# Modelling temporal uncertainty and eruptive vent clustering at Campi Flegrei caldera (Italy)

A. Bevilacqua[1, 2], F. Flandoli[3], R.Isaia[4], A. Neri[1], S. Vitale[5] and D. Quaranta[3].

(1) Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Pisa, Pisa, Italy
(2) Scuola Normale Superiore, Pisa, Italy
(3) Università di Pisa, Dip.to di Matematica, Pisa, Italy
(4) Istituto Nazionale di Geofisica e Vulcanologia, Osservatorio Vesuviano, Napoli, Italy
(5) Università di Napoli Federico II, Dip.to di Scienze della Terra, dell'Ambiente e delle Risorse, Napoli, Italy

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# Outline of the presentation

- Campi Flegrei caldera
- Research objective
- Model of epistemic uncertainty on the eruptive record
- Self-exciting probabilistic model for vent clustering
- Results of the models and their sensitivity to different volcanological assumptions
- Summary and conclusions

# Campi Flegrei caldera (Italy)

The Campi Flegrei caldera is a volcanic area with a diameter of about 12 km and the town of Pozzuoli at its center. In the last 15 ka, intense and mostly explosive volcanism and deformation have occurred within and along the caldera boundaries.

Eruptions were closely spaced in time, over periods from a few centuries to a few millennia, with periods of quiescence lasting several millennia.

Activity has been subdivided into three distinct epochs:
Epoch I [15 - 10.6 ka BP]
Epoch II [9.6 - 9.1 ka BP]
Epoch III [5.5 - 3.8 ka BP]
Monte Nuovo [477 years BP]
(estimates from Smith et al. [2011]).


**Figure 1.** The Campi Flegrei caldera and surrounding areas (including the city of Naples on the west) [Bevilacqua et al., 2015].

# Research objective

We produce a long-term probabilistic temporal model for vent opening at Campi Flegrei (CF) based on available information concerning past activity, quantifying its uncertainty and developing a probability function to reproduce the main system features observed.

The study is aimed at further developing the results of Bevilacqua et al. [2015] and Neri et al. [2015], by including temporal scales into the spatial hazard assessments produced.

