The ocean wave dynamo: a source of magnetic field fluctuations

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Abstract
The alternating magnetic dynamo field of sea surface waves, a consequence of their Lorentz electric field, has been observed with a pair of simultaneously operated, closely spaced tri-axial magnetometers. Measurements from a magnetometer located in the centre of a tiny, uninhabited island served to compensate measurements from a near-shore magnetometer for magnetic pulsations of ionospheric origin, leaving the ocean wave dynamo field, effective close to shore only, as the dominant residual magnetic field. Amplitude and frequency of waves and swell were recorded with a vertical accelerometer (wave rider buoy) floating nearby on the sea surface. A spectral analysis was performed on ten nighttime intervals of three hours length each, and for every interval, the peak power of the surface waves (obtained from the wave rider) was compared with the peak power of the residual horizontal magnetic field (after the background field had been removed). The results suggest that the dual-sensor magnetic field observations yield, within the limits of statistical significance, a good quantitative description of the amplitude and frequency of sea surface waves and swell.

Key words ocean waves – surface wave dynamo – magnetic field oscillations

1. Introduction
ULF fluctuations of the geomagnetic field result predominantly from the combined effects of primary electric currents flowing in the upper atmosphere and magnetosphere of the earth, and secondary (induced) electric currents flowing in the Earth’s crust and upper mantle. Geomagnetic measurements performed in the sea, on the sea bottom, or at low altitude above the sea, often reveal additional small magnetic field oscillations which are associated with the sea surface wave dynamo. Sea water that moves with a velocity $v$ across the geomagnetic main field, $B_0$, generates an electric $v \times B_0$ or Lorentz field which provides for an electromagnetic dynamo force. The high electrical conductivity of sea water (typically 3 to 6 $\text{Sm}^{-1}$) permits the dynamo to drive an alternating electric current which is via Ampère’s law associated with an alternating magnetic field.

Fostered by improvements in the robustness and sensitivity of magnetic field sensors and by new developments in sensor concepts, the magnetic effect of ocean waves received considerable scientific attention during the 1960’s and 1970’s. A quantitative theoretical treatment of surface-wave associated magnetic variations observed above the sea surface was published by Crews and Futterman (1962). Their work was supplemented by Warburton and Caminiti (1964) who calculated the subsurface magnetic field of sea surface waves.
Weaver (1965) used a novel approach to calculate the magnetic field of sea surface waves above and below the surface of oceans with infinite and finite depth. Beal and Weaver (1970) followed with a calculation of the magnetic effects of internal waves in oceans of infinite and finite depth. Pedne (1975) worked out a comprehensive theory of the magnetic field perturbations associated with surface and internal waves in oceans of infinite and finite depth. Russian theoretical work on the subject was briefly summarized by Schel'nikov (1985).

In the wake of the theoretical work, several attempts were undertaken to measure the magnetic field of sea water waves. A number of successful measurements were made with total field sensors. They possess the advantage of suffering minimally or not at all from rotational noise, i.e., from apparent magnetic fluctuations produced by mechanical oscillations of the sensor when following the oscillatory movement of the water mass elements. Macure et al. (1964) measured simultaneously magnetic variations in deep water with a rubidium vapor magnetometer suspended from a spar buoy, and surface wave period and amplitude with an accelerometer floating on the surface. Fraser (1965) deployed a proton precession magnetometer and an echo sounder on the sea bottom at 40 m depth and showed that, under the assumption of a mean wavecrest length of at least 300 m, power spectra of water waves and magnetic field variations were consistent with theoretical predictions. Podney and Sager (1979a,b) exploited the high sensitivity of SQUID magnetometers to measure the vertical gradient of the east-west magnetic field variation above shallow water. They showed that the magnetic gradient fluctuated in accordance with surface and internal waves which they recorded with pressure sensors, and current meter and thermistor chains, respectively. Ochadlik (1989) analysed simultaneously recorded magnetic and oceanographic measurements from two different experiments. In one experiment, a helium vapor magnetometer was mounted on a research tower placed in shallow water, and a pressure transducer was installed on the sea bottom. In the other experiment, a dual-sensor helium vapor magnetometer was towed by a plane flying at low altitude over large-amplitude ocean swell. Ochadlik found his results to be consistent with Weaver's theory.

