

1     **BISTROP: Bayesian Inversion of Spectral levels Ratios and P wave**  
2                   **polarities for focal mechanism determination**

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29 **Abstract**

30 The analysis of earthquakes focal mechanism provides information about  
31 the stress regime, fault geometry and deformation processes acting in a  
32 given region. Generally, the techniques aimed at determining focal  
33 mechanism are designed to work in specific magnitude range operating both  
34 in the time and frequency domain, and using different data (e.g., P-  
35 polarities, S-wave polarization, S/P amplitude ratios, etc.). In this paper we  
36 present a new method named BISTROP that can be applied to both small  
37 and moderate-to-large magnitude events. BISTROP uses a Bayesian  
38 approach to jointly invert the long-period spectral level P/S ratios and the P-  
39 polarities to infer the fault plane solutions. We apply this method to analyze  
40 synthetic data as well as those generated by real earthquakes. We find that  
41 the obtained solutions for moderate earthquakes are comparable with those  
42 obtained by using moment tensor inversion, and they are more constrained  
43 with respect to the solutions obtained using only P-polarity data for small  
44 earthquakes.

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55 **Introduction**

56 A thorough understanding of earthquake focal mechanisms gives  
57 information about the stress regime that originates the occurrence of the  
58 earthquakes and their relation to the regional tectonics. The currently  
59 available techniques for the focal mechanism estimation are designed to  
60 work on specific magnitude ranges. In particular, for moderate-to-large  
61 earthquakes waveforms or spectral data inversions is performed to yield  
62 stable solutions (Dreger and Helmberger, 1993; Dreger, 2003; Delouis,  
63 2014; Fojtíková and Zahradník, 2014). The same approach cannot be easily  
64 applied to small earthquakes due to the higher noise level and to the  
65 generally inadequate description of the medium response at the wavelengths  
66 at which micro-earthquakes radiate. However, source characterization of  
67 small earthquakes can be a useful tool to obtain information on the  
68 seismogenic structures, the stress field and its variations in time and space.  
69 This is of particular relevance in those areas where seismicity rate is low  
70 due to long inter-seismic periods or in areas where seismicity is induced by  
71 man-made activities. In addition, in many seismic regions worldwide  
72 current seismic networks are densely spaced and equipped with high  
73 dynamic sensors and data loggers. This has significantly decreased the  
74 minimum detectable magnitude and has greatly increased the number of  
75 recordings for small earthquakes. As a consequence, it is necessary that the  
76 techniques can be used for a massive data analysis, reaching the limit of the  
77 real-time analysis.

78 The determination of the focal mechanism for small earthquakes is  
79 hampered by several factors. For example, moment tensor inversion, that is  
80 the more stable and robust technique for moderate-to-large earthquakes  
81 analysis, may not be suitable for small earthquakes, since it requires a  
82 precision on source location, and details about the propagation medium  
83 (Green's function) at a scale that is rarely achievable. Moreover, the number  
84 of stations that record small events reduces with distance from the source  
85 making inefficient, for example, the application of methods based on the use  
86 of the P-wave polarities. Also the techniques that use P- and S-waves  
87 amplitudes cannot be easily used because, being the recorded signal richer  
88 in high-frequencies, it is contaminated by wave diffraction at small-scale  
89 crustal heterogeneities. In a recent study, Fojtíková and Zahradník (2014)  
90 have proposed a method called CSPA (Cyclic scanning of the Polarity  
91 Solutions), aimed at reducing the uncertainty of the solutions obtained by  
92 methods based on the inversion of the P-wave polarities. For a given  
93 distribution of observed P-wave polarities, the solutions obtained by means  
94 of the inversion code inversion FOCMEC (Snoke, 2003), one by one are  
95 used as input to the inversion of the waveform at a single station. The best  
96 model will be the one characterized by the minimum variance. Since this  
97 method uses a single station the solution might strictly depend on the quality  
98 of the input waveform.

99 Several studies have proposed the use of the S/P amplitudes ratio including  
100 Kisslinger (1980), Kisslinger et al. (1981), Julian and Foulger (1996), Rau et  
101 al. (1996) and Shen et al. (1997) and more recently Hardebeck and Shearer

102 (2003). These latter authors have used the S/P amplitudes ratio in addition to  
103 polarities, to better constrain the focal mechanism of earthquakes with  
104 magnitude lower than 4. The S/P ratio enables to neglect the magnitude and  
105 the geometrical attenuation, and to consider only site effects and the  
106 difference between the P-waves and S-waves anelastic attenuation.

107 A stable and robust alternative is the joint use of P-polarities and S-wave  
108 polarization, which are less affected by propagation effects compared to the  
109 amplitudes. In this respect, Zollo et al. (1991) have proposed a non-linear  
110 inversion method for studying the earthquake mechanism by computing the  
111 posterior probability of parameters (strike, dip and slip fault angles) for the  
112 given observational data sets by using a Bayesian approach. The method  
113 allows to locate the maximum likelihood solutions, and to get a reliable  
114 estimate of the parameter correlation and resolution. Subsequently, several  
115 authors have applied the same or a similar technique to a variety of  
116 earthquake studies (e.g., Iannaccone et al., 1994; Walsh et al., 2009).  
117 However, the method relies on the capability to precisely measure the S-  
118 wave polarization, which can be a difficult task in case of noisy records or  
119 in presence of dominant multi-path and anisotropy effects. Recently,  
120 Imanishi et al. (2016), have proposed a technique for focal mechanism  
121 determination of nonvolcanic tremor using S-wave polarization after  
122 removing anisotropy effects.

123 Zahradnik et al. (2001) have proposed an inversion method to determine the  
124 focal mechanism of small earthquakes (M 3.4–3.7), called ASPO  
125 (Amplitude Spectra and Polarities) that operates in the frequency domain.

126 The method is based on the comparison between the amplitude spectrum of  
127 the whole recorded signal and the one calculated from synthetic  
128 seismograms for different sets of strike, dip and rake, and for different  
129 values of the scalar seismic moment and hypocenter. The polarities are used  
130 in the final step to accept or reject the obtained solution. As for most of  
131 other methods, the performance of this technique depends on the level of  
132 knowledge about the propagation medium and the capability to simulate the  
133 high frequency spectral content of real earthquakes.

134 In the present study we propose a method for estimating the focal  
135 mechanism of earthquakes modeled as a double couple (DC) point source  
136 shear-dislocation able to work with both moderate-to-large and small  
137 events. The technique, named BITROP, which stands for Bayesian  
138 Inversion of Spectral levels Ratios and P-wave polarities, jointly inverts  
139 the ratio between P- and S-wave long-period spectral levels and the P-wave  
140 polarities in a Bayesian framework. Concerning the input data, spectral  
141 levels are easier to be measured compared to other observables (e.g., S-  
142 wave polarization or P- and S-waves amplitudes) and, as a consequence,  
143 they are generally available for a broad range of magnitude. Formally, the  
144 method is based on the relationship between the theoretical radiation pattern  
145 and the long-period spectral level calculated from the first P- or S-waves at  
146 different stations. Unlike inversion methods that use only the polarity of the  
147 first arrivals, whose identification is not always easy because of the  
148 background noise, BISTROP does not impose the need to identify the first  
149 arrival with the same precision; it will be sufficient that the portion of the

150 seismogram used for the spectral inversion contains the selected first arrival  
151 and does not include other seismic phases. However this does not exclude  
152 the need of few P-polarities to better constrain the mechanism. Moreover,  
153 working with the ratio between low-frequency part of the P- and S-wave  
154 spectra allows to reduce, in first approximation, if they are not known, the  
155 effects of the geological soil condition (site effect) and of both geometric  
156 and anelastic attenuation. Note that, the probabilistic formulation of the  
157 inverse problem allows obtaining a thorough description of the inferred  
158 parameters (i.e., uncertainty and correlation).

159 As example of application to a moderate event, we analyzed the 23  
160 December 2013,  $M_W$  5.0, Matese, Italy, earthquake. This event has been  
161 recorded by a high number of stations of different seismic networks with a  
162 good azimuthal coverage and its focal mechanism has been accurately  
163 obtained from moment tensor inversion.

164 Regarding the small magnitude earthquakes, optimal conditions in terms of  
165 station distribution and signal-to-noise ratio are less frequently verified. As  
166 a consequence, we first set up a synthetic test on a simulated  $M_L$  2.9  
167 earthquake recorded at a network reproducing the geometry of the Irpinia  
168 Seismic Network (ISNet) (Weber et al., 2007). Next we applied it to two  
169 different earthquakes occurred in Southern Italy and characterized by  
170 different focal mechanisms and recorded at seismic networks with different  
171 azimuthal coverage: the 27 May 2009,  $M_L$  2.9, Laviano earthquake and the  
172 27 September 2012,  $M_L$  4.1, Benevento earthquake.

173

174 **Method**

175 Seismic moment can be measured from the far-field P-wave displacement  
176 spectra using the relation (Aki and Richards, 1980):

177

$$M_0 = \frac{4\pi\rho_s^{1/2}\rho_r^{1/2}\alpha_s^{5/2}\alpha_r^{1/2}R'\Omega_0^P}{F\langle|\mathcal{R}_{\theta\varphi}^P|\rangle} \quad (1)$$

178

179 where  $\rho_s$  and  $\rho_r$  are the medium densities at the source and at the receiver,  
180 respectively;  $\alpha_s$  and  $\alpha_r$  are the P-wave velocities at the source and at the  
181 receiver, respectively;  $F$  is the free-surface effect;  $\Omega_0^P$  is the long-period  
182 spectral level of the P-wave and  $\langle\mathcal{R}_{\theta\varphi}^P\rangle$  is the average P-wave radiation  
183 pattern;  $R'$  is the geometrical spreading estimated for a linear variation of  
184 velocity with depth (Ben-Menahem and Singh, 1981):

$$R' = \sqrt{\frac{\rho_r\alpha_r}{\rho_s\alpha_s}} \frac{R}{\sin i_h} \quad (2)$$

185 where  $R$  is the epicentral distance and  $i_h$  is the take-off angle.

186 Equivalently, for the S-wave it is possible to write a similar expression,

187

$$M_0 = \frac{4\pi\rho_s^{1/2}\rho_r^{1/2}\beta_s^{5/2}\beta_r^{1/2}R'\Omega_0^S}{F\langle|\mathcal{R}_{\theta\varphi}^S|\rangle} \quad (3)$$

188

189 where  $\beta$  is the S-wave velocity (at the source  $\beta_s$  and at the receiver  $\beta_r$ ,  
190 respectively),  $\Omega_0^S$  is the long-period spectral level of the S-wave and  $\langle\mathcal{R}_{\theta\varphi}^S\rangle$



191 is the S-wave average radiation pattern. The formulation for the geometrical  
192 spreading  $R'$  is as reported in eq. (2) but using the  $\beta$  values.

193 The long-period spectral level can be measured from the observed  
194 displacement spectrum assuming a given source and attenuation model.  
195 According to Boatwright (1980), the displacement source spectrum can be  
196 described as:

197

$$S(f) = \frac{\Omega_o}{\left[1 + \left(\frac{f}{f_c}\right)^\zeta\right]^{1/n}} \quad (4)$$

198 where  $f$  is the frequency,  $f_c$  the corner frequency,  $\zeta$  the high-frequency fall  
199 off rate, and  $n$  is a constant. As for the attenuation, we assumed the  
200 exponential decay filter:

201

$$Q(f) = e^{-\pi f \frac{T}{Q_f}} \quad (5)$$

202

203 where  $T$  is the travel-time,  $Q_f$  has a general formulation given by  $Q_f = Q_o f^n$   
204 where  $Q_o$  is the quality factor evaluated at the reference frequency of 1Hz  
205 (Morozov, 2008). In the present study we assumed  $n=0$ , that is, a frequency  
206 independent quality factor.

