Intermittency and Passive Scalar Nature of Electron Density Fluctuations in the High-Latitude Ionosphere at Swarm Altitude

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Key Points:

- The ionospheric electron density fluctuations show an intermittent and passive scalar behavior
- Plasma irregularities exhibit a complex dynamic behavior similar to a turbulent velocity field
- Turbulence is a key point to understand the occurrence of large local electron density gradients

Plain Language Summary

Turbulence phenomena represent a key element for understanding the ionospheric dynamics and have a great impact on the features of both magnetic and electric fields and plasma density. It has been suggested that turbulence can be one of the mechanisms responsible for the generation of ionospheric irregularities, which may strongly affect the satellite navigation, positioning and communication systems. In such a framework the characterization of turbulent fluctuations of ionospheric plasma parameters paves the way for a better modeling of space weather related phenomena.

The ESA-Swarm mission provides a unique opportunity to investigate the relevance of turbulence phenomena in the topside ionosphere. Using data recorded on board Swarm A satellite at high latitudes, the intermittent and passive scalar features of electron density fluctuations are investigated. The obtained results support the idea that plasma density irregularities exhibit a complex dynamic behavior similar to that of a turbulent field. Turbulence is therefore confirmed as a fundamental element to be taken into account in order to fully understand the occurrence of large local electron density gradients.

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Abstract

Some physical processes due to Sun-Earth interaction can influence the configuration and the dynamics of the high-latitude ionospheric plasma, particularly during geomagnetically disturbed periods. A possible consequence of this interaction is the occurrence of turbulent fluctuations that can be observed both in magnetic and electric fields and plasma density. Here, we focus on the intermittent nature of high-latitude ionospheric electron density fluctuations during geomagnetically disturbed periods as observed by one of the satellites of the ESA-Swarm constellation. The most obvious finding emerging from this study is the strong intermittent character of electron density fluctuations and the existence of an agreement between the anomalous scaling features of electron density fluctuations and those expected from a passive scalar quantity in fluid turbulence. This latter result supports the view of a passive scalar behavior of electron density in the high-latitude ionosphere that can have significant implications in the field of Space Weather studies.

1 Introduction

A large and growing body of literature has established that most space plasma processes involving interplanetary, magnetospheric and ionospheric plasma media, exhibit a turbulent and intermittent character (*Bruno and Carbone*, 2013; *Carbone et al.*, 2018; *De Michelis et al.*, 2015; *Kintner and Seyler*, 1985; *Tsunoda*, 1988). Turbulence plays a central role in several physical processes occurring in the interplanetary and near-Earth space regions and affects the plasma transport from the interplanetary space to the Earth's magnetosphere and ionosphere. Concerning the Earth's ionosphere, the turbulent nature of ionospheric plasma can play an important role in the formation and dynamics of plasma irregularities (*Hysell*, 2020). These can have a great impact on those technologies primarily related to telecommunications and satellite-based positioning systems. Indeed, plasma density irregularities can strongly influence the quality of electromagnetic signals that propagate in space. They are expected to be responsible for the ionospheric plasma scintillations, which strongly affect the Global Navigation Satellite System (GNSS), e.g., the Global Positioning System (GPS), and radio propagation (*Kelley*, 1990; *Knepp and Coleman*, 2020; *Wheelon*, 1960). Thus, it is of great importance to study and provide a better understanding of the processes that can affect ionospheric magnetized plasma density. In the past, it was clearly shown how in the high-latitude polar ionosphere the electrostatic turbulent fluctuations can significantly influence the plasma features, and in particular the electron density generating irregularities in the ionospheric medium (*Kintner and Seyler*, 1985; *Pécseli*, 2015). As clearly stated by *Kintner and Seyler* (1985), the ionospheric electron density (and more in general the plasma density) may respond to the generation of electrostatic turbulence as a passive scalar quantity. In such a framework, the study of the features of electron density fluctuations can provide information on the physical processes responsible for the rise of turbulence in the ionospheric plasma. Several previous studies suggested different mechanisms for the rise of turbulence in the high-latitude ionosphere, such as, for instance, the gradient drift instability (GDI), the convective current instability (CCI), the Kelvin-Helmholtz instability (KHI), etc. (see, e.g., *Basu et al.*, 1990; *Carlson et al.*, 2007; *Cerisier et al.*, 1985; *Keskinen et al.*, 1988; *Mounir et al.*, 1991). Thus, the investigation of the scaling features and intermittency of electron density fluctuations in the ionosphere are of central interest to unveil the origin of the plasma irregularities. The characterization of these features during geomagnetically disturbed periods is the central issue of this work.