The theory of the dynamo effect of ocean waves had been comprehensively developed by the end of the 1970's for the case of a horizontally unbound, horizontally stratified and otherwise uniform ocean and freely propagating gravity waves. A number of measurements, employing either a total field magnetometer or a uni-directional gradiometer and conducted sufficiently far away from the coast, confirmed the effect of the ocean wave dynamo quantitatively. To our knowledge, no theoretical treatment or numerical modelling of a three-dimensional topography has been published yet. Although the above cited theoretical papers were not meant to cover a more complex topographic situation, we have attempted to apply them to our 3D setting. We report here for the first time, to our knowledge, about measurements of the ocean wave dynamo made with a pair of vector magnetometers operated on land, thereby avoiding problems related to the sensitivity of such devices to mechanical motions, and even in this case we find our measurements consistent with theoretical predictions.

2. Measurements

Between 13 and 28 September 1995, magnetic and oceanographic measurements were conducted simultaneously on and near the tiny, uninhabited island of Formica Grande (370 m long, 230 m wide, located at 43.6°N, 10.9°E, 14 km off the coast of Southern Tuscany). It is the largest of three rocky islands called Formiche di Grosseto which are aligned on a northwest-southeast striking rock bank rising rather sharply from an otherwise flat and gently sloping sea floor. The transition from the inclined island flanks to the almost level sea floor occurs about 300 m northeast of the island at 100 m depth and 450 m southwest at 115 m depth. An autonomously operating lighthouse constitutes the only significant civilisation landmark on Formica Grande. It is
powered by solar energy collected with an array of silicon panels and stored in truck batteries. The lighthouse building served as our field laboratory and housed our technical equipment, including a small Diesel generator. Except for the generator, which was operated during daytime hours only, no known source of anthropogenic electromagnetic noise exists on the island.

Two tri-axial fluxgate magnetometers were operated on the island at various locations. A continuously recording floating vertical accelerometer buoy (‘wave rider’) was moored some 600 m southwest of Formica Grande. The magnetometer and wave rider measurements were recorded digitally with 48.25 Hz and 2.56 Hz sampling rate, respectively. For this paper we selected only data which were collected at night when the generator was shut off and all instruments were powered by batteries. We further restrict our study to the first five nights when one of the sensors was located in the centre of the island (site M2) and the other at its southeastern tip (site M1), close to shore and 160 m away from M2 (fig. 1). The magnetic fluctuations recorded at M2 which were not affected by the ocean wave dynamo (as we will see further below) were used to compensate the recordings from M1 for magnetic variations of ionospheric origin. The compensation is necessary because the amplitude of the magnetic fluctuations induced by swell was almost always of a lesser order of magnitude than the average amplitude of the magnetic fluctuations of ionospheric origin at the same frequency. A compensation is possible because the horizontal scale length of the ionospheric fluctuations is several orders of magnitude larger than that of the swell, with the consequence that the spatial gradient of the ionospheric fluctuations is much smaller than that of the swell dynamo magnetic field.

Figure 2 shows a ten-minute interval of the northward and eastward components of the magnetic fluctuations observed at the nearshore site, M1, the magnetic field difference between M1 and M2, and the same difference enlarged by a factor of ten. The curves are offset along the ordinate for reasons of graphical distinction. Obviously, the magnetic field

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**Fig. 1.** Formica Grande, the northwestmost island of the Formiche di Grosseto group, with lighthouse and magnetometer sites.