207 Equations (1) and (3) assume an average radiation pattern coefficient 0.52  
208 and 0.63 for P- and S-wave, respectively (Aki and Richards, 1980).

209 Actually, for a double-couple source model, the radiation pattern depends  
210 on the fault orientation, through the angles strike  $\phi$  and dip  $\delta$ , on the rake  
211 angle  $\lambda$ , and on the take-off angle  $i_h$  (Aki and Richards, 1980). Thus, for a

212 given earthquake, if the focal mechanism and the source-station relative  
 213 position are known, the functions  $\mathcal{R}^P = \mathcal{R}^P(\phi, \delta, \lambda, i_h, \phi_R)$  and  $\mathcal{R}^S =$   
 214  $\mathcal{R}^S(\phi, \delta, \lambda, i_h, \phi_R)$ , being  $\phi_R$  the source-receiver azimuth, can be  
 215 substituted in equations (1) and (3). Using the new formulation for eq.(1)  
 216 and eq.(3), , it results that:

217

$$\frac{\mathcal{R}^P(\phi, \delta, \lambda, i_h, \phi_R)}{\mathcal{R}^S(\phi, \delta, \lambda, i_h, \phi_R)} = \left( \frac{\alpha_s^2 \alpha_r}{\beta_s^2 \beta_r} \right) \frac{\Omega_0^P}{\Omega_0^S} \quad (6)$$

218

219 For a given earthquake that has been located by using  $N$  recording stations  
 220 and a specific velocity model the variables  $i_h$  and  $\phi_R$ , as well as the  $\alpha$  and  $\beta$   
 221 are known quantities. Thus, at least in principle, if it is possible to  
 222 accurately measure the long-period spectral levels, the ratio  $\Omega_0^P/\Omega_0^S$  plotted  
 223 as a function of the source-receiver azimuth would match the ratio between  
 224 the radiation pattern coefficients.

225 The aim of the present study is to jointly invert the observed  $\Omega_0^P/\Omega_0^S$  ratios  
 226 along with the observed P-wave polarities, to infer the focal mechanism  
 227 parameters  $(\phi, \delta, \lambda)$ , by using a Bayesian framework. The approach is  
 228 similar to that proposed by Zollo and Bernard (1991) but uses different  
 229 proxies. Specifically, given the vector of model parameters  $\mathbf{m}$  whose  
 230 components are the angles  $(\phi, \delta, \lambda)$ , and a vector of observed data  $\mathbf{d}$ , we aim  
 231 at inferring an *a-posteriori* probability density function  $q(\mathbf{m}|\mathbf{d})$  given by:

$$q(\mathbf{m}|\mathbf{d}) = \frac{f(\mathbf{d}|\mathbf{m})p(\mathbf{m})}{\int_M f(\mathbf{d}|\mathbf{m}')p(\mathbf{m}') d\mathbf{m}'} \quad (7)$$

232 where  $f(\mathbf{d}|\mathbf{m})$  is the conditional probability function that represents the  
 233 probability density function (pdf) associated with possible data realizations  
 234 for a fixed parameter vector  $\mathbf{m}$  in the model parameter space  $\mathbf{M}$ , and  $p(\mathbf{m})$  is  
 235 the *a-priori* pdf. In the present study the data vector  $\mathbf{d}$  is composed by  $N_L$   
 236 measurements of  $\Omega_0^P/\Omega_0^S$  ratios ( $\mathbf{d}^L$ ) and  $N_P$  measurements of P polarities  
 237 ( $\mathbf{d}^P$ ). As a consequence, given that P-wave polarities and spectral ratios are  
 238 independent data sets, the conditional probability function may be written as  
 239  $f(\mathbf{d}|\mathbf{m})=f(\mathbf{d}^L|\mathbf{m})f(\mathbf{d}^P|\mathbf{m})$ .

240 Assuming that the  $N_L$   $\Omega_0^P/\Omega_0^S$  observations are independent and normally  
 241 distributed, the  $f(\mathbf{d}^L|\mathbf{m})$  can be formulated as

$$\begin{aligned}
 f(\mathbf{d}^L|\mathbf{m}) &= f(d_1, \dots, d_{N_L}|\mathbf{m}) \\
 &= \prod_{i=1}^{N_L} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{\{d_i - [G(\mathbf{m})]_i\}^2}{2\sigma_i^2}\right) \quad (8)
 \end{aligned}$$

242 where  $G(\mathbf{m})$  represents a functional relationship between models and data  
 243 and corresponds to equation 6. Assuming homoscedasticity for the analyzed  
 244 variables (that is the observables have the same finite variance) eq.(8) is  
 245 written as:

246

$$\begin{aligned}
 f(\mathbf{d}^L|\mathbf{m}) &= \frac{1}{(\sqrt{2\pi}\sigma)^{N_L}} \exp\left(-\frac{\sum_{i=1}^{N_L}\{d_i - [G(\mathbf{m})]_i\}^2}{2\sigma^2}\right) \quad (9)
 \end{aligned}$$

247 The  $\sigma$  value is estimated from the uncertainty on spectral measure and the  
 248 term in the summation for the  $i$ -th observation is

249

$$w_i \left[ \left( \frac{\mathcal{R}^P}{\mathcal{R}^S} \right)_i^{Obs} - \left( \frac{\mathcal{R}^P}{\mathcal{R}^S} \right)_i^{Theo} \right]^2 \quad (10)$$

250

251 The coefficient  $w_i$  is a weighting factor that depends on the take-off angle  
 252 value. In practice, one can decide to give less weight to data observed at  
 253 stations for which the  $i_h$  values are close to  $90^\circ$ .

254 As regard  $f(\mathbf{d}^P | \mathbf{m})$  we use the formulation proposed by Brillinger et al.  
 255 (1980):

$$f(\mathbf{d}^P | \mathbf{m}) = \prod_{i=1}^{N_P} \frac{1}{2} [1 + \psi(\mathcal{R}_i^P, \gamma_i, \rho_0) Y_i \text{sign}(\mathcal{R}_i^P)] \quad (11)$$

256 where  $\psi(\mathcal{R}_i^P, \gamma_i, \rho_0) = (1 - 2\gamma_i) \text{erf}(|\rho_0 \mathcal{R}_i^P(\mathbf{m})|)$  and  $\text{erf}(\ast)$  is the error  
 257 function.

258 The quantity in square brackets in eq.(11) gives the probability that the  
 259 observed  $i$ th polarity  $Y_i$  is coherent with the one computed from the model  
 260  $\mathbf{m}$ .  $\mathcal{R}_i^P$  is the theoretical  $\mathbf{P}$  amplitude at station  $i$  for a given fault and  
 261  $\text{sign}(\mathcal{R}_i^P)$  is its polarity. The parameters  $\rho_0$  and  $\gamma$  control the shape of the pdf  
 262 and range between  $[0, \infty)$  and  $[0, 1/2]$ , respectively. They take into account  
 263 for the errors in ray tracing (and medium velocity) and the uncertainty on  
 264 polarity reading. For imprecise polarity data  $\gamma$  is near 0.5 or  $\rho_0$  near 0;  
 265 precise data correspond to small  $\gamma$  and large  $\rho_0$  ( $\rho_0 > 20$ ). The formulation of  
 266 the pdf proposed by Brillinger (1980) has the property that the larger the  
 267 magnitude of  $\mathcal{R}_i^P$ , the higher is the probability of measuring the correct  
 268 polarity.

269 The pdf  $p(\mathbf{m})$  represents the *a-priori* probability associated with a given  
 270 model parameter in the model parameter space  $M$ . The selection of a proper  
 271  $p(\mathbf{m})$  is crucial and must enclose all the available information about the  
 272 models before gathering the data. In the method implemented in the present  
 273 study, it is possible to select both a *null informative* probability, which  
 274 corresponds to a uniform pdf on  $M$  or a tridimensional Gaussian pdf  
 275 centered on a given point of  $M$  with a prescribed degree of correlation  
 276 among the parameters. The latter formulation could be used for example to  
 277 restrict the explored mode space when, according to the stress-regime of a  
 278 given area, some focal mechanisms are more likely than others.

279 For each analyzed event, the best model  $\mathbf{m}_b$  corresponds to the maximum a  
 280 posteriori (MAP) model of the *a-posteriori* pdf (Aster et al., 2013). A mere  
 281 graphical representation of the pdf as a function of the model parameters  
 282 can provide global information about the solution quality, highlighting for  
 283 example the presence of secondary maxima and the possible correlation  
 284 among the parameters. However, the availability of the *a-posteriori* pdf  
 285 allows to quantitatively evaluate the uncertainty level associated to the MAP  
 286 model. Since a Bayesian approach is used, the 68%, 90% and 95%  
 287 "Bayesian confidence intervals" or simply "confidence intervals" are  
 288 computed (Carlin and Louis, 2009). By definition a  $100 \times (1 - \alpha)\%$  credible set  
 289 for  $\mathbf{m}$  is a subset  $C$  of the space model parameters such that:

$$1 - \alpha \leq \int_C q(\mathbf{m}|\mathbf{d}) d\mathbf{m}$$

290 where  $\alpha$  is the significance level. In addition, the posterior marginal PDFs  
291 on each component of  $\mathbf{m}$  are computed by integrating the  $q(\mathbf{m}/\mathbf{d})$  on the  
292 remaining nuisance components.

293

#### 294 **Data analysis**

295 In the present study we analyzed recordings of velocimeters and/or  
296 accelerometers of three different networks that are ISNet, Rete  
297 Accelerometrica Nazionale (RAN) managed by the Italian Civil Protection  
298 Department and Italian Seismic Network managed by Istituto Nazionale di  
299 Geofisica e Vulcanologia (INGV). In order to compute the displacement  
300 spectra, we have manually windowed all the available waveforms by  
301 considering a different duration for P- and S-wave. The selected P window  
302 includes at least 0.5 s of pre-P signal and the whole P-wave train. For the S  
303 window, the pre-S signal is shorter to avoid possible overlap with the P-  
304 wave train, and contains the whole S-wave train. Figure 1 shows an  
305 example of the selected P and S windows on the waveforms recorded at  
306 LIO3 station (Figure 2a) during the  $M_L$  2.9, Laviano earthquake, together  
307 with the observed displacement spectra, the fitting curves and the spectrum  
308 of pre-P noise. We tested the sensitivity of the retrieved focal mechanism to  
309 the window length and verified that the results are almost insensitive to a  
310 variation up to  $\sim 30\%$  of the initial window length. Figures 1b and 1c show  
311 the comparison between displacement spectra computed on the initial  
312 selected time window (Fig. 1a) and those computed by a reduction of 30%

313 of the window length for P- and S-wave, respectively. Examples for other  
314 stations are shown in Figures S1 and S2, available in the electronic  
315 supplement to this article.

316 Each windowed time series is then tapered by using a 5% cosine taper  
317 function and zero padded before computing the Fourier amplitude spectra.  
318 The spectra have been converted to displacement spectra (through a single  
319 or double integration in the frequency domain) and smoothed by applying  
320 an average moving window with a two-point half width.

321 We considered only those displacement spectra whose large majority  
322 (~80%) of spectral amplitudes are 3 times larger than the noise level in the  
323 considered frequency band, assigning a lower weight to the part of the  
324 spectrum with low S/N in the inversion process. As for the noise, according  
325 to Abercrombie (1995) and Zollo et al. (2014), we have considered the pre-  
326 P noise in a time-window whose length is equal to the selected signal  
327 window length.

328 Both for P- and S-wave we have inverted the displacement spectra for  
329 estimating the long-period spectral level spectral  $\Omega_0$ . We used a  $\omega^{-2}$   
330 displacement source model (i.e.,  $\zeta = 2$  and  $n = 1$  in eq.4) and have inverted  
331 also for attenuation parameters ( $Q_0$ ). Note that, if the attenuation model is  
332 known or a reasonable assumption can be made for the study area, it can be  
333 fixed and only the source spectrum is inverted.

