and altering atmospheric composition, stability, and motion. While the precise relationship between turbulence and climate change is an ongoing area of research, there are several factors that suggest a potential connection.

Most on the research done on climate change induced turbulence focuses on jet streams which are strictly related to the occurrence of turbulence. Jet streams are indeed driven by velocity shears and temperature gradients: the larger the shear/gradient, the stronger the jets and the more turbulence (Lee et al., 2019). Increasing anthropogenic greenhouse gases in the atmosphere is leading increasing temperature gradients with a consequent intensification of shear-driven clear-air turbulence (CAT) episodes. CAT has a practical everyday implication on flights (see Figure 3): it induces more turbulent flights, especially along transatlantic routes (Williams & Joshi, 2013).

CAT, that is, cruising-phase (about 10 km from the surface) aircraft bumpiness in free-cloud regions and far-away from thunderstorms, is one of the largest causes of weather-related aviation incidents (Sharman et al., 2006). The major drawback of CAT is that it cannot be seen neither by pilots nor by radar and it cannot even be predicted

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Figure 3. Schematic illustration of the role of the jet stream and velocity shears in driving aircraft turbulence. Green, orange, and red texts report the three different categories of aircraft turbulence together with the expected bumpiness in meters.

by using operational turbulence forecasts via numerical models. Indeed, aviation turbulence occurs on scales of a few hundreds of meters but the current computational power is not sufficient to explicitly describe these scales (Sharman et al., 2006). Thus, several diagnostic indices have been developed to likely and possibly identify CAT as the Ellrod and Knapp turbulence index (Ellrod & Knapp, 1992), the Richardson number, the potential vorticity, and so on (Jaeger & Sprenger, 2007). It has been recently reported that the frequency of CAT episodes increases along the globe at aircraft cruising altitudes over the past four decades (up to 55%), in line with the observed effects due to climate change (Prosser et al., 2023). This is likely projected to increase in the future with 10%–40% increase in low-turbulence episodes and a 40%–170% increase for moderate-or-greater turbulence ones (Williams & Joshi, 2013). Current challenges for scientists include to improve climate models to capture these complex interactions in order to predict and mitigate the impacts of climate change and turbulence on various sectors, including aviation, transportation, and weather forecasting. All the above mentioned aspects have also profound economic implications since the effects of climate change on turbulence may need to modify take-off conditions (Gratton et al., 2020) as well as may lead to longer flight times with increased fuel consumption and emissions, requiring to search for fuel-optimized strategies (Wells et al., 2021).

Turbulence in the atmosphere also influences cloud formation and dissipation and accurately representing clouds in climate models is vital for predicting future climate changes. However, there are various challenges in representing clouds in climate models, including the complex interactions between turbulence and cloud microphysics (Liu et al., 2023). Turbulence affects cloud formation and dissipation, and the interactions between turbulence and cloud particles can be highly non-linear and difficult to model accurately (Shaw, 2003). One of these interactions is related to the cloud microphysics since turbulence can influence the collision and coalescence of cloud droplets, ice crystal formation, and the distribution of cloud particle sizes. Furthermore, turbulence also enhances the mixing of environmental air into a cloud that can impact the cloud's properties, including its size, density, and lifespan (Liu, 2019). Addressing knowledge gaps related to turbulence-cloud interactions and turbulent mixing processes is essential for improving the accuracy of climate models since clouds remain a major source of uncertainty in climate projections. Small changes in cloud properties or behavior can have significant effects on the magnitude and regional distribution of future climate changes.

Another crucial aspect of climate change is the increase in the intensity of extreme weather events, referring to severe and unusual weather conditions that deviate from the average climate patterns (Masson-Delmotte et al., 2021). These events can include hurricanes, tornadoes, heatwaves, intense precipitations, droughts, and floods, among others. Climate change is believed to be contributing to the increase in extreme weather events by altering atmospheric conditions and causing shifts in weather patterns (Faranda et al., 2023). For example, warmer air temperatures can enhance the formation and intensity of hurricanes, while higher atmospheric moisture content can lead to heavier rainfall and increased flood risks (Faranda et al., 2022). Extreme weather events and turbulence are both influenced by atmospheric conditions and can sometimes be interconnected. For example, thunderstorms, which are extreme weather events, can generate areas of severe turbulence due to the convective activity and rapidly changing wind patterns associated with them. In this case we refer to convective turbulence since it arises from convective processes in the atmosphere and it occurs when there are rapid vertical movements of air masses due to temperature gradients. Convective turbulence within thunderstorms may result in rapid changes in wind speed and direction, leading to severe turbulence that poses risks to aircraft, especially during takeoff, landing, or when flying in or near convective weather systems. As for CAT also convective turbulence can be challenging to predict accurately due to its localized and rapidly evolving nature, although if associated with thunderstorms it can be detected via weather radars and numerical weather prediction models. Mitigating convective turbulence risks is essential for aviation safety. Pilots may alter their flight paths, change altitudes, or request deviations from air traffic control to avoid areas of known or expected turbulence. Airlines and aviation authorities also employ turbulence reporting systems to share real-time information about turbulence encounters, helping to improve forecasts and enhance safety measures. Convective turbulence is also challenging for numerical models at convection-permitting scales (from a few km) since there is no theory for simulating turbulence, thus requiring for future studies to devise parameterization strategies for unresolved physics as turbulence and sub-grid-scale clouds (Coppola et al., 2021; Moeng, 2014; Prein et al., 2015).

