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Forecast of solar maximum and minimum dates for solar cycles 23 to 29

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Abstract
The solar cycle length for cycles 23 to 29 are forecasted. Two methods are analysed. In the first one, the solar cycle length is separated into its two phases – the rise time and the fall off time – and a multiple regression method is applied to each phase using lagged values as independent variables. In the second method, the multiple regression is fitted directly to the solar cycle length. The minimum and maximum solar activity dates are listed for the cycles predicted with the latter method which proves to be more accurate. Two lagged values appear in the multiple regression adjusted to the solar cycle length. One is associated with the Gleissberg period, also observed in the maximum sunspot number, and the other is coincident with one of the periodicities in the C^{14} time record, which is associated with solar activity variation.

Key words *solar cycle – solar activity periodicities – sunspot forecast*

1. Introduction

Xanthakis (1966) and King Hele (1963) found a recurring tendency of seven cycles in the rise time of the solar cycle, T_R. Adler and Elias (1996) found the same periodicity in T_R and a different one in the fall off time of the solar cycle, T_F. They forecasted the solar cycle length, SCL, until cycle 25, through forecasted T_R and T_F values.

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An alternative method to that proposed by Adler and Elias (1996) is presented in this work. It consists in forecasting SCL through SCL past values, that is not splitting the solar cycle into its two phases T_R and T_F.

Both methods: SCL prediction through T_R and T_F (method A) and through SCL itself (method B), are compared in this work. Method B proves to be more efficient and the forecast for solar cycles 23 to 29 is therefore made with this method.

2. Data and method

T_R values were taken from Xanthakis (1967) and T_F values were calculated from tabulated values of maximum and minimum dates given by Friis-Christensen and Lassen (1993). Dates of maximum and minimum solar activity levels for solar cycles previous to cycle 4 are not considered reliable enough (King-Hele, 1963), so they are not included in the present calculations.

Two methods are analysed in order to forecast SCL for future solar cycles. The first one, method A, uses T_R and T_F and the second, method B, uses SCL directly.

Method A

A multiple linear regression of the form

$$y_t = \sum_{i=1}^{k} \alpha_i x_i + \beta \qquad (2.1)$$

where α_1, ..., α_n and β are constants, was fitted to T_R and T_F values separately. y_t is, in turn, T_R or T_F for solar cycle t, and the independent variables, x_i, are lagged values of y_t.

To choose the lagged values to be considered in eq. (2.1), an autocorrelation analysis of each series was made, finding not just one combination of lagged values which optimises eq. (2.1), but a set of them. To choose one of these sets and to test the efficiency of the method, part of the series was left out in order to see the fitness degree of the method predictions by comparison with these left out cycles.

The parameters of the multiple regression equation were calculated by the least squares method using the data series from cycle 4 to cycle 20 (cycles 21 and 22 were left out). With the equation obtained, the values for the left out cycles were estimated.

The error of prediction, *Ep*, was calculated as the sum of the square difference between the experimental value and the predicted one, that is

$$Ep = \sum_{t=21}^{22} (y_t - \hat{y}_t)^2 \qquad (2.2)$$

where y_t is the observed value of T_R or T_F and $\hat{y}_t$ is the one calculated with eq. (2.1). The equations with the least error of prediction are:

$$T_R(t) = 0.61\ T_R(t-3) + 0.16\ T_R(t-7) - 0.41\ T_R(t-12) + 2.64 \qquad (2.3)$$

$$T_F(t) = 0.66\ T_F(t-3) + 0.40\ T_F(t-9) - 0.26\ T_F(t-12) + 1.46. \qquad (2.4)$$

The correlation coefficient, r, of both of these multiple regressions estimated with the observed data set from cycle 4 to 20, are 0.98 and 0.99 for T_R and T_F respectively.

To estimate the significance level of regressions (2.3) and (2.4) – the probability that there is a true correlation and that r might have not been obtained by chance – the f ratio was calculated as

$$f = \frac{r^2 \sum (y_t - \bar{y})^2 / k}{\sum (y_t - \hat{y}_t)^2 / (n-k-1)}$$

(Walpole and Myers, 1993) where n is the number of y_t values, k is the number of independent variables and $\bar{y}$ is the mean value of y_t. f has an F-distribution with n_1 and n_2 degrees of freedom, where $n_1 = k$ and $n_2 = n-k-1$. The significance level was estimated then from tabulated values of the F-distribution.

Both values of r are very high, but in the case of T_R the significance level of r is less than 95%. In T_F case it is greater than 99%.

Method B

The same procedure described in method A has been applied to SCL determined from sunspot minimum to minimum as well as from maximum to maximum. The multiple regression estimated with SCL from cycle 4 to 20 with the least *Ep* is

$$\mathrm{SCL}(t) = 0.31\ \mathrm{SCL}(t-7) - 0.29\ \mathrm{SCL}\ (t-12) + 10.21 \qquad (2.5)$$

with a correlation coefficient r equal to 0.96 at a significance level α greater than 99%.