334 Finally, following Abercrombie (1995), both for P- and S-wave the final  
335 value of  $\Omega_0$  is given by the vector composition of the single values  
336 determined on each component, that is

337

$$\Omega_0 = \sqrt{\Omega_{0Z}^2 + \Omega_{0NS}^2 + \Omega_{0EW}^2} \quad (12)$$

338

339 The ratio of  $\Omega_o^P$  to  $\Omega_o^S$  values as function of the station azimuth and take-off  
340 angle (obtained from event location) thus represents the input data vector  $\mathbf{d}^L$   
341 for the Bayesian inversion described in the previous section. For all the  
342 analyzed earthquakes the model parameter space  $M$  is sampled at  $2^\circ$  step.

343 Events are located by using the manual P- and S-picks and a proper 1D  
344 velocity model. The P-polarities are used to obtain an independent estimate  
345 of the focal mechanism by using the FPFIT code (Reasenber and  
346 Oppenheimer, 1985). Specifically, for the 23 December 2013,  $M_W$  5.01,  
347 Matese earthquake we used the model proposed by Scognamiglio et al.  
348 (2009), for the synthetic test and for the 27 May 2009,  $M_L$  2.9, Laviano  
349 earthquake the 1D velocity model proposed by Matrullo et al. (2013), and  
350 for the 27 September 2012,  $M_L$  4.1, Benevento earthquake the model by Del  
351 Pezzo et al. (1983). Figure 2 displays the location of the events and the  
352 corresponding recording stations.

353

## 354 **Results**

### 355 *The 23 December 2013, $M_W$ 5.0, Matese earthquake*

356 From late 2013 to early 2014 a sequence of about 250 events occurred in the  
357 Matese Massif that is a part of the Southern Apennines (Italy). Almost all  
358 the events occurred at 10-20 km depth (<http://cnt.rm.ingv.it>, last accessed 30



359 July 2015). The 23 December 2013,  $M_w$  5.0, Matese earthquake represents  
360 the mainshock of the sequence. It occurred at a depth of about 20 km and is  
361 the largest event analyzed in the present study. As reported by INGV, the  
362 event was characterized by normal focal mechanism ( $\phi = 324^\circ$ ,  $\delta = 50^\circ$  and  
363  $\lambda = -78^\circ$  (<http://cnt.rm.ingv.it/tdmt/>, last accessed 30 July 2015); see Figure  
364 3) oriented NW-SE, parallel to the Apennine chain axis and represents our  
365 reference model.

366 The Matese earthquake has been recorded at 38 stations belonging to RAN  
367 and Italian Seismic Network. After a selection based on the spectral fit  
368 quality the number of available spectral ratios is reduced to 25, with  
369 epicentral distances ranging between 7 - 60 km (Figure 2a). Figure 4 shows  
370 the observed polarities and spectral ratio levels.

371 The retrieved MAP model for the Matese event is  $\phi = 358^\circ$ ,  $\delta = 46^\circ$  and  $\lambda =$   
372  $-66^\circ$  ( $\phi = 145^\circ$ ,  $\delta = 49^\circ$  and  $\lambda = -112^\circ$ ) (Figure 3) that is in agreement with  
373 the model reported by INGV. The MAP model is plotted using a lower-  
374 hemisphere stereographic projection in Figure 4a together with the observed  
375 P-polarities. Figure 4b displays the comparison between observed and  
376 predicted  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios. The 68%, 90% and 95% Bayesian confidence  
377 intervals together with the 1D posterior marginal PDFs are shown in Figure  
378 5. The slices representation of the *a-posteriori* pdf is shown in Figure S3  
379 (available in the electronic supplement to this article) and indicates that the  
380 inferred solution is very well constrained. Note that, due to the azimuthal

381 distribution of the stations on the focal sphere the dip angle is highly  
382 constrained.

### 383 *Synthetic test*

384 Since, for small magnitude earthquakes the low signal-to-noise ratio  
385 generally prevents to have a high density of data and a good azimuthal  
386 coverage, we performed a synthetic test to evaluate the applicability of the  
387 technique to such earthquakes.

388 The synthetic waveforms were computed reproducing the features of the 27  
389 May 2009,  $M_L$  2.9, Laviano earthquake. We set the depth of the event at  
390 13.5 km, seismic moment  $M_0 = 2.8 \times 10^{13}$  Nm, and strike, dip and rake to  
391  $280^\circ$ ,  $40^\circ$ ,  $-100^\circ$  (see Figure 3 for the beach ball representation). We  
392 generated synthetic seismograms using Axitra code (Cotton and Coutant,  
393 1997) based on the discrete wavenumber approach for 1D layered model.  
394 We selected a triangle source-time function with 0.1s duration according to  
395 results of Stabile et al. (2012). The maximum simulated frequency was 50  
396 Hz. We used the 1D velocity model proposed by Matrullo et al. (2013), a  
397 sampling rate of 0.005 s and a stations configuration reproducing the ISNet  
398 seismic network (Figure 2b). The epicentral distances cover a range of 8 -  
399 47 km. Synthetics were manually picked (both P and S arrival times and P-  
400 wave polarities were identified) and the time-windows enclosing P- and S-  
401 waves arrival times were also manually selected. The windowed time-series  
402 were processed according to the procedure described in the Data analysis  
403 section. Figure 6b shows the observed  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios as function of the

404 station azimuth together with the same values obtained for the reference  
405 focal mechanism.

406 In order to test the Bayesian inversion procedure we used three different  
407 data sets: (a)  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios and polarities, (b)  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios, (c) all  
408 the  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios and only 8 polarities, respectively (for this latter  
409 condition standard codes based only on the polarities generally provide  
410 unconstrained solutions).

411 The obtained results indicate that, when both all the  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios and  
412 all the polarities are used, the Bayesian approach identifies the MAP model:  
413  $\phi = 274^\circ$ ,  $\delta = 40^\circ$  and  $\lambda = -112^\circ$  that is well consistent with the values  
414 assumed for the simulations. The  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios corresponding to the  
415 MAP model together with the stereographic projection of the solution are  
416 shown in Figure 6a. The 68%, 90% and 95% Bayesian confidence intervals  
417 together with the 1D posterior marginal PDFs are shown in Figure 7a. In  
418 addition, the slices representation of the *a-posteriori* pdf at different dip  
419 values shows only two areas with probability different from zero (Figure S4,  
420 available in the electronic supplement to this article). Overall, these results  
421 indicate that the implemented procedure is able to accurately constrain both  
422 the principal and the auxiliary nodal plane

423 When only the  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios are inverted, since P-polarity data are not  
424 used, there is an intrinsic ambiguity (i.e., the sign of  $\lambda$  cannot be solved) that  
425 leads to identify as MAP models two solutions  $\phi = 274^\circ$ ,  $\delta = 40^\circ$  and  $\lambda =$   
426  $\pm 112^\circ$  with equal probability and produce large Bayesian confidence

427 intervals together with broad 1D posterior marginal PDFs (Figure 7b).  
428 Moreover, in this case the *a-posteriori* pdf, in addition to the MAP models,  
429 shows a number of secondary maxima (Figures 7b and S5, available in the  
430 electronic supplement to this article).

431 Generally, for small earthquakes the number of available P-polarities is  
432 small and the inferred solution might be highly uncertain. As an example,  
433 for the event simulated in this study, the solution obtained using only 8 P-  
434 polarities (Figure 2b) and FPFIT code is  $\phi = 260^\circ$ ,  $\delta = 58^\circ$  and  $\lambda = -130^\circ$ ,  
435 which significantly differs from the reference model featuring a strike-slip  
436 component. However if these polarities are jointly inverted with the  
437  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios in the Bayesian approach, the solution is expected to be  
438 improved. Indeed, the usage of all the  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios and only 8  
439 polarities over the 21 available provides as MAP model:  $\phi = 274^\circ$ ,  $\delta = 40^\circ$   
440 and  $\lambda = -112^\circ$  fitting the reference model but with slightly larger Bayesian  
441 confidence intervals (Figure 7c) compared to those obtained when all data  
442 are used (Figure 7a). The slices representation of the *a-posteriori* pdf at  
443 different dip values is shown in Figure S6, available in the electronic  
444 supplement to this article.

445 Figure 3 summarizes as case a, case b, and case c the inferred focal  
446 mechanisms together with their beach ball representation.

447

448 *The 27 May 2009,  $M_L$  2.9, Laviano earthquake*

449 As regards the Laviano earthquake we have analyzed waveforms recorded at  
450 15 stations of the ISNet network with epicentral distances between 8 - 46  
451 km (Figure 2c).

452 We compared our solution with the model determined by Stabile et al.  
453 (2012) from the analysis of 26 P-wave polarities identified on waveforms  
454 recorded at both INGV and ISNet stations. The model is given by  $\phi = 287^\circ$ ,  
455  $\delta = 38^\circ$  and  $\lambda = -102^\circ$  (Figure 3). Note that, for the present study the  
456 waveform for the stations AND3, BEL3, AVG3 and PGN3 of ISNet were  
457 not available. However, in order to be consistent with synthetic test we  
458 added in our inversion the polarities provided by Stabile et al. (2012).

459 The technique proposed in the present study identifies both the principal and  
460 the auxiliary planes and, in particular, the MAP model corresponds to  $\phi =$   
461  $294^\circ$ ,  $\delta = 40^\circ$  and  $\lambda = -100^\circ$  (Figure 3) that agrees with the model proposed  
462 by Stabile et al. (2012). Thus a reduced number of P-polarities with respect  
463 to that used by Stabile et al. (2012), but jointly inverted with the  
464  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios, allowed constraining the focal mechanism. The MAP  
465 model is coherent with the expected focal mechanism of the earthquakes  
466 occurring in the southern Apennines. In fact, being this portion of the  
467 Apennines undergoing to an extensional regime, the larger portion of the  
468 events feature a normal focal mechanism (Matrullo et al., 2013). Figure 8b  
469 also shows the comparison between the observed  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios and  
470 those corresponding to the MAP model and to the model by Stabile et al.  
471 (2012), together with the P-polarities (Fig. 8a). The Bayesian confidence

472 intervals and marginal PDFs are shown in Figure 9. Notably, the slices  
473 representation of the *a-posteriori* pdf displays only two small areas of high  
474 pdf values in correspondence of the two nodal planes, indicating that the  
475 solution is very tightly constrained (Figure S7, available in the electronic  
476 supplement to this article).

477

#### 478 *The 27 September 2012, $M_L$ 4.1, Benevento earthquake*

479 The Benevento earthquake occurred in the northernmost part of the southern  
480 Apennines. Due to a transpressive stress regime acting in the area, focal  
481 mechanisms are characterized by a pronounced strike-slip component. This  
482 is the case of the 1930 event (Pino et al., 2008) and of two events occurred  
483 in 1962 (Westaway, 1987).