The statistical properties of fluctuations in turbulent media (fluids and/or plasmas) can be investigated via the classical approach of scaling analysis of structure functions (see, e.g., *Frisch*, 1995; *Schmitt and Huang*, 2016), which are particularly suitable in studying scaling features dealing with turbulent signals and in investigating the occurrence of intermittency (anomalous scaling features and multifractality).

Given a time series $\theta(t)$, the q-th order generalized structure function at scale τ, $S_q(\tau)$, is defined as

$$S_q(\tau) = \langle |\theta(t+\tau) - \theta(t)|^q \rangle, \tag{1}$$

where the angular brackets $< ... >$ correspond with an ensemble average. In the case of scale-invariant signals/media the S_q behaves like a power law, being $S_q(\tau) \propto \tau^{\zeta(q)}$ where $\zeta(q)$ is the corresponding scaling exponent. This is, for instance, what can be observed analyzing fluctuations in fluid and magnetohydrodynamic turbulence in the inertial range. The scaling properties of observed fluctuations and the set of scaling exponents $\zeta(q)$ allow characterizing the turbulence and quantifying the degree of intermittency, offering the possibility to discern among all the different models available to reproduce the observed scalings (*Frisch*, 1995; *Warhaft*, 2000). For example, according to the Kolmogorov's theory (K41) of homogeneous and isotropic turbulence, the scaling exponent associated with the first-order structure function is expected to be $\zeta(1) = 1/3$ in the inertial range and the scaling exponents associated with higher-order structure functions are expected to scale linearly with the moment order q (*Frisch*, 1995; *Schmitt and Huang*, 2016) according to the following relation:

$$\zeta(q) = q\zeta(1) = \frac{q}{3}. \tag{2}$$

When Eq. (2) holds, one scaling exponent is enough to characterize the scaling features of fluctuations at all orders, and the employed K41-like models describe a mono-fractal behavior. However, real observations often reveal a departure from the linear dependence of scaling exponents $\zeta(q)$ on the moment order q (*Frisch*, 1995; *Schmitt and Huang*, 2016). In this case, the system is characterized by anomalous scaling (multifractal features), which is the evidence of an intermittency phenomenon (*Frisch*, 1995). Of course, the higher the deviation from the straight line in Eq. (2), the stronger the degree of intermittency of the system.

The study of transport and mixing properties of vector and scalar fields, passively advected by the turbulent flow, is of great interest in fluid turbulence. For instance, the behavior of temperature and density fluctuations in a turbulent flow has been extensively investigated (*Falkovich et al.*, 2001; *Falkovich and Sreenivasan*, 2006; *Shraiman and Siggia*, 2000; *Warhaft*, 2000) and modeled via the advection-diffusion equation,

$$\frac{\partial \theta}{\partial t} + (\mathbf{v} \cdot \nabla)\theta = \chi \nabla^2 \theta + f(\mathbf{r}, t), \tag{3}$$

where θ is the passive scalar quantity, $\mathbf{v}(\mathbf{r}, t)$ is the velocity field which transports the passive scalar quantity, χ is the diffusivity and f is a forcing term necessary to attain a stationary state. According to this equation the passive scalar is a quantity driven by the flow and such to not affect the flow dynamics. Despite Eq. (3) is linear, complex features may be generated due to the coupling between θ and $\mathbf{v}$. In particular, in the case of passive quantities transported by turbulent flows it has been shown that, conversely to the case of zero molecular diffusion which implies a rigid advection of the scalar field, a small amount of diffusivity gives rise to a different behavior (*Sreenivasan*, 1991). Indeed, the observed behavior is similar to what happens in a turbulent flow in the inertial range, where scalar fluctuations cascade from large to small scales at a constant scale-independent rate ϵ_ϕ.

Although the Kolmogorov-Obukhov-Corrsin (KOC) theory (*Corrsin*, 1951; *Obukhov*, 1949) for passive scalar advection/diffusion predicts a simple scaling for the scalar increment structure functions $S_q(\delta r) = (S_2(\delta r))^{q/2}$ (where δr is a specific spatial scale), deviations from this simple scaling can be observed. They are generally interpreted as due to intermittency effects. The studies of the scaling features of passive scalar quantities (such as, for instance, the temperature) in turbulent fluid flows (see e.g. Ref. *Antonia et al.* (1984); *Ruiz-Chavarria et al.* (1996)) evidenced that the anomalous scaling of a passive quantity increments is more pronounced than that of the velocity field. This strong intermittent character of passive quantities is generally discussed in terms of a double intermittency correction for kinetic energy and passive quantity fluctuations dissipation rates (*Ruiz-Chavarria et al.*, 1996).