6. Conclusions and Perspectives

In this paper we briefly explored how turbulence has been understood and studied for a long time and all the different ways it is approached and encountered. We also discussed on open challenges to show that studying turbulence can help us learn new things and find useful and practical ways to use it. Clearly, we only tackled a reduced list of open challenges and fields of applications claiming for deeper understanding of turbulence. There are still missing aspects and applications that merit to be considered and that are briefly mentioned below as future perspectives.

One of these is the use of machine learning and artificial intelligence (AI) to reshape the study of turbulence, closure problems, and large-eddy simulations (LES). AI, indeed, excels in data-driven modeling, discerning intricate relationships from rich data sources, yielding more precise turbulence models based on neural networks (Duraisamy et al., 2019). As an example, in LES, AI contributes to represent unclosed terms, model discrepancies, and subfilter scales, emphasizing the importance of the training process in ensuring the consistency of ML augmentations with the underlying physical model (Duraisamy, 2021).

On the theoretical side renormalization techniques have emerged as a pivotal approach in addressing the intricate theoretical problems posed by turbulence, establishing themselves as a cornerstone in fluid dynamics (Yakhot & Orszag, 1986). The key aspect is the concept of coarse-graining, where the statistical characteristics of turbulent flow structures are examined at progressively larger scales, facilitating a deeper understanding of the energy cascade, scaling laws governing turbulence, and other crucial phenomena (Eyink, 1994; Verma, 2004). In recent years, notable strides have been made in refining and advancing renormalization techniques in the context of turbulence, especially in approaching the problem of inviscid limit in the presence of small-scale noise and the concept of spontaneous stochasticity (Mailybaev & Raibekas, 2023).

On the numerical point of view the need for larger numerical simulations in turbulent flows is claimed for investigating fine-scale structures in high Reynolds number flows, common in aerospace, environmental modeling, and industrial processes. Larger simulations also serve as benchmarks for validating turbulence models and numerical techniques, assisting in assessing the accuracy of turbulence closures (Benzi & Toschi, 2023; Dubrulle, 2022). Perspectives in this field should carefully consider that the pursuit of larger simulations is not without constraints. Indeed, computational resources, including processing power and storage capacity, can pose limitations; hence, practical considerations must be weighed against research objectives. In this regard, hybrid approaches emerge as a promising solution by combining various simulation techniques, such as LES and methods for modeling smaller scales, to strike a delicate balance between computational cost and simulation accuracy (Biferale et al., 2017; Fröhlich & von Terzi, 2008; Linkmann et al., 2018).

It is also noteworthy to remark that turbulence is also a significant factor in the transport of energetic particles in space plasmas, fusion devices, and space weather. Indeed, turbulence in space plasmas can scatter and accelerate energetic particles, including cosmic rays, solar energetic particles, and those found within astrophysical objects like accretion disks and pulsar wind nebulae (Horton, 2012). In fusion devices like tokamaks turbulence plays a dual role: on one side, it can lead to particle and energy loss, hindering the achievement of controlled fusion reactions; on the other side, it is fundamental in fuel mixing and increasing fusion reaction efficiency through enhanced particle transport (Walkden et al., 2022). Finally, turbulence in the solar wind and near-Earth electromagnetic environment is a key driver of space weather phenomena, transporting energetic particles toward Earth which can disrupt satellite communication, navigation systems, and power grids (e.g., Laitinen, Timo et al. (2018)).

Thus, studying turbulence is an active area of research that requires a multidisciplinary approach coming from dynamical systems, statistical mechanics, and complex systems (Alberti et al., 2023), also combining experimental, theoretical, and computational methods to tackle the challenges posed by this complex phenomenon. The relentless pursuit of turbulence understanding persists, offering exciting prospects for scientific progress and real-world advancements in the future.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

Data were not used, nor created for this research.

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