484 Waveforms available for Benevento earthquake have been recorded by  
485 ISNet, RAN and Rete Sismica Nazionale. Starting from an initial number of  
486 waveforms recorded at 16 stations, due to the quality of the recordings and  
487 the spectral fit, the number of spectral ratios reduced to 10 (Figure 2d) with  
488 epicentral distances ranging between 10 and 50 km, whereas the number of  
489 clearly identifiable P-wave polarities was 7. As a consequence this event  
490 represents an optimal case study to evaluate the performance of the  
491 proposed technique. In fact, while the number of polarities may be  
492 insufficient for using standard codes, their integration with a distribution of  
493 spectral ratios and a joint inversion can help to constrain the focal  
494 mechanism. In addition, focal mechanism for the Benevento earthquake has

495 been determined by other authors by using codes based on P-wave polarities  
496 inversion and from moment tensor inversion. In particular, Adinolfi et al.  
497 (2015) have found a strike slip focal mechanism with angles  $\phi = 267^\circ$ ,  $\delta =$   
498  $75^\circ$  and  $\lambda = 179^\circ$  (indicated as Ref a in Figure 3). The Time Domain  
499 Moment Tensor (TDMT) technique implemented by the INGV  
500 (<http://cnt.rm.ingv.it/tdmt>, last accessed 30 July 2015) provided a focal  
501 mechanism with angles  $\phi = 267^\circ$ ,  $\delta = 68^\circ$  and  $\lambda = -153^\circ$  (indicated as Ref b  
502 in Figure 3). For the same event, using the ISOLA code (Sokos and  
503 Zahradník, 2008) and waveforms recorded at 14 stations belonging to  
504 ISNet, Michele et al. (2014) have determined a normal focal mechanism  
505 with a strong strike-slip component whose angles are  $\phi = 246^\circ$ ,  $\delta = 76^\circ$  and  
506  $\lambda = -143^\circ$  (indicated as Ref c in Figure 3). Using the technique proposed in  
507 the present study we obtained the solutions:  $\phi = 358^\circ$ ,  $\delta = 52^\circ$ ,  $\lambda = -2^\circ$  and  $\phi$   
508  $= 89^\circ$ ,  $\delta = 88^\circ$ ,  $\lambda = -142^\circ$ . Figure 10a shows the stereographic projection  
509 corresponding to the MAP model together with P-polarities. The observed  
510  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios, those corresponding to the MAP model and to the  
511 reference model are shown in Figure 10b.

512 The slices representation of the a-posteriori pdf shown in Figure S8  
513 (available in the electronic supplement to this article) indicates that the *a-*  
514 *posterior* PDF is almost zero everywhere except in correspondence of the  
515 MAP model. However, looking at the Bayesian confidence intervals and  
516 marginal PDFs (Figure 11) it can be noted that the dip is characterized by  
517 the largest uncertainty.

518 In order to further test our code, we have inverted only for the spectral ratios  
519 by excluding the polarities. Aside from the ambiguity on the sign of the rake  
520 angle, the obtained mechanism (not shown here) is identical to the one  
521 determined from the joint inversion. Thus, we concluded that for the  
522 specific stations configuration relative to the Benevento earthquakes, the  
523 polarities have a minor effect on the final result with respect to the  
524 distribution of the observed spectral ratios.

## 525 **Discussion and conclusion**

526 In this study we have presented a technique that jointly inverts long-period  
527 spectral levels and P-wave polarities to infer focal mechanism in a Bayesian  
528 framework. Since the long-period spectral levels are observables usually  
529 available for both small and moderate-to-large earthquakes, this method can  
530 be used in a broader magnitude range compared to those operating only on  
531 small magnitude or only on moderate-large magnitude. The Bayesian  
532 approach offers the possibility to inherently consider additional observables  
533 as the S-polarization or P and S wave amplitudes. Moreover, it allows for a  
534 fully description of the inverted parameters through an *a-posteriori* pdf that  
535 permits to evaluate both the parameters uncertainty and correlation. This is  
536 very important for inverse problems where the relation between data and  
537 parameters is strongly non-linear as for the case of the focal mechanism. In  
538 this study, we computed the 68%, 90% and 95% Bayesian confidence  
539 intervals, the 1D *a-posterior* marginal PDFs and a slices representation of  
540 the *a-posteriori* pdf. All these analyses of the *a-posteriori* pdf provide  
541 information about the uncertainty of the inferred solution enlightening



542 possible secondary maxima and correlation among the parameters.  
543 However, the presence of at least two solutions is inherent to the assumed  
544 double-couple source model, which prescribes two equivalent nodal planes.

545 We tested the technique on events with different magnitude values, different  
546 focal mechanisms, and different network configuration (i.e., different  
547 number of stations, azimuthal gaps, ...).

548 As an example of moderate normal-fault earthquake, we considered the 23  
549 December 2013,  $M_W$  5.0, Matese earthquake. For this event, the obtained  
550 solution is fully consistent with that obtained by INGV using TDMT  
551 inversion that can be more onerous in terms of required information and  
552 computational time.

553 For small earthquakes, large azimuthal gap of the recording stations, a poor  
554 signal-to-noise ratio and the related difficulty of correctly identifying first  
555 arrival phase and its polarity, may hamper the determination of the focal  
556 mechanism. Thus, before applying it to real earthquakes, we used synthetic  
557 events to simulate different network geometries and a different combination  
558 of long-period spectral ratios and P-polarities. As examples of real small  
559 magnitude earthquakes, we analyzed the 27 May 2009,  $M_L$  2.9, Laviano  
560 earthquake and the 27 September 2012,  $M_L$  4.1, Benevento earthquake that  
561 have different focal mechanism and different azimuthal coverage. The  
562 results suggest that for all the analyzed events, the addition of the spectral  
563 ratios also when a limited number of polarities are available, allows to

564 obtain a solution that well agrees with the solution obtained by other authors  
565 with different techniques.

566 The results suggest that the approach is promising and could be applied in  
567 those situations where the focal mechanisms of small magnitude  
568 earthquakes may help to understand the stress regime and deformation  
569 processes acting in an area. Moreover, since the long-period spectral levels  
570 and P-polarities can be automatically measured, BISTROP could be  
571 implemented for real-time applications. In the latter case, the selection of a  
572 proper time window enclosing P- or S-wave is a key aspect, which may  
573 require a preliminary calibration on a set of earthquakes of different  
574 magnitude and location.

575

## 576 **Data & Resources**

577 Seismograms used in this study were obtained from: ISNet Data  
578 Management Center at [www.isnet.infn.it](http://www.isnet.infn.it) (last accessed November  
579 2015); from the Italian Seismological Instrumental and Parametric  
580 Data-Base (ISIDE) available at  
581 <http://iside.rm.ingv.it/iside/standard/index.jsp> (last accessed  
582 November 2015); from Italian acelerometric network (RAN) at  
583 [www.protezionecivile.gov.it](http://www.protezionecivile.gov.it) (last accessed November 2015).

584 Figures have been generated with the Generic Mapping Tools (GMT,  
585 Wessel and Smith, 1991).

586 The BISTROP code can be required contacting the corresponding author.

587

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590 reviewers for their comments that helped to improve the manuscript.

591

## 592 **References**

593 Abercrombie, R. E. (1995). Earthquake source scaling relationships from  $-1$   
594 to 5 ML using seismograms recorded at 2.5 km depth, *J. Geophys.*  
595 *Res.* **100**, 24,015–24,036.

596 Adinolfi, G. M., R. De Matteis, A. Orefice, A. Zollo, R. De Nardis, R., and  
597 G. Lavecchia (2015). The September 27, 2012, ML 4.1, Benevento  
598 earthquake: A case of strike-slip faulting in Southern Apennines  
599 (Italy), *Tectonophysics*, **660**, 35-46,  
600 [doi.org/10.1016/j.tecto.2015.06.036](https://doi.org/10.1016/j.tecto.2015.06.036).

601 Aki, K., and P. G. Richards (1980). Quantitative Seismology, Theory and  
602 Methods, W.H. Freeman, San Francisco.

603 Aster, R. C., B. Borchers, and C. H. Thurber (2013). Parameter estimation  
604 and Inverse Problems, second edition, Elsevier, Amsterdam, 360 pp.

605 Ben-Menahem, A., and S. J. Singh (1981). Seismic Waves and Sources,  
606 1108 p, Springer-Verlag, New York.

607 Boatwright, J. (1980). A spectral theory for circular seismic sources; simple  
608 estimates of source dimension, dynamic stress drop, and radiated  
609 seismic energy, *Bull. Seismol. Soc. Am.*, **70** (7), 1–27.

610 Brillinger, D. R., A. Udías, and B. A. Bolt (1980). A probability model for  
611 regional focal mechanism solutions, *Bull. Seism. Soc. Am.* **70**, 149-  
612 170.

613 Carlin, P. B., and T. A. Louis (2009). Bayesian Methods for Data Analysis,  
614 Chapman and Hall/CRC Press, Boca Raton.

615 Cotton, F., and O. Coutant (1997). Dynamic stress variations due to shear  
616 faults in a plane-layered medium, *Geophys. J. Int.* **128** (3), 676–688.

617 Delouis, B. (2014). FMNEAR: Determination of Focal Mechanism and  
618 First Estimate of Rupture Directivity Using Near-Source Records and  
619 a Linear Distribution of Point Sources, *Bull. Seismol. Soc. Am.* **104**, 3,  
620 1479-1500.

621 Del Pezzo, E., G. Iannaccone, M. Martini, and R. Scarpa (1983). The 23  
622 November 1980 southern Italy earthquake, *Bull. Seismol. Soc. Am.* **73**,  
623 187–200.

624 Dreger, D. S. (2003). 85.11 TDMT\_INV: time domain seismic moment  
625 tensor inversion, *Int. Geophys.* **81**, 1627.

626 Dreger, D. S., and D. V. Helmberger (1993). Determination of source  
627 parameters at regional distances with three-component sparse network  
628 data, *J. Geophys. Res.* **98** 8107-8125.

629 Fojtíková, L., and J. Zahradník (2014). A new strategy for weak events in  
630 sparse network: The first-motion polarity solutions constrained by  
631 single-station waveform inversion, *Seism. Res. Lett.* **85**, Number 6.

632 Hardebeck, J., and M. Shearer (2003). Using S/P Amplitude Ratios to  
633 Constrain the Focal Mechanisms of Small Earthquakes, *Bull. Seism.*  
634 *Soc. Am.* **93**, 6, pp. 2434–2444, December 2003.

635 Julian, B. R., and G. R. Foulger (1996). Earthquake mechanisms from  
636 linear-programming inversion of seismic-wave amplitude ratios, *Bull.*  
637 *Seism. Soc. Am.* **86** (4), 972-980.

638 Kisslinger, C. (1980). Evaluation of S to P amplitude ratios for  
639 determining focal mechanisms from regional network observations,  
640 *Bull. Seism. Soc. Am.* **70** (4), 999-1014.

641 Kisslinger, C., J. R. Bowman, and K. Koch (1981). Procedures for  
642 computing focal mechanisms from local (SV/P) z data, *Bull. Seism.*  
643 *Soc. Am.* **71** (6), 1719-1729.

644 Iannaccone, G., A. Zollo, R. De Matteis, and A. Deschamps (1994). Rupture  
645 characterization of a low magnitude earthquake of central Apennines  
646 (Italy), *Phys. Earth Plan. Int.* **82**, 157-165.

647 Imanishi, K., T. Uchide, and N. Takeda (2016). Determination of focal  
648 mechanisms of nonvolcanic tremor using S wave polarization data  
649 corrected for the effects of anisotropy, *Geophys. Res. Lett.* **43**, 611–  
650 619, doi:10.1002/2015GL067249.

651 Matrullo E., R. De Matteis, C. Satriano, O. Amoroso, and A. Zollo (2013).  
652 An improved 1D seismic velocity model for seismological studies in

653 the Campania-Lucania region (Southern Italy), *Geophys. J. Int.* **195**,  
654 Issue 1, pp.460-473, doi: 10.1093/gji/ggt224.

655 Michele, M., S. Custódio, and A. Emolo (2014). Moment tensor resolution:  
656 case study of the Irpinia Seismic Network, Southern Italy, *Bull.*  
657 *Seismol. Soc. Am.* **104**, 1348-1357, doi: 10.1785/0120130177.

658 Morozov, I. B. (2008). Geometrical attenuation, frequency dependence of  
659 Q, and the absorption band problem, *Geophys. J. Int.* **172**, 239–252,  
660 doi:10.1111/j.1365-246X.2008.03888.x.

