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Real time Gutenberg–Richter *b*-value estimation for an ongoing seismic sequence: an application to the 2022 marche offshore earthquake sequence (M_L 5.7 central Italy)

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SUMMARY

We estimate the *b*-value parameter of the Gutenberg–Richter law for earthquake magnitudes in the early stage of the Costa Marchigiana (Italy) seismic sequence, starting on 2022 November 9, with an M_L 5.7 event in the Adriatic sea. In particular, we estimate both the completeness magnitude M_c and the *b*-value within the first 4 and 7 d after the initial strong event in the sequence. Our work represents a practical example of *b*-value estimation in 'true' real time, that is, during the seismic sequence, and its possible interpretation in terms of short-term forecasting. We highlight some critical issues to consider both in estimating/intepreting the *b*-value, and in evaluating the real time estimation of M_c. These issues are mainly due to the fact that preliminary catalogues available in real time are quite different from the revised ones, which are usually delivered after a few months. The criticalities are linked to the raw data recorded at an early-stage, an unreliable evaluation of the M_c with statistical approaches, the Short Term Aftershock Incompleteness entailed after the initial strong event, and the magnitude binning. Our results show that real time estimation of the *b*-value can give insights into the evolution of an ongoing seismic sequence, when attention is paid to data quality and quantity.

Key words: *b*-value; Probabilistic forecasting; Statistical methods; Statistical seismology.

1 INTRODUCTION

The frequency–magnitude distribution (FMD) of earthquakes is well described by the Gutenberg–Richter law (GR, Gutenberg & Richter 1944); the parameter of this law that rules the proportion between small and large earthquakes is called *b*-value. Great interest has been shown in the literature about this parameter, as some authors claim that its variability can be interpreted in terms of physical processes and, therefore, as a possible precursor for strong seismic events (Smith 1981; Gulia & Wiemer 2019). The global increase of the coverage by seismic stations and the development of algorithms for detecting earthquakes have enormously expanded the number of events in instrumental seismic catalogues (Ross *et al.* 2019). This large availability of events led to an increase in the interest about spatial and temporal variations of the *b*-value (Wiemer & Wyss 2002; Schorlemmer 2005; Gulia *et al.* 2016).

A few authors highlighted that the *b*-value varies significantly in space and time, during some Italian seismic sequences, such as the 1997 Colfiorito, the 2009 L'Aquila and the 2016 Amatrice sequences; the common feature of these three sequences is that the *b*-values were anomalously low especially in the areas that were affected by the strongest shocks (Murru *et al.* 2004; De Gori *et al.* 2012; Montuori *et al.* 2016). Gulia & Wiemer (2019) suppose that the evolution of the *b*-value can be analysed as a proxy for the average stress conditions of a fault and, on this basis, they propose a traffic light classification to assess the probability of a subsequent large event in real time. The proposed method has already been applied and debated in the literature (Dascher-Cousineau *et al.* 2020; Gulia & Wiemer 2021; Dascher-Cousineau *et al.* 2021). Some evidences against the interpretation of the *b*-value as a reliable precursor have been found (Lombardi 2022), and some adjustments were proposed to account for transient changes in catalogue completeness (van der Elst 2021), by which the *b*-value is strongly affected (Marzocchi *et al.* 2020).

The active debate about the *b*-value variability reflects the importance of estimating this parameter in real time, that is, during the seismic sequence (using an early-stage, not revised catalogue), still bearing in mind that several sources of bias may lead to a wrong interpretation of the estimates obtained (Marzocchi *et al.* 2020; Geffers *et al.* 2023). One above all, it is known that preliminary catalogues contain a lower number of events with respect to the catalogues revised at a later stage; also the magnitudes can slightly vary after the revision (Cattania *et al.* 2018).

We contribute to this debate by performing the real time estimation of the *b*-value for the still ongoing earthquake sequence on the Costa Marchigiana (Pesaro-Urbino, Italy), that started with an M_w

5.5 (M_L 5.7) event occurred in the sea at about 30 km NE from Fano (Pesaro-Urbino municipalities, Italy) on 2022-11-09 06:07:25 UTC. An M_L 5.2 event has followed this earthquake about one minute later, and two events with $M_L > 4$ occurred in the following seven days, during which a total of about 365 shocks, with a minimum magnitude M_L 0.9, were recorded ($M_L \geq 4$ events in Table 1). Here, we estimate the *b*-values for the first 4 and 7 d since the initial strong event, we highlight the possible problems associated with the estimation in real time, and we interpret the *b*-value's temporal variability.