Here, the scaling features of electron density and its passive scalar nature are investigated applying the structure function analysis on the ionospheric electron density measured at high latitudes by one of the three satellites of the ESA-Swarm constellation during two geomagnetically disturbed periods. In particular, a comparison between the scaling features of electron density and those of passive scalar quantities in turbulent media is presented. A brief discussion on the relevance of this study for the magnetosphere-ionosphere coupling and its impact on the field of Space Weather studies are also provided.

2 Data set

To investigate the scaling features and the intermittent nature of the high-latitude electron density fluctuations we use the plasma measurements provided by the Langmuir probes on-board Swarm A satellite. This satellite, which is one of the three satellites of Swarm constellation, flies in a near-polar orbit at an altitude of about 460 km allowing the study of those processes that mainly occur in the topside F region of the ionosphere. We consider local electron density, n_e, measurements collected during the high-latitude crossings of Northern ionosphere (QD Lat $> 60^\circ$ N) in two different geomagnetically disturbed days (17 March 2015 and 25 October 2016). Each crossing lasts approx. 20 min (i.e., approx 2k points) and considering that Swarm A completes 15 orbits in a day, the measurement dataset consists of $\sim 60k$ points. Here, we use the quasi-dipole (QD) magnetic reference system (*Laundal and Richmond*, 2017). This latitude range has been chosen to focus on the auroral oval and polar cap regions where plasma precipitation is expected to greatly increase during geomagnetically disturbed periods. The two selected days refer to periods of mid-high geomagnetic disturbance, being the root-mean squared value of the auroral electrojet geomagnetic index (AE) (*Davis and Sugiura*, 1966) for 17 March 2015 and 25 October 2016 equal to $\mathrm{AE}_{RMS} \simeq 830$ nT and $\mathrm{AE}_{RMS} \simeq 750$ nT, respectively. Electron density measurements at a rate of 2 Hz have been selected from the ESA ftp repository (*ftp://swarm-diss.eo.esa.int*).

Figure 1 shows a sample of the electron density, n_e, measurements and the corresponding increments δn_e (where $\delta n_e(\delta t) = n_e(t+\delta t) - n_e(t)$) calculated for $\delta t = 0.5$ s relative to one of the high-latitude crossings of the polar ionosphere on 17 March 2015. Similar behaviors are observed for the other considered crossings. A clear increment of electron density fluctuation amplitudes can be observed over the high latitudes.

3 Analysis and Results

We start our study analyzing the power spectral features of electron density fluctuations.

Figure 2 displays the average Power Spectral Densities (PSDs) of the electron density n_e measured during the high-latitude crossings of the northern ionosphere in the two selected periods. Each PSD is normalized by dividing it for the variance $\sigma^2_{n_e}$ of the electron density n_e of the correspondent period. Both PSDs are characterized by a power-law behavior, $S(f) \sim f^{-\alpha}$, with a scaling exponent $\alpha \sim 5/3$ over more than two orders of magnitude. These spectral features can be considered the signature of a turbulent character of the observed fluctuations, and agree with previous findings (see, e.g.:

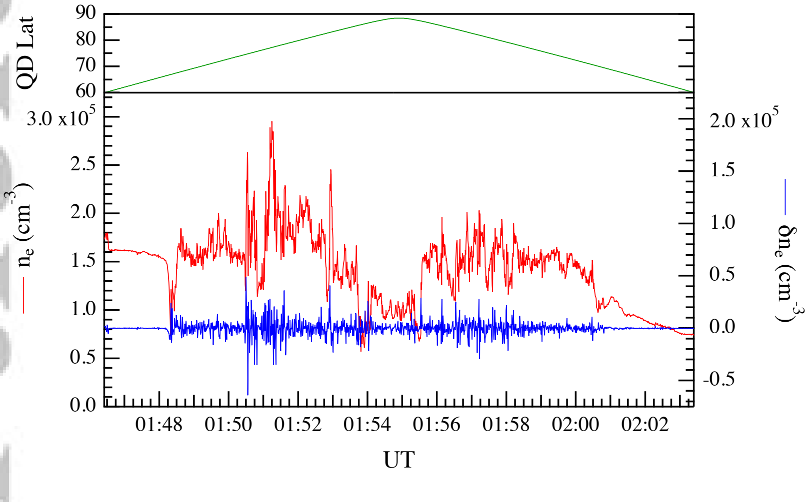

Figure 1. A sample of electron density, n_e, and electron density increments δn_e at the scale $\delta t = 0.5$ s during one of the high-latitude crossings of the Northern ionosphere on 17 March 2015. The upper panel shows the quasi-dipole magnetic latitude (QD Lat).