661 Pino, N. A., B. Palombo, G. Ventura, B. Perniola, and G. Ferrari (2008).  
662 Waveform modeling of historical seismograms of the 1930 Irpinia  
663 earthquake provides insight on “blind” faulting in Southern Apennines  
664 (Italy), *J. Geophys. Res.* **113**, no. B5, doi: 10.1029/2007JB005211.

665 Rau R. J., F. T. Wu, and T.C. Shin (1996). Regional network focal  
666 mechanism determination using 3D velocity model and SH/P  
667 amplitude ratio, *Bull. Seismol. Soc. Am.* **86** (5), 1270-1283.

668 Reasenber, P. A., and D. Oppenheimer (1985). FPFIT, FPLOT, and  
669 FPPAGE:Fortran computer programs for calculating and displaying  
670 earthquake fault-plane solutions, U.S. Geol. Surv. Open-File Rep. 85-  
671 739.

672 Scognamiglio, L., E. Tinti, and A. Michelini (2009). Real-time  
673 determination of seismic moment tensor for the Italian region, *Bull.*  
674 *Seismol. Soc. Am.* **99** (4), 2223-2242.

675 Shen Y., D. W. Forsyth, J. Conder, and L. M. Dorman (1997). Investigation  
676 of microearthquake activity following an intraplate teleseismic swarm

677 on the west flank of the southern East Pacific Rise, *J. Geophys. Res.*  
678 **102** (B1), 459-475.

679 Snoke J. A. (2003). FOCMEC: focal mechanism determinations,  
680 International Handbook of Earthquake and Engineering Seismology,  
681 1629-1630.

682 Sokos, E. N., and J. Zahradník (2008). ISOLA a Fortran code and a Matlab  
683 GUI to perform multiple-point source inversion of seismic data,  
684 *Computers and Geosciences* **34**, 967-977.

685 Stabile T.A., C. Satriano, A. Orefice, G. Festa, and A. Zollo (2012).  
686 Anatomy of a microearthquake sequence on an active normal fault,  
687 *Sci. Rep.* **2**. doi:10.1038/srep00410.

688 Zahradník, J., J. Janský, and K. Papatsimpa (2001). Focal mechanisms of  
689 weak earthquakes from amplitude spectra and polarities, *Pure and*  
690 *applied geophysics*, **158**: 647-665. Birkhäuser Verlag, Basel.

691 Zollo, A., and P. Bernard (1991). Fault mechanism from near source data:  
692 joint inversion of S polarizations and P polarities, *Geophys. J. Int.*  
693 **104**, 441-451.

694 Zollo, A., A. Orefice, and V. Convertito (2014). Source Parameter Scaling  
695 and Radiation Efficiency of Microearthquakes Along the Irpinia Fault  
696 Zone in Southern Apennines, Italy, *J. Geophys. Res.* **119**, 4, 3256–  
697 3275. DOI: 10.1002/2013JB010116.

698 Walsh, D. R. Arnold, and J. Townend (2009). A Bayesian approach to  
699 determining and parametrizing earthquake focal mechanisms,

700 *Geophys. J. Int.* **176**, 235-255, DOI: 10.1111/j.1365-  
701 246X.2008.03979.x.

702 Weber, E., V. Convertito, G. Iannaccone, A. Zollo, A. Bobbio, L. Cantore,  
703 M. Corciulo, M. Di Crosta, L. Elia, C. Martino, A. Romeo, and C.  
704 Satriano (2007). An advanced seismic network in southern Apennines  
705 (Italy) for seismicity investigations and experimentation with  
706 earthquake early warning, *Seismol Res. Lett.* 78:622–634.

707 Wessel, P., and W. H. F. Smith (1991). Free software helps map and display  
708 data, *EOS Trans. AGU*, 72, 445–446.

709 Westaway, R. (1987). The Campania, southern Italy, earthquakes of 1962  
710 August 21, *Geophys. J. Roy. Astron. Soc.* **88**, no. 1, 1–24, doi:  
711 10.1111/j.1365-246X.1987.tb01366.x.

712

### 713 **Figure Captions**

714 **Figure 1:** a) Example of analysed waveforms together with the manually  
715 selected time windows enclosing P- and S-waves picking. b) left panel:  
716 displacement spectrum of the P-wave (grey line) computed on the selected  
717 window, the fitting curve (black line) and the pre-P noise displacement  
718 spectrum (dashed grey line); right panel: same as left panel but for a  
719 window length reduced by 30%. c) Same as panel b) but for S-wave.

720 **Figure 2:** Geographic distribution of seismic stations (triangle) and  
721 epicenter location (star) of the analyzed events. a) the 23 December 2013,  
722  $M_W$  5.0, Matese earthquake; b)  $M_L$  2.9 synthetic earthquake; the black  
723 triangles identify the stations whose P polarities have been used in the test c



724 described in the paragraph “*Synthetic test*”; c) the 27 May 2009,  $M_L$  2.9,  
725 Laviano earthquake; d) the 27 September 2012,  $M_L$  4.1, Benevento  
726 earthquake.

727 **Figure 3:** MAP models obtained from the BISTROP procedure for the  
728 analyzed events (black beach-balls). As for the synthetic event, the MAP  
729 models for the three tests are indicated as case a, case b and case c (see the  
730 text for details). For each event, the gray beach-balls represent the solutions  
731 by other authors obtained by using different data and techniques.

732 **Figure 4:** a) stereographic projection of the solution corresponding to the  
733 MAP model. P and T axes are identified by grey diamond and grey hexagon  
734 respectively. Black squares represent compression first motion while empty  
735 gray squares represent dilatational first motion. The symbol size is  
736 proportional to the observed  $|\mathcal{R}^P|/|\mathcal{R}^S|$  ratios; b) observed  $|\mathcal{R}^P|/|\mathcal{R}^S|$   
737 ratios (black cross) as a function of stations azimuth compared with those  
738 predicted by using the MAP model (grey circle) and those predicted by  
739 using the model proposed by the authors indicated in the text and also  
740 reported in Figure 3 (black diamond).

741 **Figure 5:** Upper panels: 1-D marginal a-posterior pdfs on each inverted  
742 parameter. Lower panels: 68%, 90% and 95% Bayesian confidence  
743 intervals.

744 **Figure 6:** Same as Figure 4, but for the  $M_L$  2.9 synthetic earthquake and  
745 case a (see the text for details).

746 **Figure 7:** a) Left panels: 68%, 90% and 95% Bayesian confidence intervals.  
747 Right panels: 1-D marginal a-posterior pdfs on each inverted parameter for

748 the test indicated as case a (see the text for details). b) same as a) but for the  
749 test indicated as case b. c) same as a) but for the test indicated as case c.

750 **Figure 8:** Same as Figure 4, but for the 27 May 2009,  $M_L$  2.9, Laviano  
751 earthquake.

752 **Figure 9:** Same as Figure 5 but for the 27 May 2009,  $M_L$  2.9, Laviano  
753 earthquake.

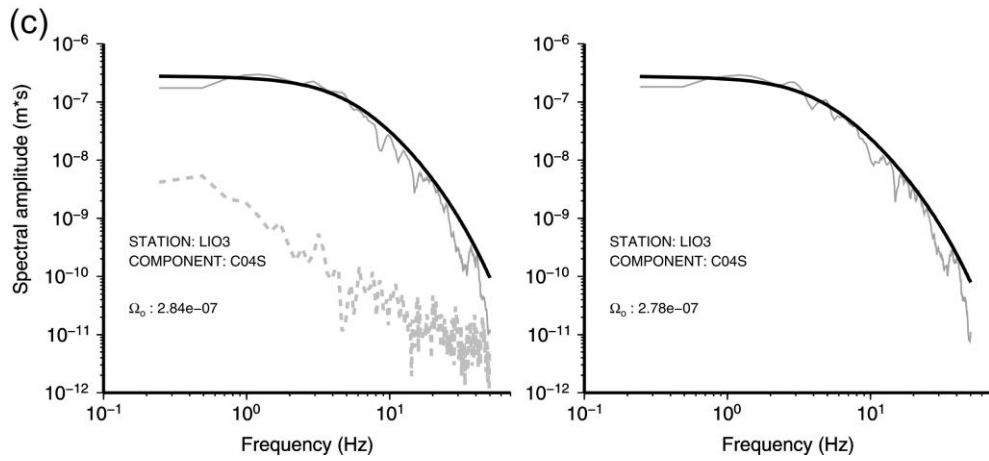
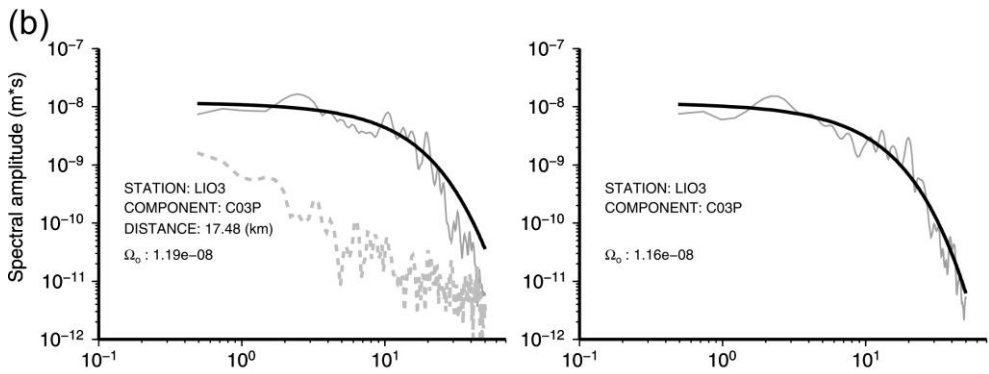
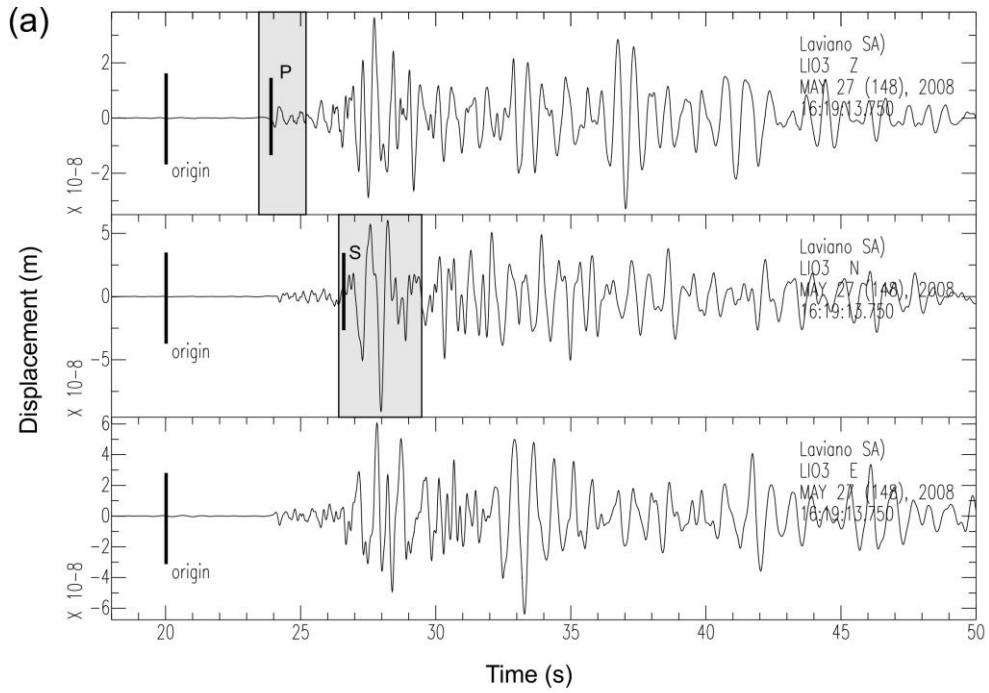
754 **Figure 10:** Same as Figure 4, but for the 27 September 2012,  $M_L$  4.1,  
755 Benevento earthquake.