2 METHODOLOGY

To estimate the *b*-values during the first days of the recent Costa Marchigiana earthquake sequence, we first compute the completeness magnitude M_c of the catalogue, which is the minimum value for detection. Several approaches can be applied to estimate the completeness value in a seismic catalogue, all based on the assessment of the magnitudes' exponentiality (i.e. the GR law). For comparison, we consider here the three apparently most often used methods, that are the Lilliefors statistical test (Lilliefors 1969; Herrmann & Marzocchi 2021), and two methods that directly use attributes of the FMD, namely the relative maximum curvature, and the 90 and 95 per cent goodness of fit of the FMD to the exponential distribution (Wiemer & Wyss 2000; Wiemer 2001). To take into account the so-called Short Term Aftershock Incompleteness (STAI, Kagan 2004; Lolli & Gasperini 2006; Stallone & Marzocchi 2019; Stallone & Falcone 2021), that is, the lack of detection of small events after large earthquakes in instrumental seismic catalogues, we used the heuristic technique of Zhuang *et al.* (2017). This approach, already used in Italian *b*-value studies (Herrmann *et al.* 2022), consists of evaluating the STAI by investigating the sequential number versus magnitude plot, which indeed allows us to highlight some small events missed just after the strong shock.

Once computed the magnitude of completeness, we estimate the *b*-value by applying the well-known maximum-likelihood estimation (MLE) technique (Aki 1965). We apply here the formulation by Marzocchi *et al.* (2020):

$$\hat{b} = \frac{N/(N-1)}{\ln(10)\left[\overline{M} - \left(M_c - \frac{\Delta M}{2}\right)\right]}, \qquad (1)$$

where N is the number of events, $\overline{M}$ is the mean magnitude and ΔM is the magnitude bin. Eq. (1) corrects the classical estimate by Aki (1965) accounting for the binning of the magnitudes, and it is asymptotically unbiased, as proposed in Ogata & Yamashina (1986). The confidence interval (CI) associated to the *b*-value estimation (Marzocchi *et al.* 2020) is given by $\mathrm{CI}_{95\text{ per cent}} = \left(\hat{b} - 1.96\,\hat{\sigma}\ ,\ \hat{b} + 1.96\,\hat{\sigma}\right)$, where $\hat{\sigma} = \hat{b}/\sqrt{N}$ is the standard error (Aki 1965), which is 10 per cent for about 100 events, and reaches 5 per cent with 400 of events. In the real time analysis, it is difficult to identify the 'true' number of events, as it is related to the correct completeness magnitude.

3 RESULTS

The data'set used to estimate the *b*-value in real time includes the events within the circular region of 30 km radius centred at the epicentre of the starting event of the Costa Marchigiana earthquake sequence (latitude 44.013 °N, longitude 13.320 °E, M_L 5.7), with maximum depth 30 km (Fig. 1). The two temporal intervals considered for the analysis start on 2022-11-09 06:07:25 UTC (starting event time), and end on 2022-11-13 16:00:00 UTC (T1, 4 d since the starting event) and 2022-11-16 16:00:00 UTC (T2, 7 d since the starting event). During T1, 291 events were recorded, increasing to a total of 365 in T2. In both cases, the minimum magnitude recorded is M_L 0.9.

The first step in this study is to analyse the FMD for the number of events (Figs 2a, c, e and f), and the presence of the STAI (Figs 2b and f) caused by the first strong event of the sequence, which could affect the results of our statistical analysis (Kagan 2004; Stallone & Marzocchi 2019). We tackle this issue by removing the first four hours after the initial event, which reduces the STAI significantly (Figs 2d and h). The subjective choice of four hours will be investigated in detail afterwards. All the FMDs show the presence of an anomalous high number of M_L 1.7 and 2.4 events in the sequence, and this is true independently of the STAI reduction.