Basu et al., 1990; *Kelley et al.*, 1980; *Kintner and Seyler*, 1985; *Spicher et al.*, 2015, and references therein) and theoretical predictions (*Kintner and Seyler*, 1985). Indeed, although the analysis is done in the temporal domain, we can assume that the observed fluctuations are mainly spatial being the low-frequency temporal fluctuations principally the effect of Doppler-shifted and stationary spatial variations (*Basu et al.*, 1990; *Consolini et al.*, 2020). Under this assumption the observed PSDs are compatible with what is expected in the case of fluid strong turbulence at least for wavenumbers $k = f/v_s$ in the range $[3, 60] \cdot 10^{-3}$ km, being the satellite velocity $v_s \sim 7.8$ km/s.

Considering that the PSD spectral exponents are in the range $\alpha \in (1, 2)$, the signal is clearly nonstationary but its increments are stationary (weak stationarity) (*Consolini et al.*, 2020; *Davis and Sugiura*, 1966) and consequently the requirements of structure function analysis are satisfied.

Another interesting feature of electron density fluctuations/increments, $\delta n_e(\delta t)$, is their non-Gaussian statistics at short timescales ($\delta t < 50$ s). Figure 3 shows the Probability Density Functions (PDFs) of the electron density increments normalized to the standard deviation ($\sigma_{\delta n_e}$) for timescales $\delta t < 50$ s as obtained considering the whole dataset. In particular, the normalization of the increments to the standard deviation is necessary to remove the dependence on the different geomagnetic activity level. The PDFs exhibit a large departure of δn_e from the Gaussian statistics, being the PDFs characterized by a leptokurtic shape (i.e., a distribution with a kurtosis higher than 3) (*De-Carlo*, 1997). The PDFs of the electron density increments are analogous to those found by evaluating the PDFs of velocity increments in turbulent fluid flows in the low-end inertial range (*Frisch*, 1995), where large departures from Gaussian statistics are observed at small scales towards the dissipation scale. Furthermore, the PDFs display a shape that is dependent on the timescale δt, i.e., the shape of the PDF is not scale invariant (PDFs collapsing is poor). This feature is a signature of intermittency, as it occurs in turbulence. We will return on this point later in the discussion.

We first investigate the occurrence of scaling features in structure functions by studying their dependence on the moment order q. The left panel of Figure 4 displays the dependence of q^{th}-order structure functions of electron density on time increments δt. We limit the investigation of scaling features to moment orders $q \leq 4$ due to the available

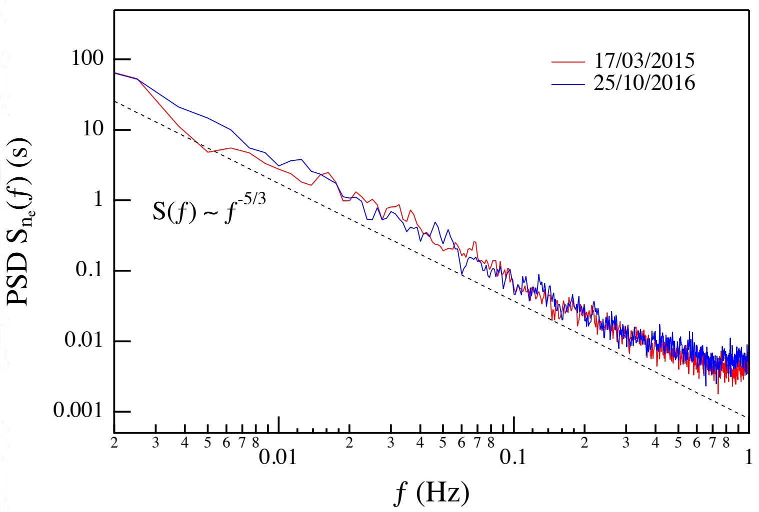

Figure 2. The average Power Spectral Density of the electron density n_e for the two selected periods. The dashed line is a power law with a scaling exponent $-5/3$, plotted for comparison.