756 **Figure 11:** Same as Figure 5 but for the 27 September 2012,  $M_L$  4.1,  
757 Benevento earthquake.

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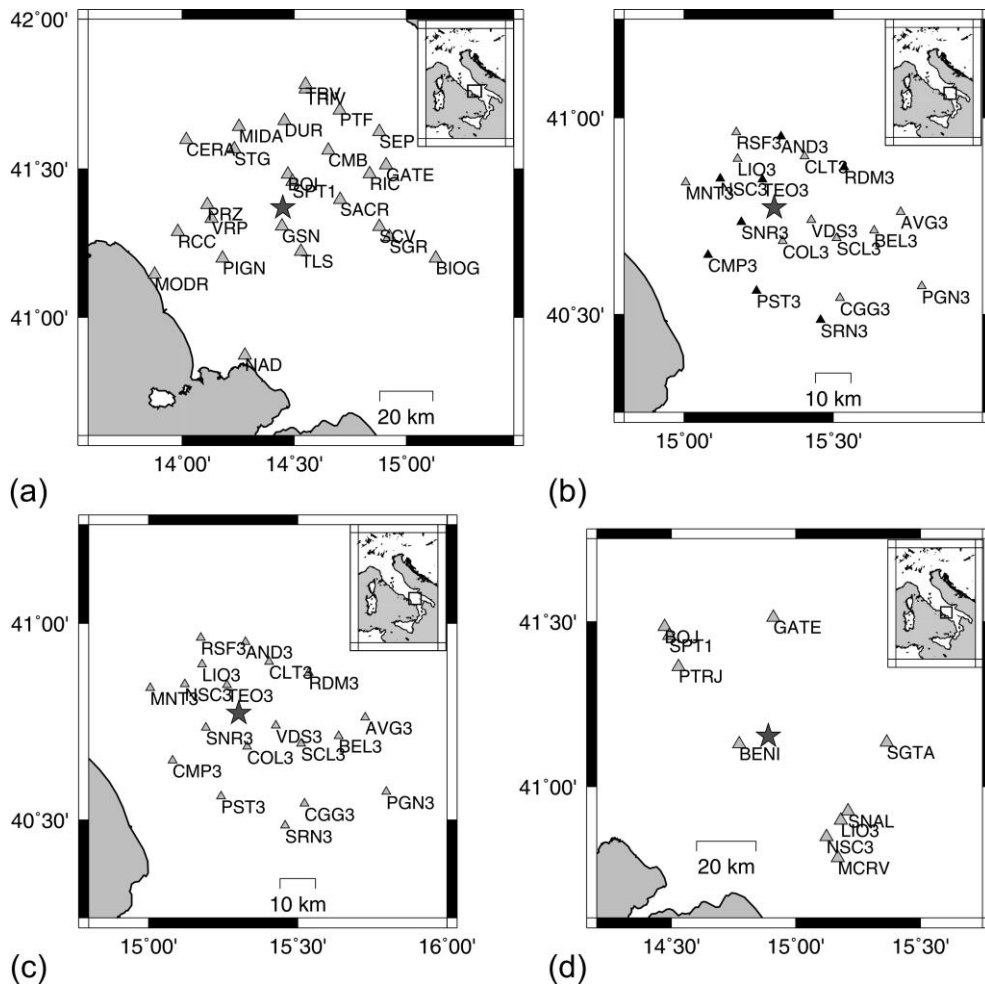
764

Figure 1

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Figure 2

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Earthquake	MAP model	Other Models
Matese	 Case a $\phi = 274^\circ$ $\delta = 40^\circ$ $\lambda = -112^\circ$  Case b $\phi = 358^\circ$ $\delta = 46^\circ$ $\lambda = -66^\circ$  Case c $\phi = 324^\circ$ $\delta = 50^\circ$ $\lambda = -78^\circ$	 (INGV)  (Theoretical model)
Synthetic	 $\phi = 274^\circ$ $\delta = 40^\circ$ $\lambda = -112^\circ$  $\phi = 274^\circ$ $\delta = 40^\circ$ $\lambda = -112^\circ$  $\phi = 280^\circ$ $\delta = 40^\circ$ $\lambda = -100^\circ$	 (Stabile et al., 2012)
Laviano	 $\phi = 294^\circ$ $\delta = 40^\circ$ $\lambda = -100^\circ$	 $\phi = 287^\circ$ $\delta = 38^\circ$ $\lambda = -102^\circ$
Benevento	 $\phi = 358^\circ$ $\delta = 52^\circ$ $\lambda = -2^\circ$  $\phi = 267^\circ$ $\delta = 75^\circ$ $\lambda = 179^\circ$	 Ref a (Adinolfi et al., 2015) $\phi = 267^\circ$ $\delta = 68^\circ$ $\lambda = -153^\circ$  Ref b (INGV) $\phi = 246^\circ$ $\delta = 76^\circ$ $\lambda = -143^\circ$  Ref c (Michele et al., 2014)

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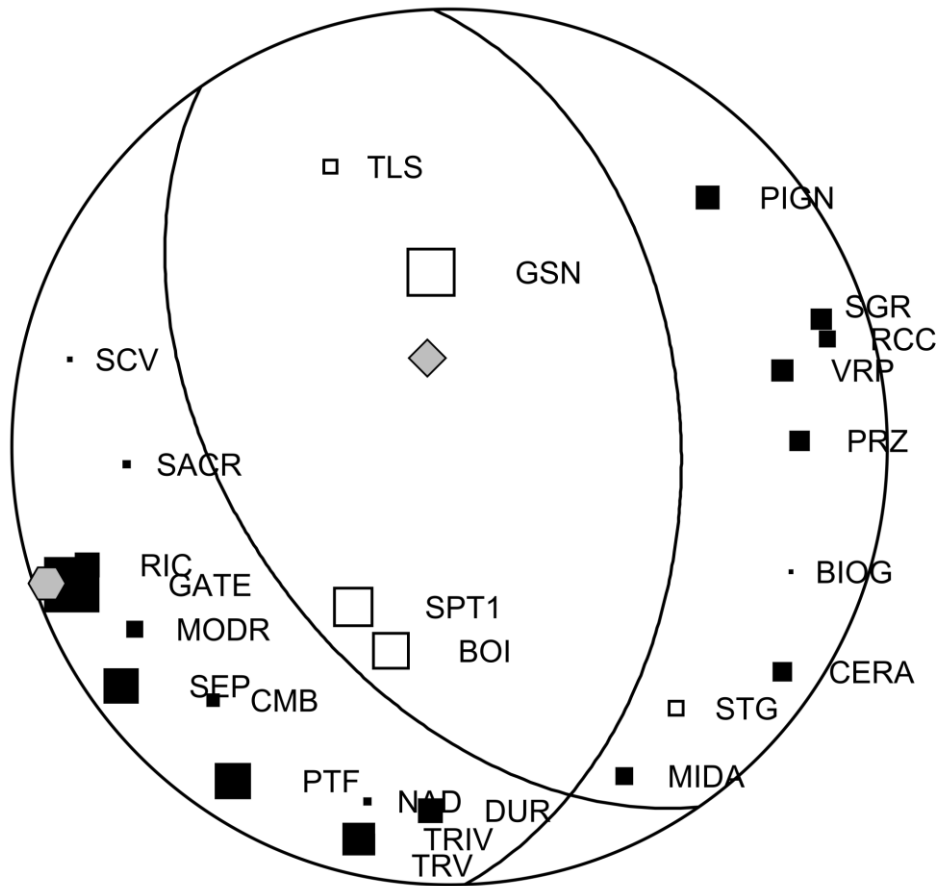
779

Figure 3

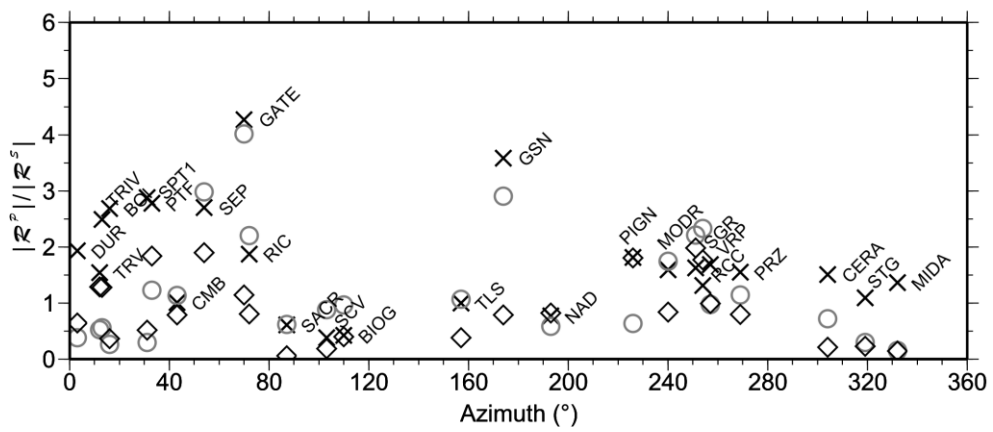
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(a)



(b)



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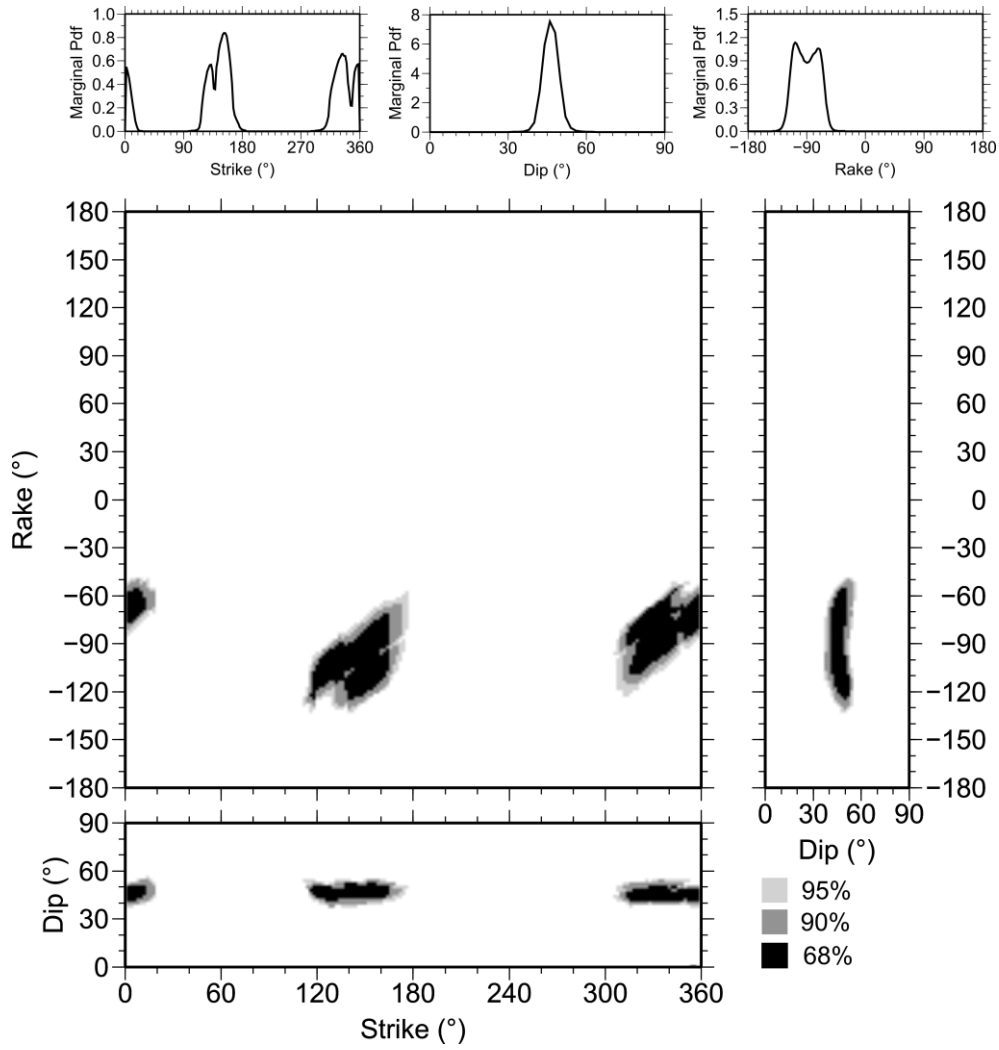
Figure 4