To estimate the *b*-value, we first compute the completeness magnitudes for the catalogues in T1 and T2, with and without STAI, using the different methods introduced in the previous section. The M_c values vary between 1.8 and 2.2 (see Table 2). By applying the 95 per cent fit of the FMD to the exponential distribution, we do not get any estimated value, due to the low number of data considered; therefore, this method is not present in Table 2. We also apply the maximum curvature method with a correction factor of 0.2, that is, $M_c = M_c(\mathrm{MAXC}) + 0.2$, because the classical MAXC is shown to underestimate M_c by this factor (Woessner & Wiemer 2005; Gulia & Wiemer 2019). The maximum curvature method with the 0.2 correction is the most conservative one (the minimum for reliability), and probably the most reasonable among those obtained. Still, the corresponding estimate for M_c appears too low, as already shown for this method in Roberts *et al.* (2015). Indeed, recalling that M_c is estimated as the minimum value for the magnitudes to be exponentially distributed, a proper completeness threshold should be the value corresponding to the right end of the hump appearing in the incremental FMDs, or larger (Fig. 2, left-hand column, red dots). It then follows that the statistical methods adopted to estimate M_c are poorly reliable in our case of analysis, owing to two aspects. First, the number of events is not enough to guarantee statistical reliability. The sequence we are considering has not been preceded by any significant seismic activity, thus impeding the consideration of past history in the estimation. Second, the catalogue contains an anomalous high number of events with a specific M_L value (1.7 and 2.4), aspect that could be related to the raw data (i.e. primary, not revised data), to the binning adopted for the magnitudes or to the uncertainty in the magnitude estimation (Taroni 2022). The magnitudes' bin in our catalogue is 0.1, higher that the usual value of 0.01 considered in high-definition seismic catalogues (Herrmann & Marzocchi 2021). The larger bin considered may undermine the exponentiality of the magnitudes themselves. In fact, methods like the Lilliefors test account for a uniform noise that may affect the results of exponentiality testing when the bin is larger than 0.01 (Herrmann & Marzocchi 2021).

Acknowledging the high sensitivity of the *b*-value estimation to the completeness magnitude, as well as the low reliability of the completeness magnitudes we obtained, we decided to perform the *b* estimation with several values of M_c. More precisely, we consider an array of 'reasonable' completeness thresholds in the interval [2.2, 2.5]. We ignore the results from Lilliefors and 90 per cent tests because they appear unreliably low from the visual inspection of the FMDs (Figs 2a, c, e and f); besides, a completeness magnitude lower than 2 just after the occurrence of a strong shock is unrealistic. For both T1 and T2, we then estimate the corresponding *b*-value together with the 95 per cent CI (Fig. 3); these estimations are made

Table 1. M_L 4.0+ events occurred in the first seven days of Costa Marchigiana earthquake sequence.

Time (UTC) yyyy- mm-dd	hh:mm:ss	Latitude °N	Longitude °E	M_L	Depth (km)
2022-11-09	06:07:25	44.01	13.32	5.7	5.0
2022-11-09	06:08:28	43.91	13.35	5.2	8.0
2022-11-09	06:12:57	43.97	13.30	4.0	5.0
2022-11-10	17:54:12	43.92	13.34	4.1	6.0

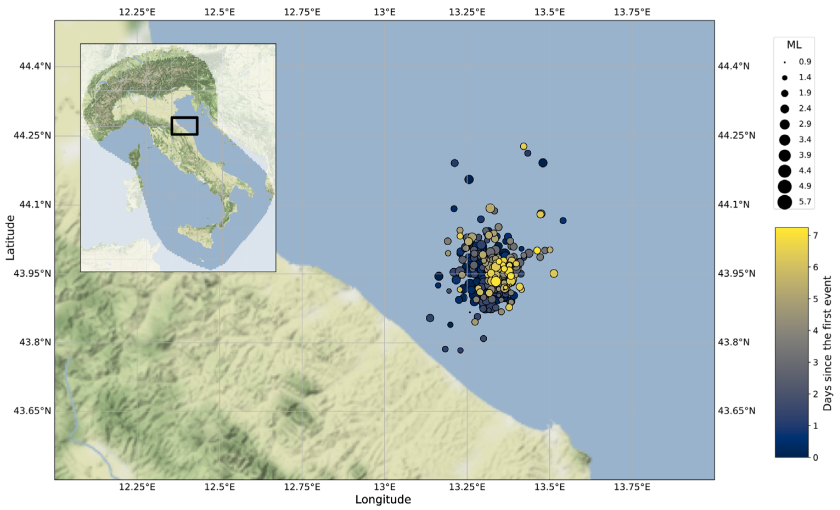

Figure 1. Seismic map of Costa Marchigiana earthquake sequence during its first 7 d. Circles colour varies with the time since the initial M_L 5.7 event, while the size increases with the magnitude.

excluding the STAI periods, to avoid bias in the estimations. The results show that the *b*-values obtained are highly uncertain. In fact, for about all the M_c considered, the CIs are quite large, not allowing to identify a well-constrained estimate of the *b*-value.