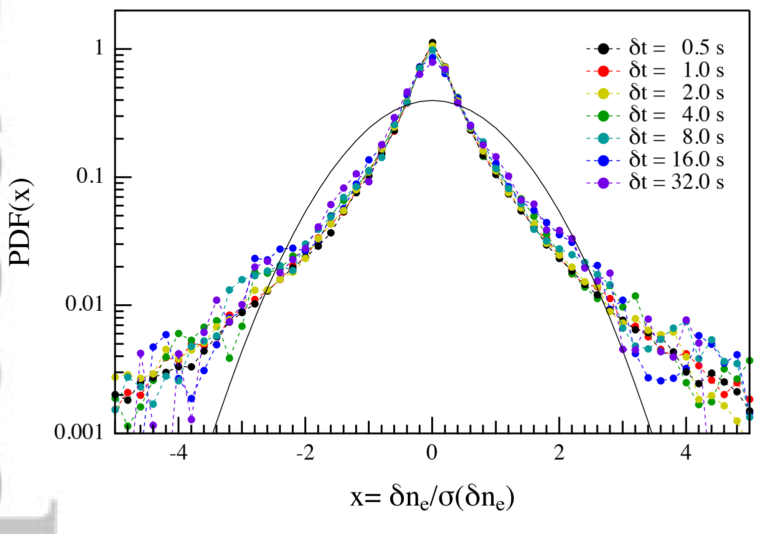

Figure 3. The PDFs of the electron density increments, δn_e, for 0.5 s $\leq \delta t \leq$ 32 s. Data have been scaled to the respective standard deviation. The black curve is a unit-variance Gaussian distribution for reference.

number of measurements (see, e.g., *Dudok de Wit*, 2004). A clear power-law scaling, $S_q(\delta t) \sim \delta t^{\zeta(q)}$ is found over more than 2 orders of magnitude, supporting that electron density fluctuations/increments have a self-affine structure over a wide range of scales, and thus they show scale-invariance.

Let us now move to the relative scaling of the q^{th}-order structure function, $S_q(\delta t)$, versus the 2^{nd}-order one, $S_2(\delta t)$, which is expected to scale as follows:

$$S_q(\delta t) = (S_2(\delta t))^{\xi(q)}\,, \tag{4}$$

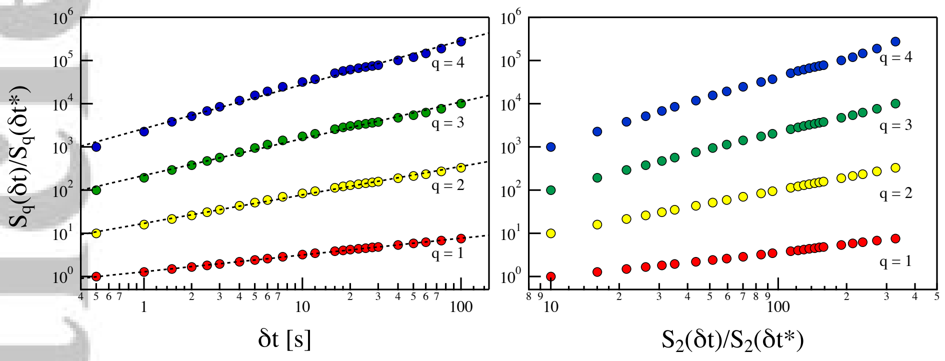

Figure 4. Left Panel: Scaling of structure function, $S_q(\delta t)$, as a function of δt for $q = 1, 2, 3,$ and 4. Dashed lines are nonlinear power-law best fits. Right Panel: The relative scaling of the q^{th}-order structure function, $S_q(\delta t)$, as a function of the 2^{nd}-order one, $S_2(\delta t)$. Structure functions are normalized to the corresponding value for timescale $\delta t^* = 0.5$ s and scaled by a factor 10^{q-1} for convenience.

and according to *Benzi et al.* (1993) is named Extended Self-Similarity (ESS) analysis. For mono-scaling (monofractal) signals $\xi(q)$ is expected to scale linearly with the moment order q, being $\xi(q) = \zeta(q)/\zeta(2) = q/2$. Differently, anomalous scaling features, i.e., a non-linear dependence on q, are expected in the case of multi-scaling (multifractal) signals, i.e., signals whose scaling features are no-longer homogeneous but acquire a local dependence with a spreading of local scaling exponents (*Benzi et al.*, 1984; *Schmitt and Huang*, 2016).

The right panel of Figure 4 displays the relative scaling of the q^{th}-order structure function, $S_q(\delta t)$ of the electron density increments $\delta n_e(\delta t)$ with $\delta t \in [0.5, 100]$ s versus the 2^{nd}-order one, $S_2(\delta t)$. The observed dependence of q^{th}-order structure functions on the 2^{nd}-order one agrees with the predictions provided by Eq. 4 over 1.5 decades.