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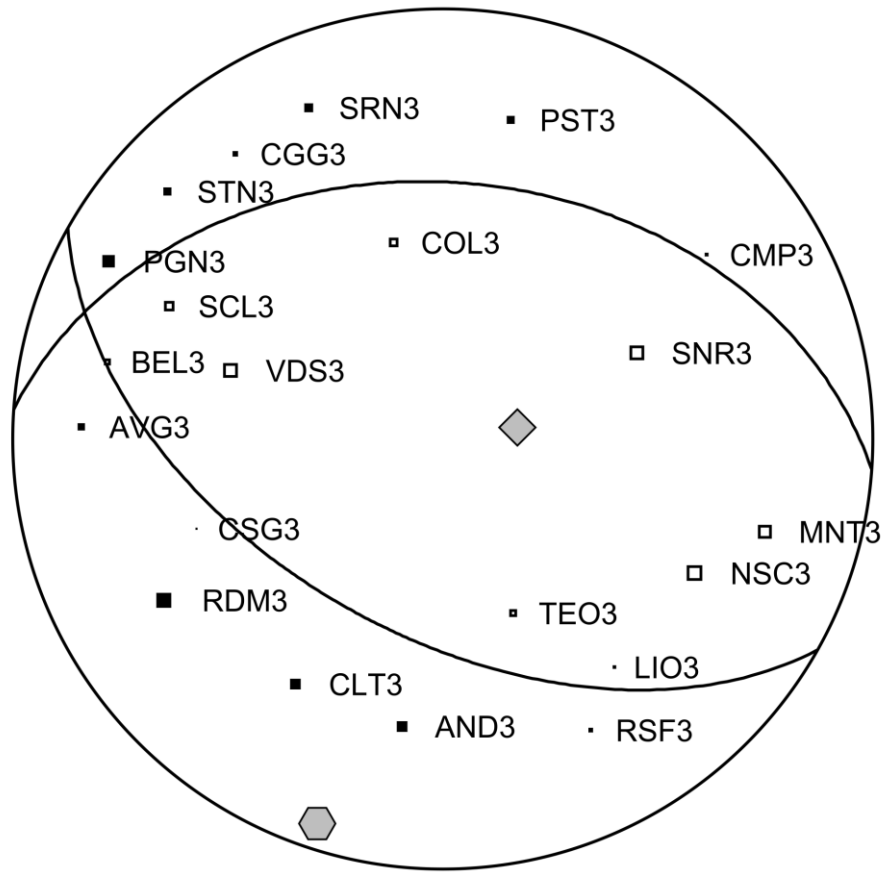
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**Figure 5**

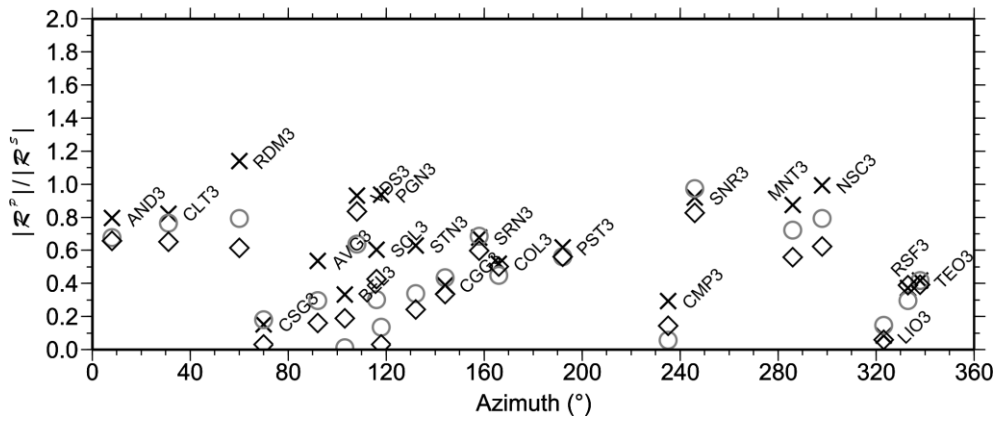
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(a)



(b)



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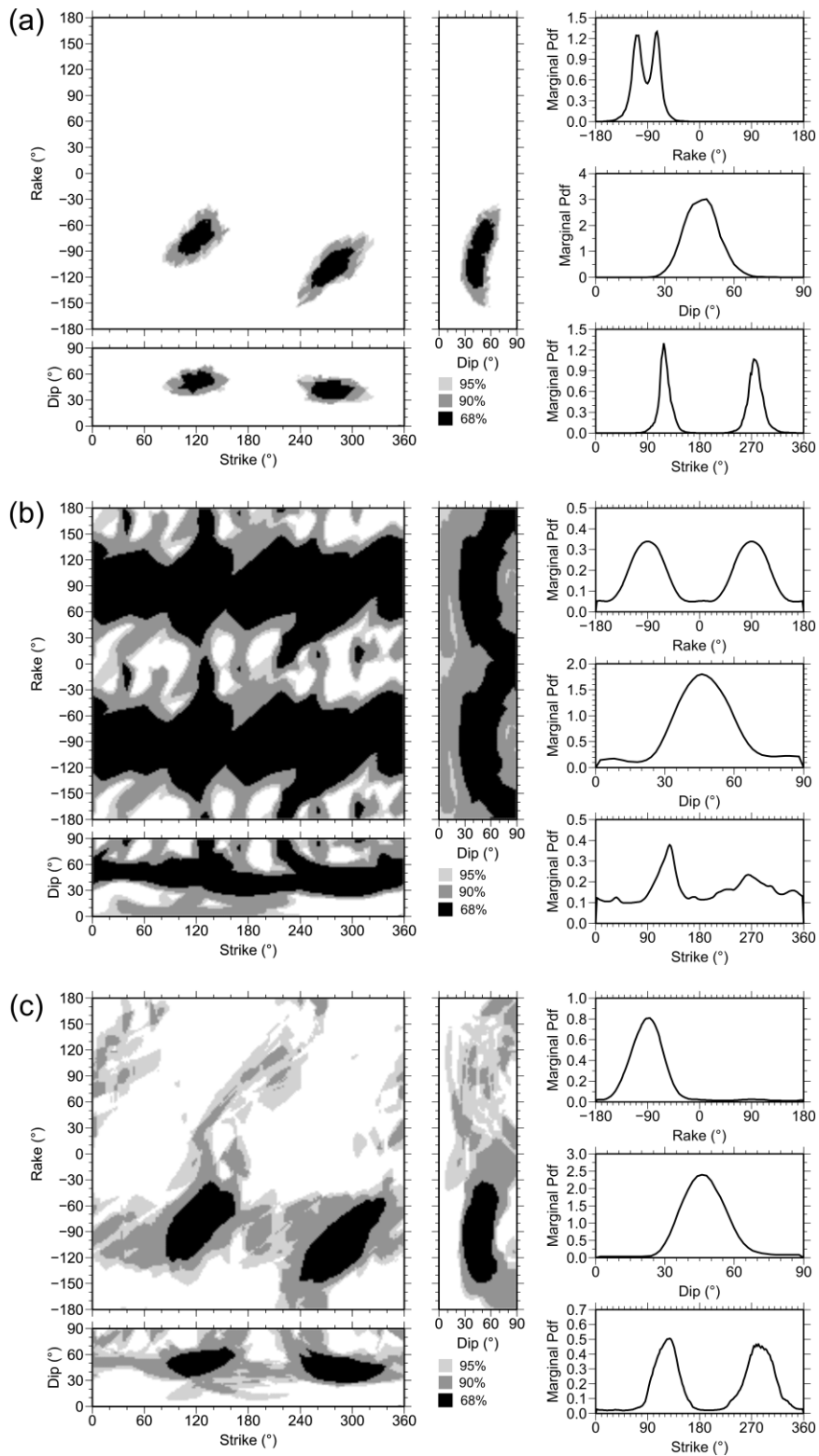
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Figure 6





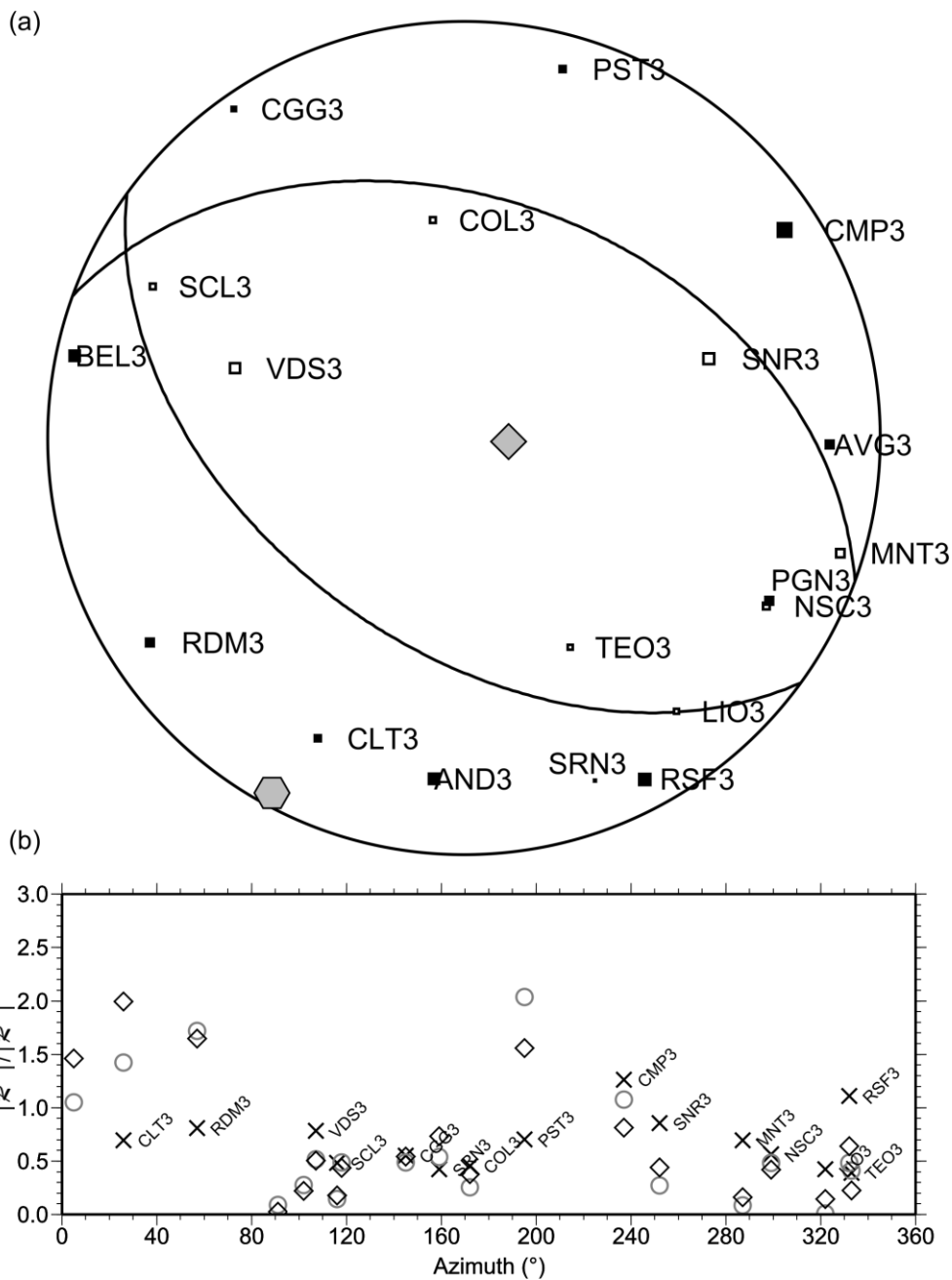
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Figure 7