To minimize our subjective choices and ensure the stability of the results obtained, we finally perform the above analysis by considering also an earthquake catalogue with a different length of STAI (from 2 to 8 hr), and a different radius for the study area (30, 40 and 50 km). Results are given in the Supporting Information, and show no substantial difference with the conclusions drawn above.

4 DISCUSSION

The problems encountered to get the proper completeness value are straightforwardly inherited by the *b*-value, whose estimate clearly suffers from the lack of data. The MLE of this parameter could also have a bias related to the dynamic magnitude range, that is the difference between the largest magnitude observed and the completeness M_c, as the dynamic range is connected to the number of events (Geffers *et al.* 2023). For the magnitudes in the interval considered in this paper, the dynamic range equals 3.2 or larger, greater than the critical value of 2.7 found in Geffers *et al.* (2023), therefore it does not affect the estimation of *b* in our case.

Another issue to be considered in the estimation of both the *b*-value and M_c is that events recorded in real time are revised after a certain period of time. Specifically, the Seismic Surveillance and Earthquake Monitoring in Italy updates the catalogue every 4 months (Margheriti *et al.* 2021). The revision improves the quality of the associated variables (time, location, magnitude and depth), possibly invalidating any early-stage analysis. Besides, events not recorded in the early stage may be added to the catalogue later on, possibly with a different temporal origin. The events' magnitude type is not always homogeneous in the entire catalogue (M_w for the stronger events, usually), or the type is modified in the revision (Margheriti *et al.* 2021). These issues highlight the difficulty of conducting a real time study of the statistical properties associated with an ongoing seismic sequence.

The results we illustrated and discussed for the Costa Marchigiana sequence are likely representative for any 'comparable' Italian earthquake sequence, to date recorded by the Italian seismic network. We expect that the problems we highlighted in this paper would have occurred also in other cases, for example, during the Central Italy Amatrice-Norcia sequence (2016–2017), started in Amatrice with an M_w 6.0 event on 2016 August 24, and not preceeded by a relevant seismic activity (Lombardi 2022). To overcome these issues in the real time *b*-value estimation, there are two possible options: try to use an estimation method less affected by the incompleteness, as the '*b*-positive' methodology presented by van der Elst (2021), or increase the number of events available in the seismic catalogue, by using for example an automatic detection of seismic events (Patanè *et al.* 2003).

5 CONCLUSION

The question of how reliable the real time estimation of the *b*-value (i.e. the parameter of the GR for earthquake magnitudes) is, represents one of the most actual and interesting debate in the scientific literature about seismic forecasting. We took the opportunity to address this question by considering the recent (and still ongoing) Costa Marchigiana (Italy) seismic sequence, that started with an M_L 5.7 event on November 9. This study represents an example of how to proceed and what we could expect when performing a real time estimation of the completeness magnitude M_c and the *b*-value. To understand the possible temporal variability of these parameters