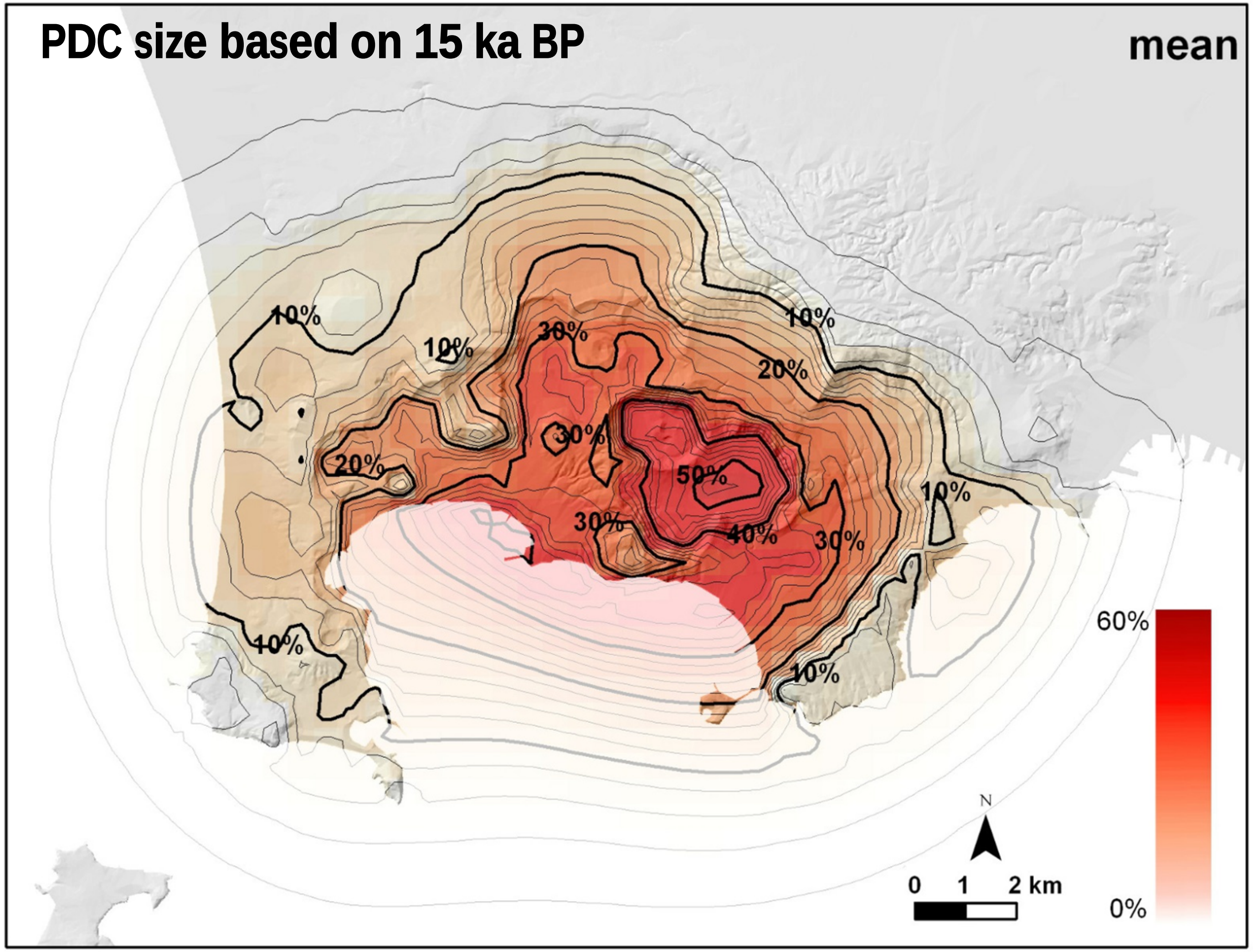



**Figure 2.** Mean probability map of PDC invasion hazard, modified from Neri et al. [2015]. Contours and colours indicate the percentage probability of PDC invasion conditional on the occurrence of an explosive eruption.

# Methodology

1) A probability model for epistemic uncertainty on past record, concerning the uncertainty estimation on the sequence of times, the location and the erupted volume of past events.

2) A probability model for the representation and replication of the main eruptive activity features in time-space, such as the vent clustering, and incorporating the effects of the sources of epistemic uncertainty considered.

The two models are linked through a nested Monte Carlo simulation by assuming to calculate the parameters of model (2) that maximize the likelihood of each sample of model (1).

Indeed our approach is doubly stochastic, i.e. each sample is made in two steps:
A) the random choice of the epistemic uncertainty,
B) the random determination of the observables of interest conditional on A).