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**Figure 8**

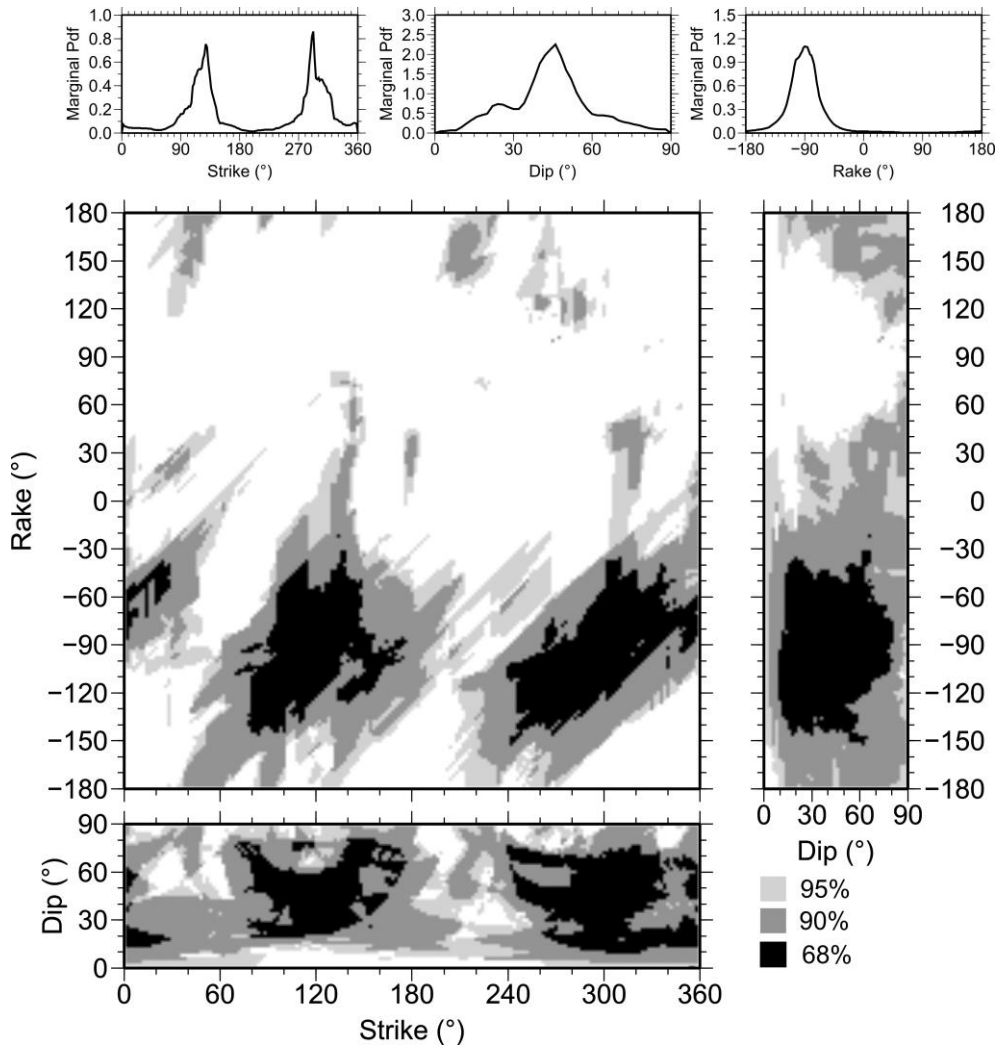
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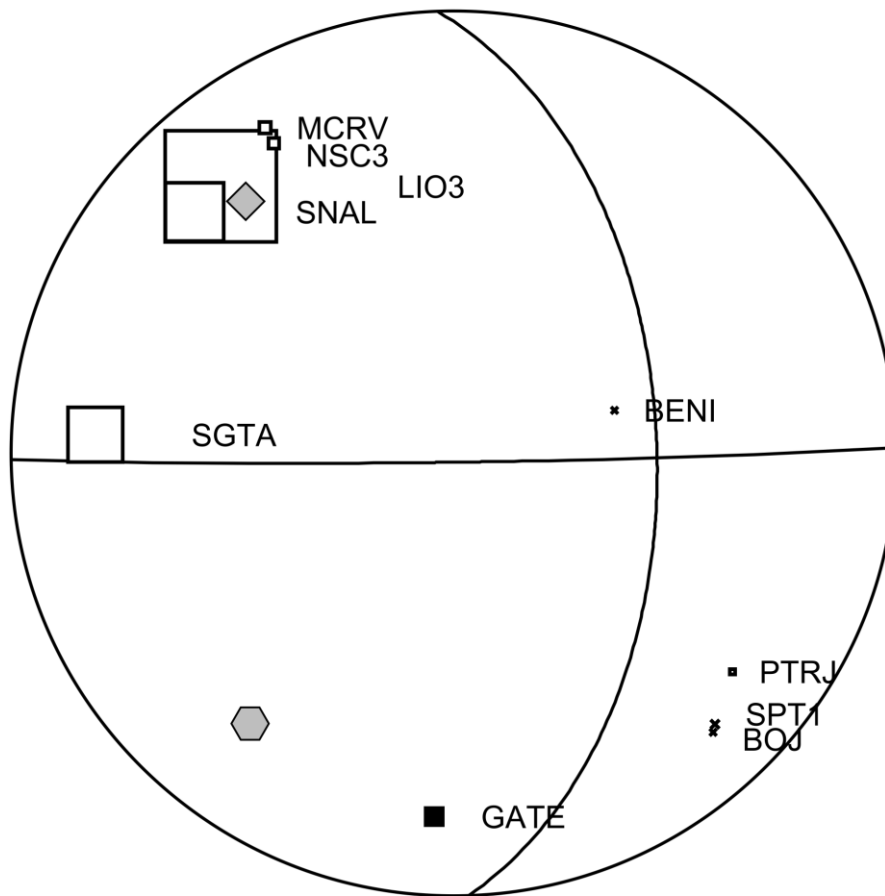
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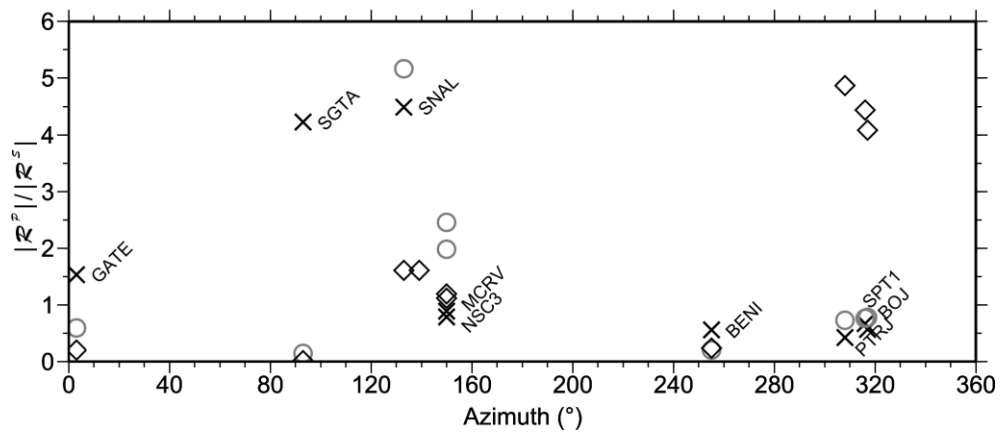
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**Figure 9**

(a)



(b)



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**Figure 10**

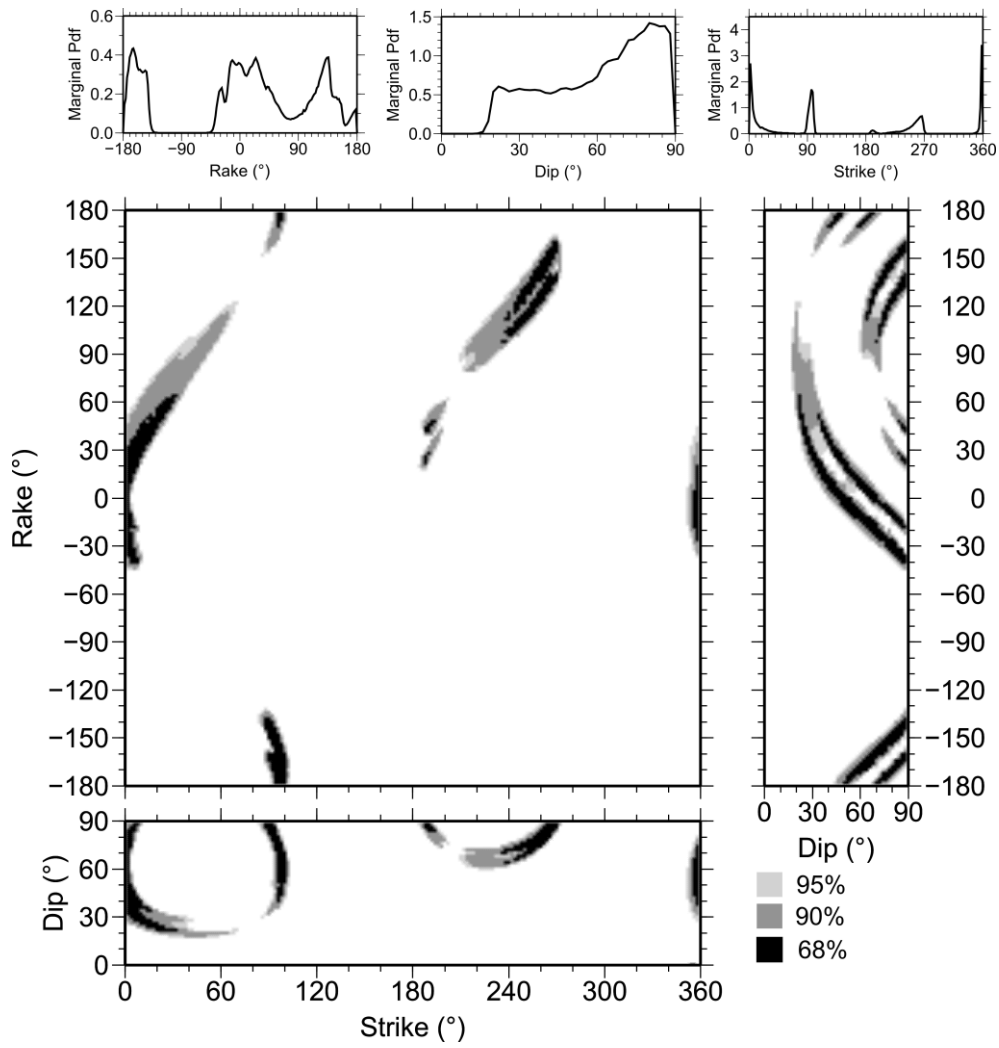
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**Figure 11**