To characterize the relative scaling features we evaluate the relative scaling exponents $\xi(q)$ for the q^{th} order electron density structure functions versus the 2^{nd}-order one. The best fit is a power-law dependence between $S_q(\delta t)$ and $S_2(\delta t)$ for timescales in the interval $\delta t \in [0.5, 50.0]$ s, which correspond to spatial scales in the range $[4, 400]$ km, taking into account the satellite speed and assuming the fluctuations to be mainly spatial (see the previous discussion on PSD). Figure 5 shows the behavior of the relative scaling exponents, $\xi(q)$, as a function of the moment order q. We considered also non-integer qs in order to better trace the scaling exponents trend. A clear nonlinear dependence of these exponents is found, suggesting that we are in presence of anomalous scaling features of the structure functions. This is the signature of the occurrence of intermittency in the case of electron density fluctuations.

In order to unveil if the observed anomalous scaling can have the same character of that observed in the case of passive scalar quantities in fluid turbulence, we have overplotted in Figure 5 the relative scaling exponents computed using results on scaling features of temperature fluctuations in fluid turbulence, which has been proven to be a passive scalar quantity. The values of the temperature scaling exponents come from the work by *Ruiz-Chavarria et al.* (1996), and they are listed in their Table 2. The agreement is excellent. Conversely, the observed anomalous scaling features seem to be different from those observed in *Spicher et al.* (2015). This could be due to the different range of scales, where the anomalous scaling features are investigated, being the analysis in *Spicher et al.* (2015) done at smaller scales, $\delta t \leq 0.1$ s.

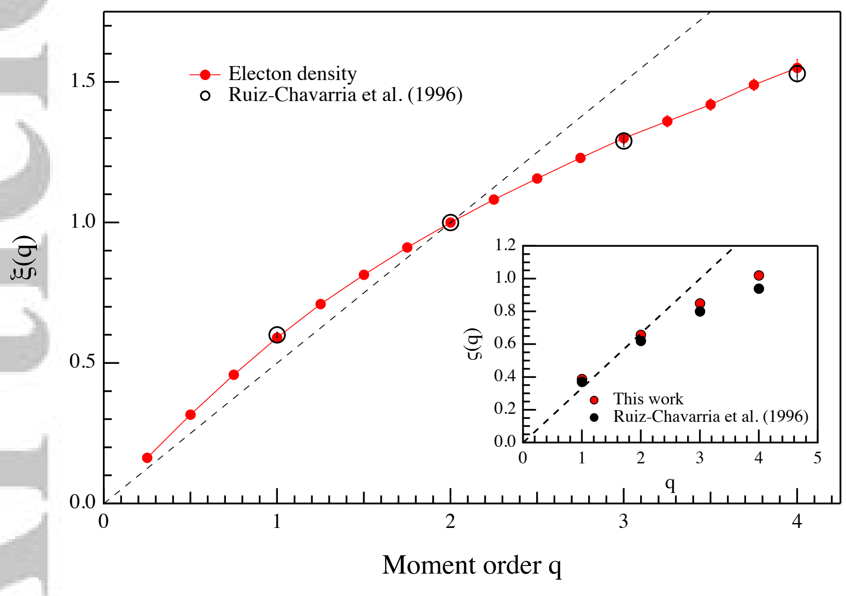

Figure 5. The relative scaling exponents $\xi(q)$ of the q^{th}-order structure function, $S_q(\delta t)$, of the electron density measurements. Empty circles refer to the same quantities computed using values from *Ruiz-Chavarria et al.* (1996). The dashed line shows the expected behavior in the case of absence of intermittency. The inset shows the comparison among the observed scaling exponents $\zeta(q)$ of the q^{th}-order structure function for this work and those reported in *Ruiz-Chavarria et al.* (1996); dashed line is Kolmogorov's K41 scaling, $\zeta(q) = q/3$.

To better underline the agreement between our results and those from *Ruiz-Chavarria et al.* (1996), we report in Table 1 the relative scaling exponents $\xi(q)$ of electron density fluctuations and the corresponding values computed from *Ruiz-Chavarria et al.* (1996). The agreement between the relative scaling exponents is very striking, suggesting that the intermittency observed in the case of electron density fluctuations is of the same universality class of intermittency observed in the case of a passive scalar quantity in fluid turbulence. We remark that up to the 4^{th} moment order the measured values of the scaling exponents, $\zeta(q)$, display a quasi-universal character as clearly shown in *Warhaft* (2000) (see Figure 11 therein) where data from different experiments are compared.

q	$\xi(q)$ This work	$\xi(q)$ from *Ruiz-Chavarria et al.* (1996)
1	0.59 ± 0.01	0.60 ± 0.01
2	1	1
3	1.30 ± 0.02	1.29 ± 0.02
4	1.55 ± 0.03	1.53 ± 0.03

Table 1. The relative scaling exponents, $\xi(q)$, of electron density fluctuations as obtained in the current work, compared to those computed from data in *Ruiz-Chavarria et al.* (1996) for temperature fluctuations. The value of the 2^{nd} order structure function scaling exponent in our case is $\zeta(2) = [0.658 \pm 0.008]$.