**Fig. 2.** Ten-minute sample of northward (upper panel) and eastward (lower panel) magnetic field fluctuations observed at site M1; north and east magnetic field differences between M1 and M2, with same amplitude scale as data from M1, and with ten times enlarged scale.
recorded at M2, in the island centre, does not completely compensate the measurements from M1. Besides low frequency residuals, more or less regular fluctuations of about 7 s period remain as the most spectacular residuals. They are associated with the dynamo field of swell, as we will see below.

3. Spectral analysis

We performed a spectral analysis on ten three-hour time intervals with simultaneous magnetometer and wave rider observations, distributed over four different nights between September 14 and 18, 1995. The selection was dictated by the level of swell or wave activity, which needed to be sufficiently high to generate a measurable dynamo field. Prior to the spectral analysis, the magnetometer and the wave rider data were low-pass filtered (with 0.4 and 0.5 Hz cutoff, respectively) and decimated to sampling rates of 1 Hz and 1.28 Hz, respectively. Power density spectra of magnetic variations in the north, east and vertical components were estimated in two different ways: a) using the Maximum Entropy (ME) method, largely in the form described by Barrodale and Erickson (1980); b) with the Welch periodogram method (Welch, 1967). ME application details are described in a different paper on the same subject (Watermann and Magunia, 1997). Smoothed periodogram estimates were obtained by averaging 167 FFT spectra from 128-point time segments with 50% overlap. Each individual time segment was centred and tapered with a 4-point Blackman-Harris window. The number of equivalent degrees of freedom of the spectrum reaches 99% of 334 (Harris, 1978), which translates into a 99% confidence interval ranging from −1.6 dB to +1.9 dB around the estimated power at each point of the spectrum. For the quantitative analysis, only the Welch periodogram estimates were used; the ME estimates served solely to confirm via an independent method the existence of significant spectral peaks. In fact, it turned out that for all ten events the ME and periodogram estimates were identical within the confidence limits.

Figure 3 shows ME auto spectra of the north, east, and vertical components of the magnetic field differences between sites M1 and M2 (computed in the time domain). To be able to assess the significance of the magnetic field residuals, we have included the upper limit of the magnetometer self noise, inferred from the technical specifications provided by the manufacturer. Also plotted are the measured (uncorrected) and the effective surface wave spectra. The latter was obtained by multiplying the measured wave height spectrum with the inverse wave rider response function.

**Formica Grande di Grosseto, 1995**

![Fig. 3. ME sample spectra of northward (dash-dotted), eastward (dashed) and vertical (solid) magnetic field differences between sites M1 and M2; uncorrected and effective corrected sea surface wave spectra for the same time interval (dotted and heavy solid lines, respectively); magnetometer system noise (heavily dotted). 0 dB is equivalent to 1 nT/Hz (magnetic field oscillations) and 1 m²/Hz (surface waves), respectively. Analysed time interval 95/09/14, 04:12-07:12 UTC.](image-url)
and a wave efficiency factor $\omega_0/\omega$ (here, $\omega_0$ denotes a reference frequency, which we chose to be $2\pi \cdot 0.14$ Hz, and $\omega$ the wave angular frequency). This factor accounts for the frequency-dependent efficiency of the wave dynamo: for a bottom depth $D > \lambda/3$ ($\lambda$ means wavelength) the surface wave dispersion relation can be approximated by $\omega^2 = gk$ (where $k$ is the wave number), so that the magnetic dynamo field scales with the factor $\alpha^2$ (Podney, 1975).

The spectra of the north and east magnetic field differences straddle the system noise over virtually the entire frequency band, except around 0.14 Hz where the spectral power exceeds the noise by some 15 dB (north component) and 20 dB (east component). This spectral peak, which coincides with the peak in the wave height spectrum, is statistically significant at a 99.9% confidence level (a number obtained from the corresponding periodogram estimates). The swell dynamo field is measurable only at the shore site and negligible in the centre of the island. It is therefore not cancelled out in the differencing process. The spectrum of the vertical component, too, shows a peak around 0.14 Hz but also much residual power at lower frequencies. We suggest that the latter represents man-made magnetic noise from an underwater cable running at 10° bearing from north, i.e. almost perpendicular to the line connecting M1 and M2, and passing the island at some 1.7 km distance in 120 m water depth.