# Uncertainty affecting past vents locations

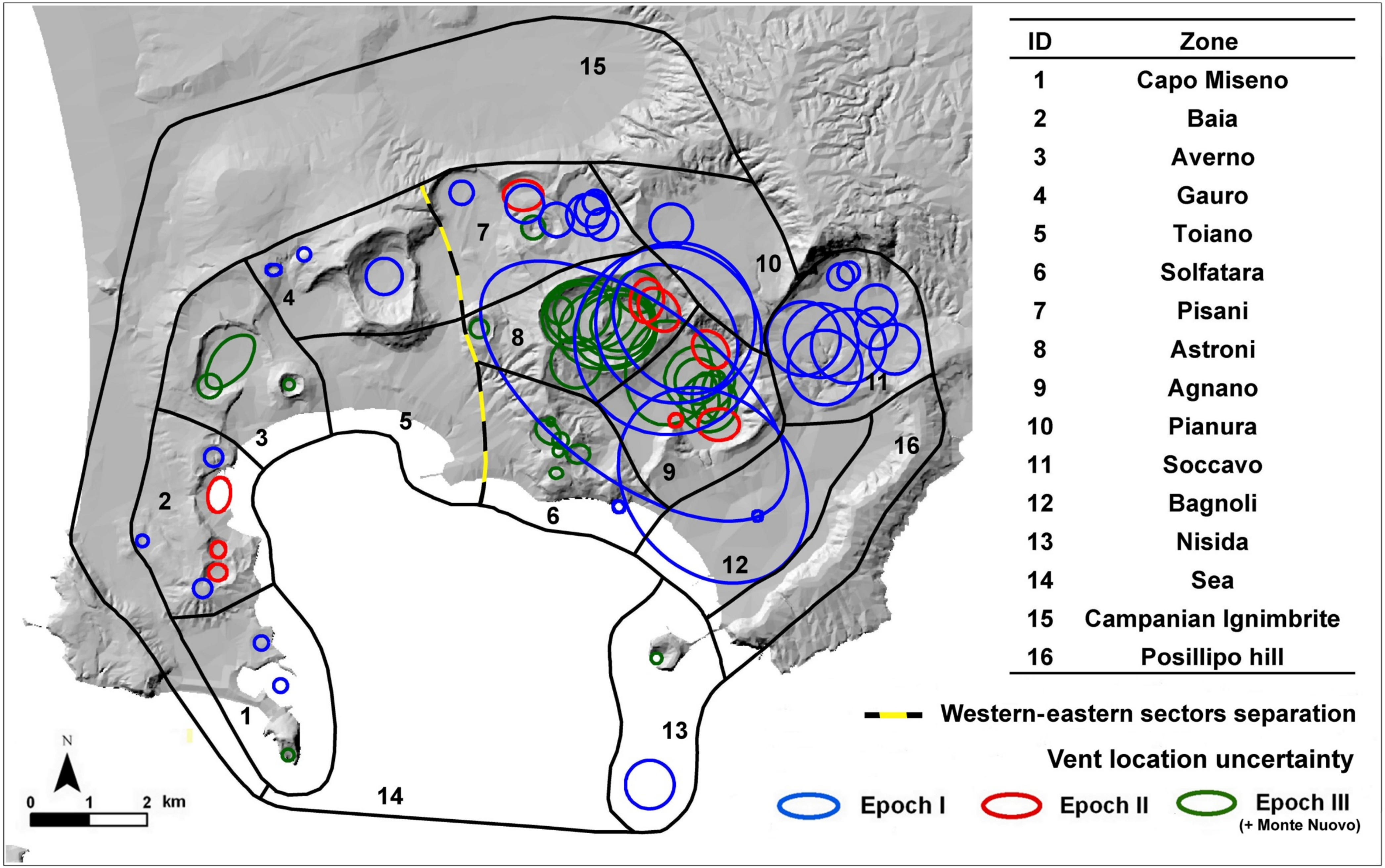



**Figure 3.** Partitioning of the caldera in 16 zones. The colours of the ellipses correspond to the epoch of activity. The yellow dashed line separates eastern and western sectors. (more details in Bevilacqua et al. [2015]).