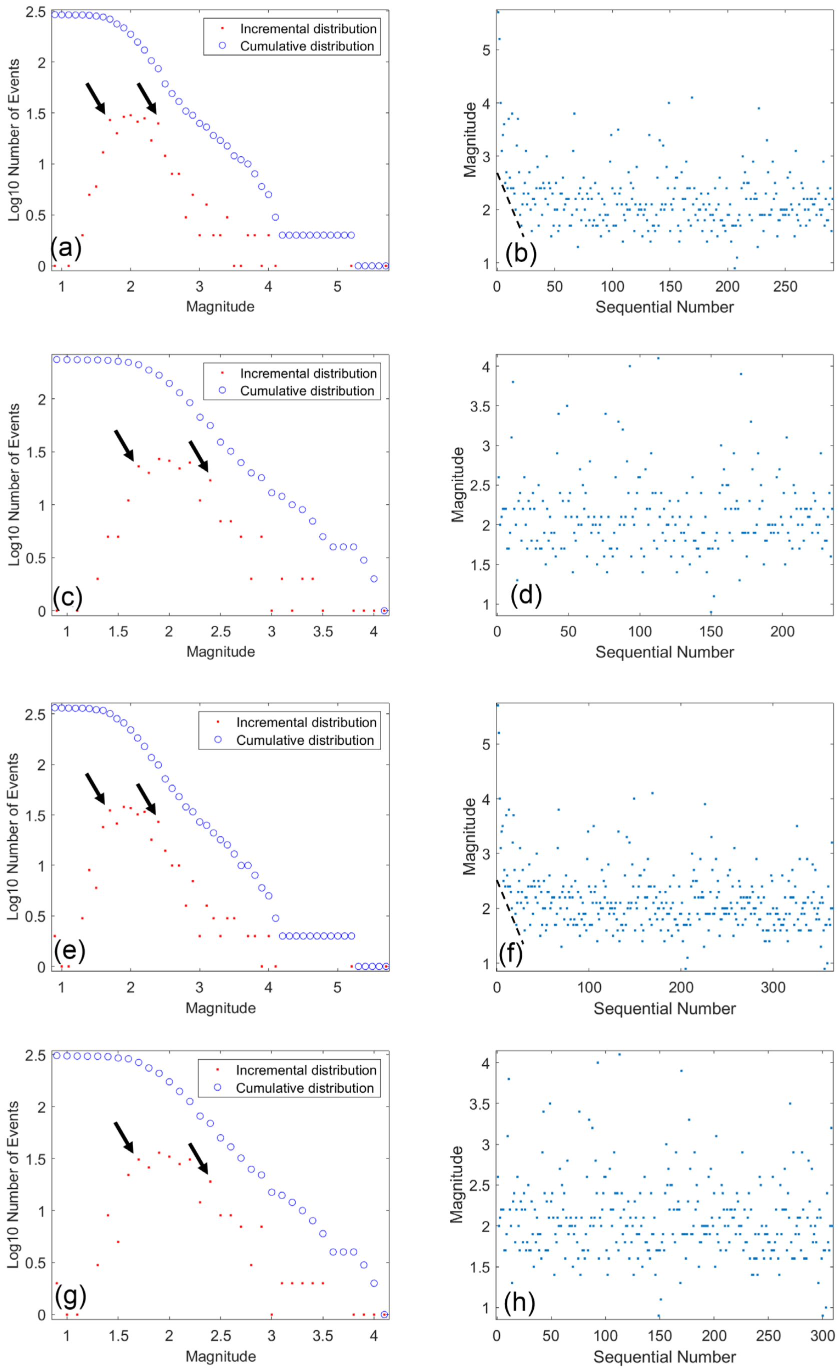

Figure 2. FMD (left-hand panels) and magnitude versus sequential number (right-hand panels) for the two examined periods, T1 and T2, with and without STAI; for the FMD, red dots represent the incremental distribution and blue circles represent the cumulative distribution. Panels (a) and (b) T1 with STAI; (c) and (d) T1 without STAI; (e) and (f) T2 with STAI; and (g) and (h) T2 without STAI. The two black arrows in panels (a), (c), (e) and (g) indicate the anomalous number of events in the bins 1.7 M_L and 2.4 M_L for the curves related to incremental distribution; and black dotted lines in panels (b) and (f) show the lack of small magnitude events due to STAI.

Table 2. Completeness magnitudes estimated for the catalogue within 4 d (T1) and 7 d (T2) since the first strong event of the Costa Marchigiana earthquake sequence.

Catalogues M_c estim. method	With STAI, T1	Without STAI T1	With STAI, T2	Without STAI T2
MAXC + 0.2	2.2	2.1	2.1	2.1
Lilliefors	1.9	1.9	1.9	1.9
90 per cent	1.8	1.8	1.8	1.8