Comparison of both methods

Methods A and B are tested comparing observed and predicted SCL values for cycles 21 to 22 – cycles not used in eq. (2.3), (2.4) and (2.5) estimation. SCL determined from the minimum of cycle i to the next minimum, which corresponds to cycle $i+1$, will be called $m(i,i+1)$, and SCL determined from the maximum of cycle i to the maximum of cycle $i+1$, $M(i,i+1)$.

In the case of method A, SCL was calculated as

$$m(i,i+1) = T_R(i)+T_F(i) \tag{2.6}$$

and

$$M(i,i+1) = T_F(i)+T_R(i+1) \tag{2.7}$$

Table I shows together with the experimental values of SCL for cycles 21 to 22, the SCL values estimated with both methods. It is clearly seen that the error of prediction with method B is the smallest.

3. Forecast of SCL for cycles 23 to 29

Method B is chosen to forecast the future SCL values given its lowest *Ep*.

According to eq. (2.5) the best set of lagged values is that composed of $t-7$ and $t-12$. The multiple regression estimated for SCL with the complete SCL record, results in:

$$\mathrm{SCL}(t) = 0.40\ \mathrm{SCL}(t-7) - 0.30\ \mathrm{SCL}(t-12) + 9.24 \tag{3.1}$$

with a correlation coefficient r equal to 0.95

Table I. SCL estimated with methods A – eqs. (2.3) and (2.4) – and B – eq. (2.5) – and the corresponding experimental SCL. The last row shows the error of prediction, *Ep*, for each method.

Period	SCL (experimental)	SCL estimated with method A	SCL estimated with method B
M (20, 21)	11.1	11.2	11.0
m (21, 22)	10.3	9.7	10.2
M (21, 22)	9.7	9.9	10.0
m (22, 23)	10.1	10.9	10.1
Error of prediction		1.05	0.11

Table II. Forecast of SCL for cycles 23 to 29 obtained with method B (eq. 3.1). The first and third columns show the period, and the second and fourth columns show the corresponding SCL forecasted values.

Period	SCL estimated with eq. (3.1)	Period	SCL estimated with eq. (3.1)
M (22, 23)	10.5 ± 0.3	m (26, 27)	10.0 ± 0.3
m (23, 24)	10.0 ± 0.3	M (26, 27)	10.5 ± 0.3
M (23, 24)	10.0 ± 0.3	m (27, 28)	11.0 ± 0.3
m (24, 25)	10.3 ± 0.3	M (27, 28)	10.5 ± 0.3
M (24, 25)	10.3 ± 0.3	m (28, 29)	10.4 ± 0.3
m (25, 26)	9.7 ± 0.3	M (28, 29)	10.5 ± 0.3
M (25, 26)	9.6 ± 0.3	m (29, 30)	10.2 ± 0.3

Table III. Dates of maximum and minimum solar activity for the forecasted cycles 23 to 29 with method B.

Solar Cycle	Date of maximum	Date of minimum
23	2000.1 ± 0.3	2006.8 ± 0.3
24	2010.1 ± 0.3	2017.1 ± 0.3
25	2020.3 ± 0.3	2026.8 ± 0.3
26	2029.9 ± 0.3	2036.7 ± 0.3
27	2040.4 ± 0.3	2047.7 ± 0.3
28	2050.9 ± 0.3	2058.8 ± 0.3
29	2061.4 ± 0.3	2068.3 ± 0.3

and a significance level α greater than 99%.

SCL is directly obtained with eq. (3.1) and the corresponding values are listed in the second column of table II. Table III lists the dates of solar maximum and minimum for the predicted solar cycles.

Figure 1 plots the smoothed SCL record after applying the Gleissberg filter to the SCL series from cycle 4 to cycle 22, plus the predicted values. The Gleissberg filter is a moving average with weights 1, 2, 2, 2, 1 applied separately to the periods from minimum to minimum solar activity and from maximum to maximum, to remove short period variations of an accidental character (Friis-Christensen and Lassen, 1993).