# Uncertainty affecting eruption times and volumes

**EPOCH III**

| ID | Name | Time 2.5%ile | Time 97.5%ile | VDRE (hm3) 5%ile | VDRE (hm3) | VDRE (hm3) 95%ile |
|---|---|---|---|---|---|---|
| 1 | Agnano 1 | 5266 | 5628 | 10 | 20 | 30 |
| 2 | Agnano 2 | - | - | 5 | 10 | 15 |
| 3 | Averno 1 | 5064 | 5431 | 10 | - | 100 |
| 4 | Agnano 3 | - | - | 95 | 190 | 285 |
| 5 | Cigliano | - | - | 25 | 50 | 75 |
| 6 | Pigniatiello 2 | - | - | 10 | 20 | 30 |
| 7 | Capo Miseno | 3259 | 4286 | 10 | 20 | 30 |
| 8 | Monte Sant'Angelo | 4832 | 5010 | 100 | - | 300 |
| 9 | Paleoastroni 1 | 4745 | 4834 | 25 | 50 | 75 |
| 10 | Paleoastroni 2 | 4712 | 4757 | 100 | - | 300 |
| 11 | Agnano Monte Spina | 4482 | 4625 | 425 | 850 | 1275 |
| 12 | St. Maria delle Grazie | 4382 | 4509 | 10 | - | 100 |
| 13 | Olibano lava dome | - | - | 0 | - | 10 |
| 14 | Paleoastroni 3 | - | - | 10 | 20 | 30 |
| 15 | Solfatara lava dome | - | - | 0 | - | 10 |
| 16 | Olibano tephra | - | - | 10 | - | 100 |
| 17 | Accademia lava dome | - | - | 0 | - | 10 |
| 18 | Solfatara | 4181 | 4386 | 15 | 30 | 45 |
| 19 | Averno 2 | * | * | 35 | 70 | 105 |
| 20 | Astroni 1 | 4153 | 4345 | 30 | 60 | 90 |
| 21 | Astroni 2 | - | - | 10 | 20 | 30 |
| 22 | Astroni 3 | - | - | 80 | 160 | 240 |
| 23 | Astroni 4 | - | - | 70 | 140 | 210 |
| 24 | Astroni 5 | - | - | 50 | 100 | 150 |
| 25 | Astroni 6 | - | - | 60 | 120 | 180 |
| 26 | Astroni 7 | 4098 | 4297 | 35 | 70 | 105 |
| 27 | Fossa Lupara | 3978 | 4192 | 10 | 20 | 30 |
| 28 | Nisida | 3213 | 4188 | 10 | 20 | 30 |

Three groups of events:
A) events with datation only;
B) events with sequence order only;
C) events with both sequence order and date;

Events in (A) and (C) were sampled with symmetrical triangular distributions with the assumed percentiles
(modified from Smith et al. [2011])

Events in (B) were sampled uniformly and independently inside intervals consistent with the sequence.

Uncertainty affecting the estimated volumes is assumed equal to ±50%, consistently with Klawonn et al. [2014].

**Table 1.** Record of times, erupted volumes and locations (eastern or western sectors and partition zones) of the events at CF during Epoch III, with uncertainty bounds.