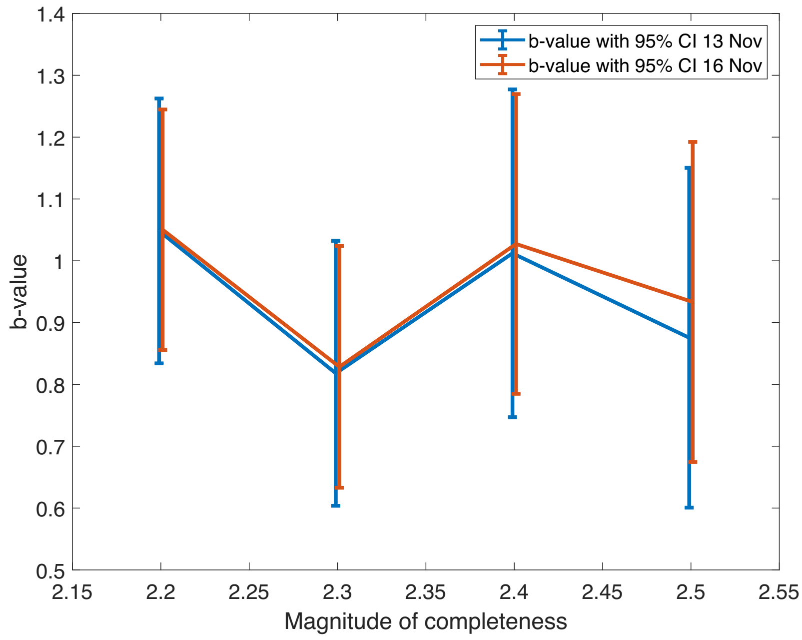

Figure 3. *b*-value and 95 per cent CI estimation for different magnitude of completeness thresholds, for the T1 catalogue (blue line) and T2 catalogue (red line).

in the early stage of the Costa Marchigiana sequence, we considered the data sets in the first 4 and 7 d after the first strong event recorded (T1 and T2). As expected, STAI was observed in the first hours after the initial event. Therefore, we performed our analysis by considering both the entire catalogue and the catalogue without the first hours.

We then estimated the completeness magnitudes of the catalogues within the two periods T1 and T2, with and without STAI, using several methods usually adopted in the literature. By looking at the FMDs, the obtained completeness values appear too low compared to the expected ones. Due to the sensitivity of the *b*-value to the completeness magnitude, we considered several completeness thresholds to estimate this parameter. As expected, also in this case, the estimated *b*-value is not well constrained, as the relative 95 per cent CIs are quite large for any M_c considered.

The results we obtained highlighted the presence of several problems likely related to the real time estimation of the completeness magnitude and the *b*-value. These are:

(i) the M_L magnitudes estimated in the first days seem to have an atypical concentration in some magnitude bins, that is, M_L 1.7 and 2.4; we expect this could be related to a shortcoming in recording the events, rather than to a 'real' effect, since the seismic sequences are not known to have some 'preferred' magnitude values;

(ii) the low number of events, the raw magnitude estimation at the early stage, and the 0.1 magnitude binning lead to unreliable estimation of the completeness magnitude, if we use the classical statistical approaches;

(iii) subjective judgments are required to assess the completeness magnitude and the STAI evaluation properly.

In principle, the real time estimation of the *b*-value can give important insights into the evolution of an ongoing sequence. Nevertheless, the possible lack of data, and the fact that data recorded at an early stage are raw, may strongly affect the estimation. In the case of the Costa Marchigiana sequence, analysed here, the *b*-values estimated during the sequence cannot help to understand the seismic evolution of the sequence itself, as the catalogue recorded in real time is affected by several problems. Improvement may come from considering different techniques to flank the classical ones, such as the b-positive, or the automatic detection of seismic events.

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DATA AVAILABILITY

The data underlying this paper were downloaded at http://terremoti.ingv.it/; Italian Seismological Instrumental and Parametric Database (ISIDe) Working Group 2007 data and code used in this paper are freely available at (https://github.com/IlariaSpassiani/Real-time_b_value_Costa_Marchigiana).

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SUPPORTING INFORMATION

Supplementary data are available at *GJI* online.

Figure S1. *b*-value and 95 per cent CI estimation for different magnitude of completeness thresholds; catalogue from November 9 to 16 with a radius of 40 km from the epicentre of the first event of the sequence and a removed STAI period of 4 hr.

Figure S2. *b*-value and 95 per cent CI estimation for different magnitude of completeness thresholds; catalogue from November 9 to 16 with a radius of 50 km from the epicentre of the first event of the sequence and a removed STAI period of 4 hr.

Figure S3. *b*-value and 95 per cent CI estimation for different magnitude of completeness thresholds; catalogue from November 9 to 16 with a radius of 30 km from the epicentre of the first event of the sequence and a removed STAI period of 2 hr.

Figure S4. *b*-value and 95 per cent CI estimation for different magnitude of completeness thresholds; catalogue from November 9 to 16 with a radius of 30 km from the epicentre of the first event of the sequence and a removed STAI period of 6 hr.

Figure S5. *b*-value and 95 per cent CI estimation for different magnitude of completeness thresholds; catalogue from November 9 to 16 with a radius of 30 km from the epicentre of the first event of the sequence and a removed STAI period of 8 hr.

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