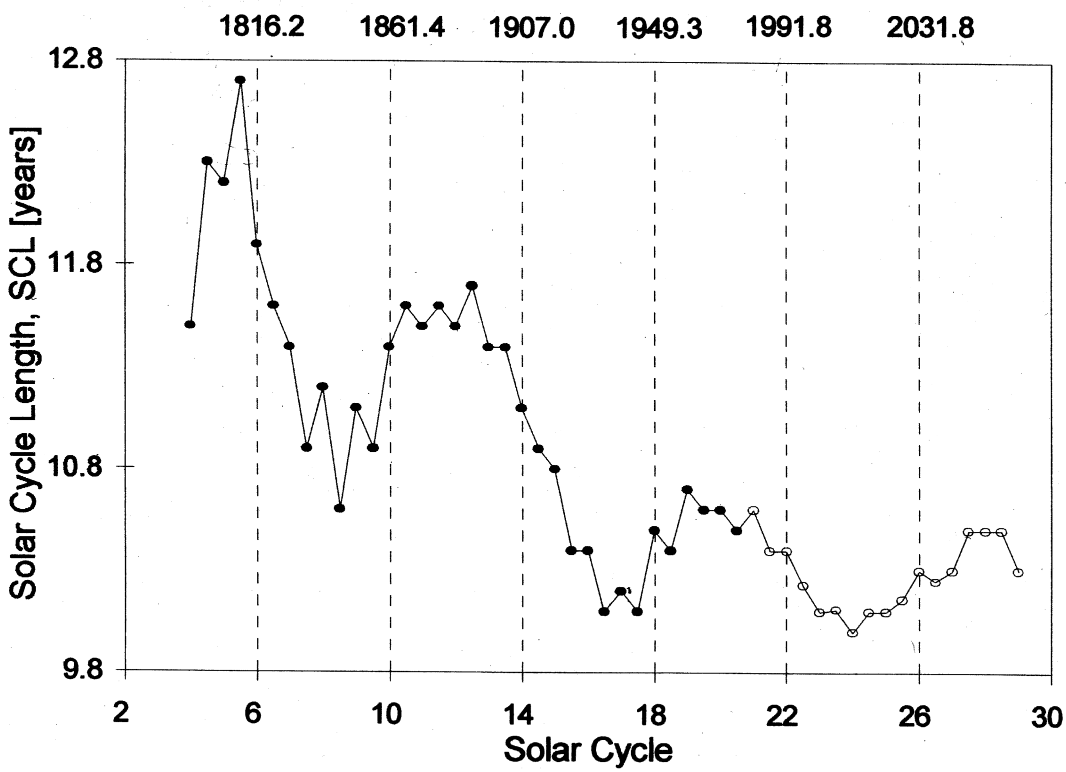

Fig. 1. Solar cycle length, SCL, from cycle 4 to cycle 29 after applying the Gleissberg filter. The filled circles are the experimental values and the empty circles are those forecasted with method B. Forecasted values begin in cycle 21 since from this cycle on, the average procedure involves predicted values. The years at the top of the figure are the central years of the corresponding solar cycle.

4. Discussion

The method applied in this work extends the forecast of SCL until cycle 29 through a pair of lagged values of SCL itself, namely lags 7 and 12.

Since the mean solar cycle length for the period 1780-1997 (solar cycle 4 to 22) is 11.2 years, lag 7 and lag 12 could be taken for periodicities of 78.4 and 134.4 years respectively.

The 78.4 years periodicity is identified with the Gleissberg period of around 80 years, observed in the maximum sunspot number. The second periodicity is close to one of the cycles seen in C^{14} records. The time variation of C^{14} activity seems to reflect changes in solar activity during the past several thousand years (Stuiver, 1961; Damon *et al.,* 1966; Suess, 1965), as far back in time as the age of wood can be determined from its rings.

In fact, changes in the C^{14} activity result primarily from a varying degree of modulation of the galactic cosmic ray flux by the sun. During periods of low solar activity the atmospheric C^{14} was rising, and at times when large sunspots were reported in historical records, C^{14} showed a tendency to decrease.

A record of C^{14} from La Jolla Radiocarbon Laboratory (San Diego, California) has a length of 8405 years extending from 6505 B.C. to the twentieth century. Sonett (1984) analyses the periodogram of this record dividing the sequence into four parts: 5995 B.C.-4005 B.C.; 3995 B.C.-1705 B.C.; 1695 B.C.-705 B.C.; 685 B.C.-1885 A.D. Although his values should be regarded as qualitative, one of the major lines in the periodogram is that with period (134 ± 9) years for the first period, (122 ± 7) years for the second, (153 ± 22) for the third, and (128 ± 6) for the fourth period, which could be identified with the longest periodicity in SCL deduced in this work.

An explanation of this long period given by Sonett (1984) is a possible long-term variation in the suns's convective zone or in the sun's core, in accordance with Hoyt and Schatten (1993) who say «The longer the timescale of the variations, the deeper the likely source for the perturbations will be».

This work forecasts the maximum of solar cycle 23 by the year 2000.1 ± 0.3, in quite good agreement with two independent estimations:

– Shatten *et al.* (1996) using a dynamo theory – based on solar magnetism properties – estimate that the next solar maximum will occur near 2000.3 ± 0.7.

– Lee *et al.* (1995), considering the correlations between total solar irradiance measurements from the Earth Radiation Budget Satellite (ERBS), the Nimbus 7, the Solar Maximum Mission (SMM) and solar magnetic activity, expect that the solar activiy will reach the next maximum level by the year 2000.

From fig. 1 it can be seen that a minimum in the smoothed SCL should be expected by the year 2012. Since the smoothed SCL appears to provide a measure of the solar irradiance (Friis-Christensen and Lassen, 1991) – with short cycles implying high solar activity and long cycles low solar activity – this would mean that a maximum in the total solar irradiance will occur around the year 2012. This forecast agrees quite well with Gilliland's (1982) prediction that a maximum in solar irradiance will occur close to the year 2010. He uses the solar diameter as a proxy data and suggests that the total solar irradiance has been increasing since 1970, when the solar diameter was maximum, and forecasts the minimum of the solar diameter near the year 2010 implying a maximum level in the solar irradiance.

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