# Examples of the time-space clusters

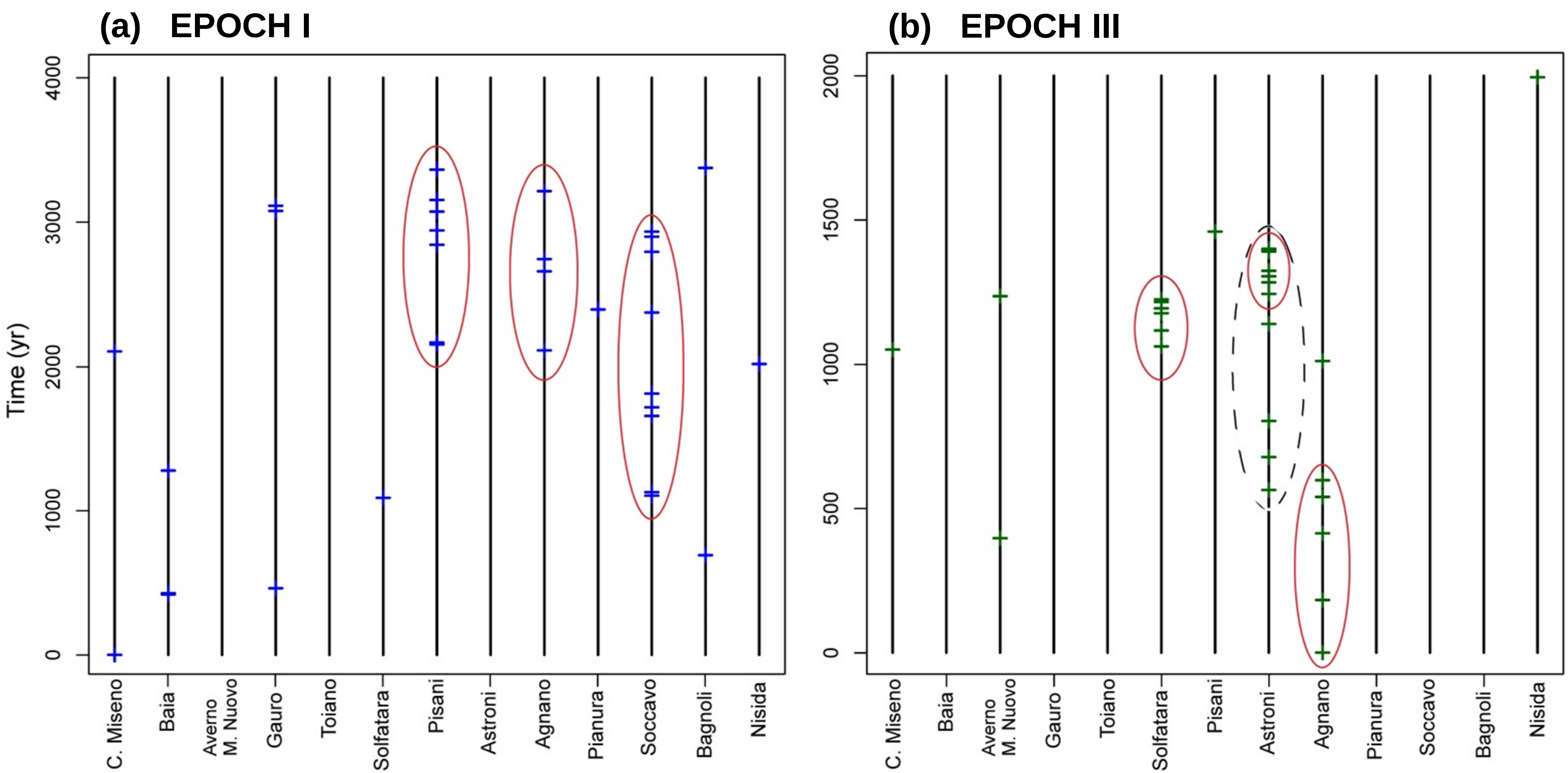