4 Conclusions

The main findings of our analysis are: 1) the signature of the occurrence of intermittency in electron density fluctuations in the auroral oval ionospheric region, 2) electron density fluctuations have the same universality class of a passive scalar quantity in fluid turbulence. To our knowledge this is the first observational evidence of the passive scalar nature of electron density in high-latitude ionospheric regions.

Turbulence has been claimed to be a very relevant phenomenon in the ionospheric medium. It can strongly affect the plasma density and, in particular, the electron density generating strong irregularities that may have a great impact on satellite navigation, positioning and communication systems. Several works (see, e.g.: *Kintner and Seyler*, 1985; *Pécseli*, 2015) have stressed the role that fluid-like turbulence generated by $\mathbf{E}\times\mathbf{B}$ gradient drift, current convective, shear flow and/or Kelvin-Helmholtz instabilities can play in generating plasma irregularities. Nevertheless it is necessary to take into consideration that also the intermittency of passive scalar quantities in fluid turbulence can be very relevant and generally stronger than that observed in the velocity field (see, e.g.: *Kraichnan*, 1994; *Ruiz-Chavarria et al.*, 1996; *Warhaft*, 2000).

Thus, our results on the intermittency and the passive scalar nature of the ionospheric electron density fluctuations strongly support the idea that turbulence is probably the most relevant phenomenon capable of generating plasma irregularities, i.e., multiscale patchy structures responsible for the occurrence of ionospheric scintillations and radio propagation anomalies in the ionospheric medium. The observed universality character of electron density scaling features and passive scalars in fluid turbulence also provides an indication that plasma velocity field is turbulent in the high-latitude ionosphere. Moreover, the great similarity between the observed scaling exponents of the electron density structure functions and those of passive scalars in fluid turbulence suggests that the turbulent nature of plasma velocity field may be fluid. This point supports the idea that this turbulent field may be generated by strong $\mathbf{E}\times\mathbf{B}$ drift velocity shears/gradients.

Further studies are clearly necessary in order to put our findings in a general theory for the formation of ionospheric plasma irregularities.

Acknowledgments

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References

Antonia R.A., E.J. Hopfinger, Y. Gagne and F. Anselmet (1984), Temperature structure functions in turbulent shear flows, Phys. Rev. A, 30, 2704.

Basu S., et al. (1990), Plasma Structuring by the Gradient Drift Instability at High Latitudes and Comparison With Velocity Shear Driven Processes, J. Geophys. Res., 95(A6), 7799.

Benzi, R., Paladin, G., Vulpiani, A. & Parisi, G. (1984), On the multifractal nature of fully developed turbulence and chaotic systems, Journal of Physics A, 17, 3521-3531.

Benzi, R., Ciliberto, S., Tripiccione, R., Baudet, C., Massaioli, F. & Succi, S. (1993), Extended self-similarity in turbulent flows, Phys. Rev. E, 48, R29-R32.

Bruno R. and V. Carbone (2013), The Solar Wind as a Turbulence Laboratory, Living Rev. Sol. Phys., 10, 2.

Carbone F., et al. (2018), Arbitrary-order Hilbert Spectral Analysis and Intermittency in Solar Wind Density Fluctuations, Astrophys. J., 859:27, doi: 10.3847/1538-4357/aabcc2.

Carlson, H. C., Pedersen, T., Basu, S., Keskinen, M, & Moen, J. (2007) Case for a new process, not mechanism, for cusp irregularity production, J. Geophys. Res.: Space Physics, 112, A11304, doi:10.1029/2007JA012384.

Cerisier, J. C., J. J. Berthelier, & C. Beghin (1985), Unstable density gradients in the high-latitude ionosphere, Radio Science, 20, 755? 761, doi:10.1029/RS020i004p00755.

Consolini G., et al. (2020), On the multifractal features of low-frequency magnetic field fluctuations in the field-aligned current ionospheric polar regions: Swarm observations, J. Geophys. Res.: Space Phys., 125, e2019JA027429, doi: 10.1029/2019JA027429.

Corrsin S. (1951), On the Spectrum of Isotropic Temperature Fluctuations in an Isotropic turbulence, J. Appl. Phys., 22, 469.

Davis A., A. Marshak, W. Wiscombe, W. and R. Cahalan, R. (1994), Multifractal characterizations of nonstationarity and intermittency in geophysical fields: Observed, retrieved, or simulated, J. Geophys. Res.: Atmosphere, 99(D4), 8055.