An electric current in the cable would produce a magnetic field perturbation the vertical component of which is 46 dB more powerful than the horizontal component, with the consequence that the magnetic signature of cable current fluctuations might be observed in the vertical component but not in the horizontal.

The magnetic north and east components reach numbers of 45 pT/Hz$^{1/2}$ and 63 pT/Hz$^{1/2}$, respectively, at 0.14 Hz. The peak value of the vertical magnetic field fluctuations cannot be determined with equal accuracy because below 0.2 Hz its power is substantially higher than the system noise, and it is difficult to distinguish between what can be attributed to the wave dynamo and what is simply incoherent background noise. We suggest that the peak reaches between 55 and 60 pT/Hz$^{1/2}$. The wave height spectrum yields a spectral peak amplitude of 1.4 m/Hz$^{1/2}$ at 0.14 Hz.

4. Discussion

Following Podney (1975), the spectral amplitude (not the spectral power) of the magnetic field fluctuations, $B_w(\omega)$, is connected to the sea surface wave amplitude, $W(\omega)$, via the electromagnetic dynamo process established by the water mass oscillating in the geomagnetic field. For a magnetic sensor placed at a height $h$ above the sea surface, Podney finds the magnitude of the magnetic dynamo field to be

$$B_w(\omega) = \frac{1}{2} \mu_0 \sigma (g/k)^{1/2} \exp (-kh) B_p W(\omega).$$ (4.1)

Here, $\sigma$ denotes the electrical conductivity of the sea water which we measured with a submerged conductivity meter and obtained 4.8 Sm$^{-1}$. $B_p$ means the projection of the geomagnetic main field on the plane in which the water mass elements oscillate. During the four nights from which we selected the ten analysis intervals, we encountered southwesterly swell with varying amplitude. We suggest that the swell originated most likely in the Straits of Bonifacio. We refer the reader to the paper by Watermann and Magnunia (1997) for details of the method of inferring the plane of the actual water mass motion (and consequently the geomagnetic field projection) from the cross spectra of the magnetic field vector oscillations. That paper explains how the relative amplitudes of the north, east and vertical components of the magnetic field oscillation and the phase shifts between them are employed to determine uniquely the plane in which the magnetic field vector and the water particles oscillate. For our present discussion it is sufficient to note that we obtained a magnetic field projection $B_p \approx 22$ $\mu$T for the ten events analysed.

The approximative dispersion relation $\omega^2 = gk$ yields $(g/k)^{1/2} = g/\omega$ so that eq. (4.1) can be
rewritten to read

\[
\frac{\omega B_w(\omega)}{W(\omega)} = \frac{1}{2} \mu_0 \sigma_g \exp \left( -\frac{\omega^2}{g} h \right) B_p. \tag{4.2}
\]

It is worth calculating at which hypothetical height \( h \) a magnetic sensor should have been placed in our experiment if we assumed that our magnetic field measurements were made above the surface of an open sea. Given the fact that we measured all variables entering eq. (4.2) except for \( h \), we can determine \( h \) from the measurements. The result is shown in fig. 4 where \( h \) is plotted against the peak swell amplitude, \( W(\omega) \). In all ten events, \( B_w(\omega) \) was computed from the magnetometer data, and \( \omega \) and \( W(\omega) \) from the wave rider data. We notice that the numbers for \( h \) do not differ substantially between the ten different events, despite the fact that the wave amplitude spanned half a decade. The mean value of \( h \) was found to be \( \bar{h} = 37.5 \) m and its standard deviation \( 5.2 \) m.