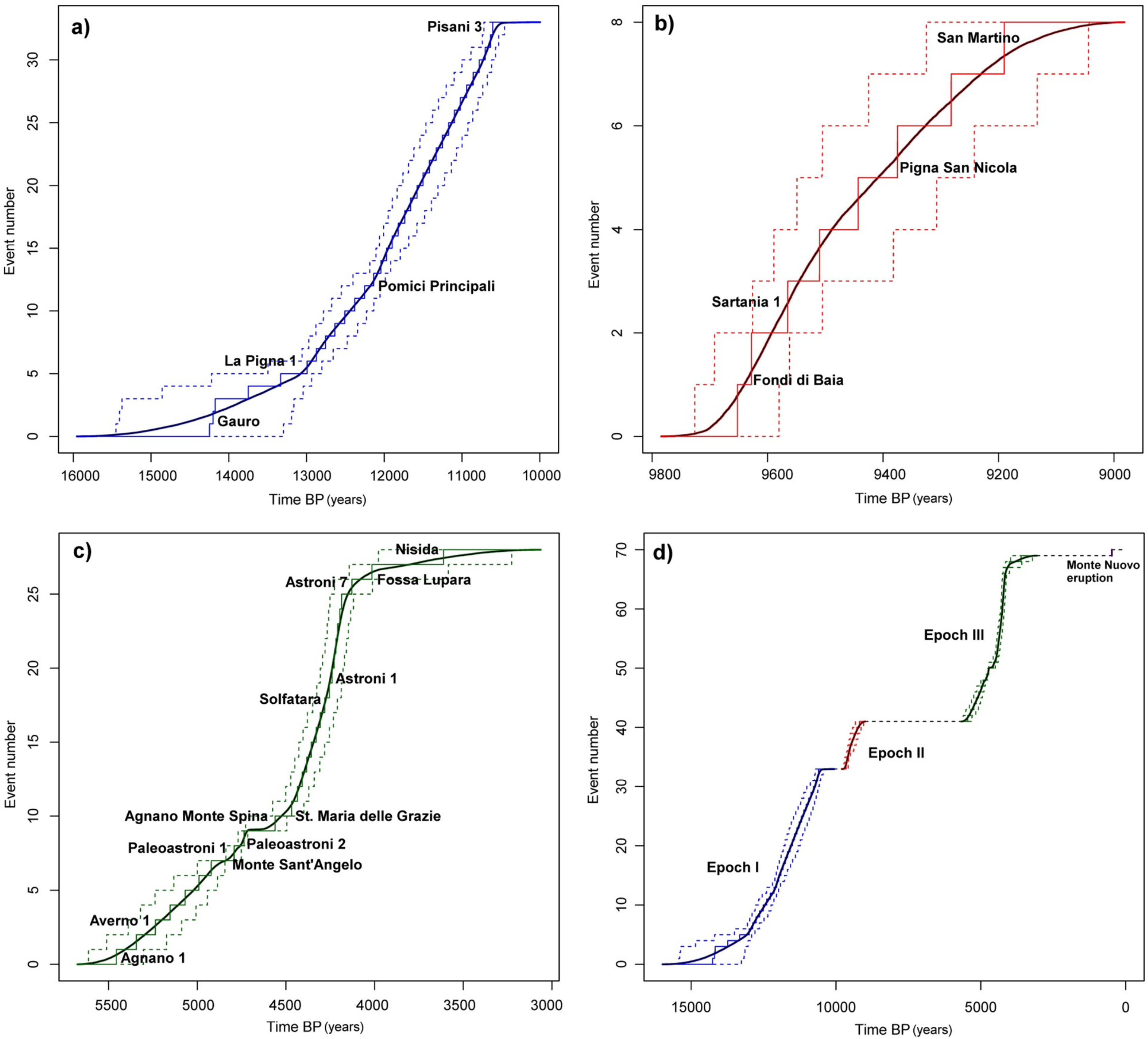