Davis T. N. and M. Sugiura (1966), Auroral electrojet activity index AE and its universal time variations, J. Geophys. Res., 71, 785.

DeCarlo L.T. (1997), On the meaning and use of kurtosis, Psychological Methods, 2, 292-307.

De Michelis P., G. Consolini, and R. Tozzi (2015), Magnetic field fluctuation features at Swarm's altitude: A fractal approach, Geophys. Res. Lett., 42, 31003105, doi:10.1002/2015GL063603.

Dudok de Wit, T. (2004) Can high-order moments be meaningfully estimated from experimental turbulence measurements?, Phys. Rev. E, 70, 055302, doi: 10.1103/PhysRevE.70.055302.

Falkovich G., K. Gawedzki, and M. Vergassola (2001), Particles and fields in fluid turbulence, Rev. Mod. Phys., 73, 913.

Falkovich G. and K.R. Sreenivasan (2006), Lessons from hydrodynamic turbulence, Phys. Today, 59, 43.

U. Frisch (1995), *Turbulence: The legacy of A.N. Kolmogorov*, Cambridge University Press, Cambridge, UK.

D.L. Hysell, in *The Dynamical Ionosphere: A System Approach to Ionospheric Irregularity*, M. Materassi et al. (eds.), Elsevier, Amsterdam (2020)

Kelley M.C. , et al. (1980), Simultaneous rocket probe, scintillation, and incoherent scatter radar observations of irregularities in the auroral zone ionosphere, Radio Sci., 15, 491.

Kelley M. C. (2009), The Earth's Ionosphere Plasma Physics and Electrodynamics, 2nd ed., Elsevier, Amsterdam.

Keskinen, M. J., Mitchell, H. G., Fedder, J. A., Satyanarayana, P., Zalesak, S. T., & Huba, J. D. (1988) Nonlinear evolution of the Kelvin-Helmholtz instability in the high-latitude ionosphere, J. Geophys. Res.-Space, 93, 137?152, doi:10.1029/JA093iA01p00137.

Kintner P. M. Jr., and C. E. Seyler (1985), The status of observations and theory of high latitude ionosphere and magnetospheric plasma turbulence, Space Sci. Rev., 41, 91.

D.L. Knepp and C.J. Coleman (2020), in *The Dynamical Ionosphere: A System Approach to Ionospheric Irregularity*, M. Materassi et al. (eds.), Elsevier, Amsterdam

Kraichnan R.H. (1994), Anomalous scaling of a randomly advected passive scalar, Phys. Rev. Lett., 72, 1016.

Laundal, K.M. & Richmond, A.D. (2017), Magnetic Coordinate Systems, Space Sci. Rev., 206, 27.

Mounir, H., Berthelier, A., Cerisier, J. C., Lagoutte, D., & Beghin, C. (1991) The small-scale turbulent structure of the high latitude ionosphere - Arcad-Aureol-3 observations, Annales Geophysicae, 9, 725-737.

Obukhov A.M. (1949), Structure of the temperature field in turbulent flows, Izv. Akad.Nauk. SSSR, Geogr. Geo z., 13, 58.

Pécseli H.L. (2015), Spectral properties of electrostatic drift wave turbulence in the laboratory and the ionosphere, Ann. Geophys., 33, 875.

Ruiz-Chavarria G., C. Baudet, and S. Ciliberto (1996), Scaling laws and dissipation scale of a passive scalar in fully developed turbulence, Physica D, 99, 369.

F.G. Schmitt and Y. Huang (2016), *Stochastic Analysis of Scaling Time Series: From Turbulence Theory to Applications*, Cambridge University Press, Cambridge, UK

Shraiman B.I. and E.D. Siggia (2000), Scalar turbulence, Nature, 405, 639.

Spicher A., W. J. Miloch, L. B. N. Clausen and J. I. Moen (2015), Plasma turbulence and coherent structures in the polar cap observed by the ICI-2 sounding rocket, J. Geophys. Res. Space Physics, 120, 110,978, doi:10.1002/2015JA021634.

Sreenivasan K.R. (1991) On local isotropy of passive scalar in turbulent shear flows, Proc. R. Soc. Lond., 434, 165-182.

Tsunoda R.T. (1988), High-latitude F region irregularities: A review and synthesis, Rev. Geophys., 26, 719.

Warhaft Z. (2000), Passive scalars in turbulent flows, Ann. Rev. Fluid. Mech., 32, 203.

Wheelon A.D. (1960), Relation of Turbulence Theory to Ionospheric Forward Scatter Propagation Experiments, J. Res. National Bureau of Standards, 64D, 301.