We need to recall that our experimental setup is different from the setup assumed in Podney’s theory. We did not measure the magnetic field at a height \( h \) above the sea surface but at a distance \( d \) from the shore, \( i.e. \) from the nearest point up to which the ocean waves can approach M1. Incidentally, \( d \) was some 40\, m in our experiment, \( i.e. \) about the size of \( \bar{h} \). Our results thus provide us with a scaling, \( d = h \), between a hypothetical sensor height \( h \), assumed in Podney’s theory, and the actual horizontal sensor distance \( d \) from the shore, valid for our experimental setup. We conclude that Podney’s theory (and eqs. (4.1) and (4.2) above, for that matter) remain approximately correct without further modification if only the sensor height \( h \) is substituted by the sensor distance \( d \) from the shore, \( i.e. \) from the nearest possible wave crest.

If this scaling law also applies to M2, in the centre of the island and 110\, m from the shore, M2 should be exposed to a dynamo field which is smaller by a factor of

\[
\exp(-k(d + 110\, \text{m})) / \exp(-kd) = \exp(-k \cdot 110\, \text{m}) < 10^{-4} \tag{4.3}
\]

compared to the field observed at M1. The dynamo effect at M2 is therefore negligible.

We must emphasize that the scaling \( d = h \) is only valid for our particular experimental setup. In a different place, or with the magnetometers located at different sites, the scaling may be different. It is not obvious at all that the scaling is linear and independent of the wavelength. One might be tempted to state a slight increase in the hypothetical sensor height with increasing wave height. Unfortunately, our data sample is too small to draw a conclusion. Statistically, the two isolated, large-amplitude events stem with about 15\% probability from the same statistical population as the eight smaller-amplitude events. The uncertainty as to a general applicability of our scaling law is no problem in principle, though. Once the scaling between \( h \) and \( d \) has been determined experimentally for the specific magnetometer setup used in an experiment, one can infer the ocean wave height from the magnetic

Fig. 4. Hypothetical magnetometer height plotted against the swell spectral peak amplitude. Each of the ten points represents one event representing a time series segment of three hours length. The ten events were selected from four different nights.
field observations without the need for direct measurements of the wave height. The method is constrained only by the condition that the wave propagation direction is known (because it is needed to determine $B_p$).

5. Conclusions

We have demonstrated that the magnetic dynamo field of sea surface waves can be measured rather accurately with a dual-magnetometer array in which one sensor, located inland, compensates the measurements from a second sensor, placed close to the sea shore, for large-scale magnetic field variations. Further, one can infer the approximate height of the sea surface waves solely from the associated magnetic field residuals using Podney's (1975) theory. It is only required that the scaling between hypothetical sensor height and actual sensor distance from shore has been determined once for each particular experimental setup, and that the wave propagation direction has been recorded.

It is not obvious from the cited theory that the vertical distance between wave crests and a magnetic sensor above the sea can be replaced by a properly scaled quasi-horizontal distance between shore-based magnetometer and nearest wave crest. Podney and his predecessors developed their theory for the geometrically simple case of an unbound ocean with a plane, strictly horizontal bottom and perfectly two-dimensional, freely propagating gravity waves. To our knowledge, the more complex topography of an island has not yet been treated analytically. Examples of certain topographic structures have, however, been studied experimentally by Miles and Dosso (1979, 1980) who employed reduced-size analogue models of various topographic features and applied consistent scaling of the physical parameters. They performed measurements of wave-induced magnetic fields on laboratory models of various coastal shapes, shelf profiles, dykes and sea mounts. They found that in shallow water, and close to topographic ocean bottom irregularities, the magnetic field of surface waves can deviate substantially from the corresponding field over a uniform ocean. A numerical simulation of wave propagation and magnetic field excitation in specific topographic settings may provide an alternative approach to dealing with complex topographies.

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