**Figure 4.** Random samples of eruption times and zones. Each coloured dash represents an event.

The red ellipses remark some qualitatively recognizable time-space clusters of activity;
the black dashed ellipse remarks an hypothetically enlarged Astroni cluster including Paleoastroni events.




# Cumulative event number with uncertainty

**Figure 5.** Event number as a function of time during Epoch I (a), Epoch II (b), Epoch III (c) and then during the entire record considered (including Monte Nuovo) (d).

The bold line is the mean value, the narrow line is the 50th percentile and the dashed lines are 5th and 95th percentiles of the uncertainty.

The labels correspond to the eruptions with both datation bounds and sequence place.

# Cumulative eruptive volume with uncertainty

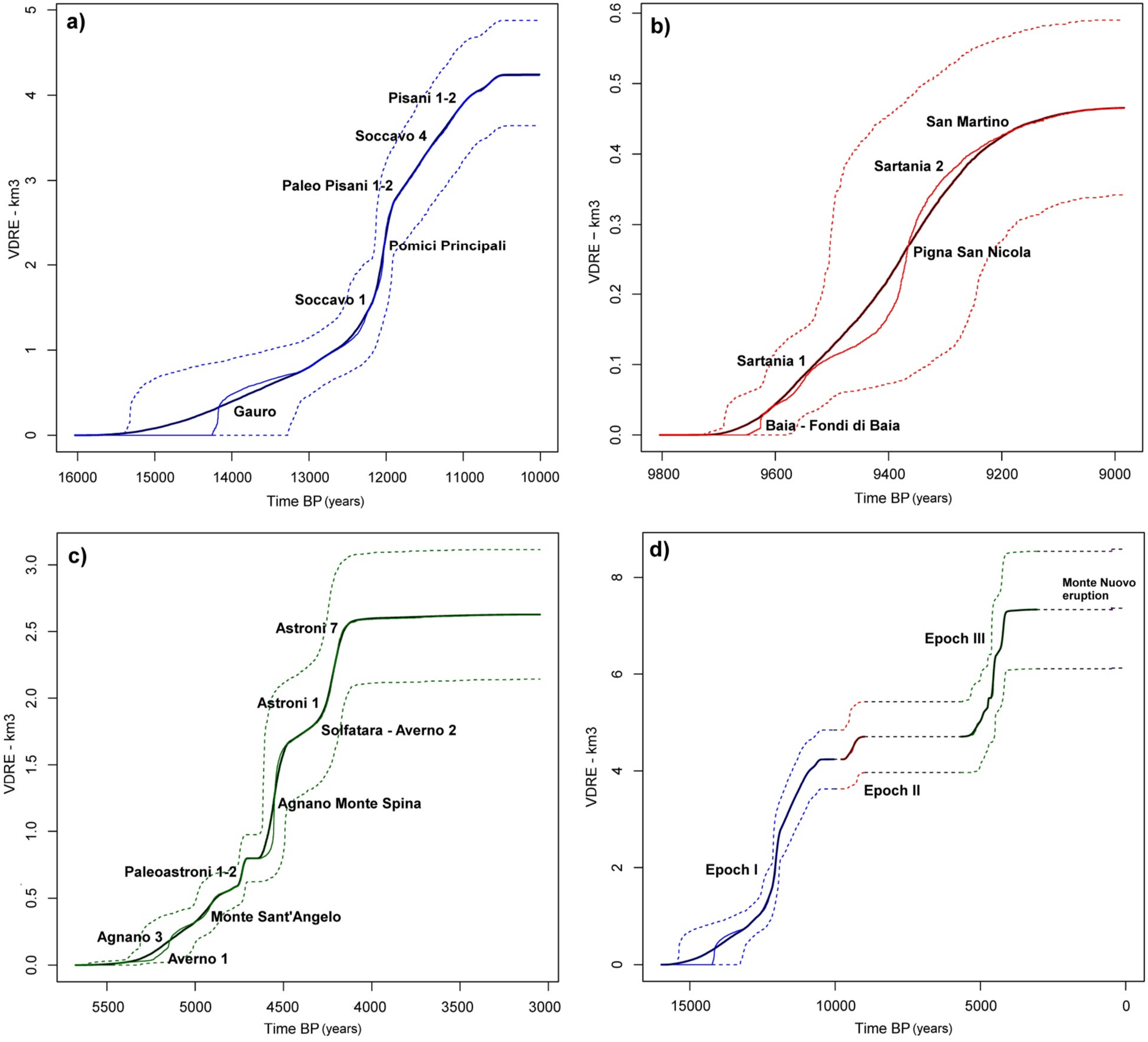



**Figure 6.** Cumulative volume erupted as a function of time during Epoch I (a), Epoch II (b), Epoch III (c) and then during the entire record considered (including Monte Nuovo) (d).

The bold line is the mean value, the narrow line is the 50th percentile and the dashed lines are 5th and 95th percentiles of the uncertainty.

The labels correspond to the largest eruptions of each epoch.

# Western vs eastern past activity

The cumulative volumes erupted by the western part of the caldera, are remarkably smaller than by the eastern. The activity in the western sector seems mainly associated to the initial phases of the eruptive epochs, coherently with considering the recent Monte Nuovo eruption as the first one of a new Epoch.

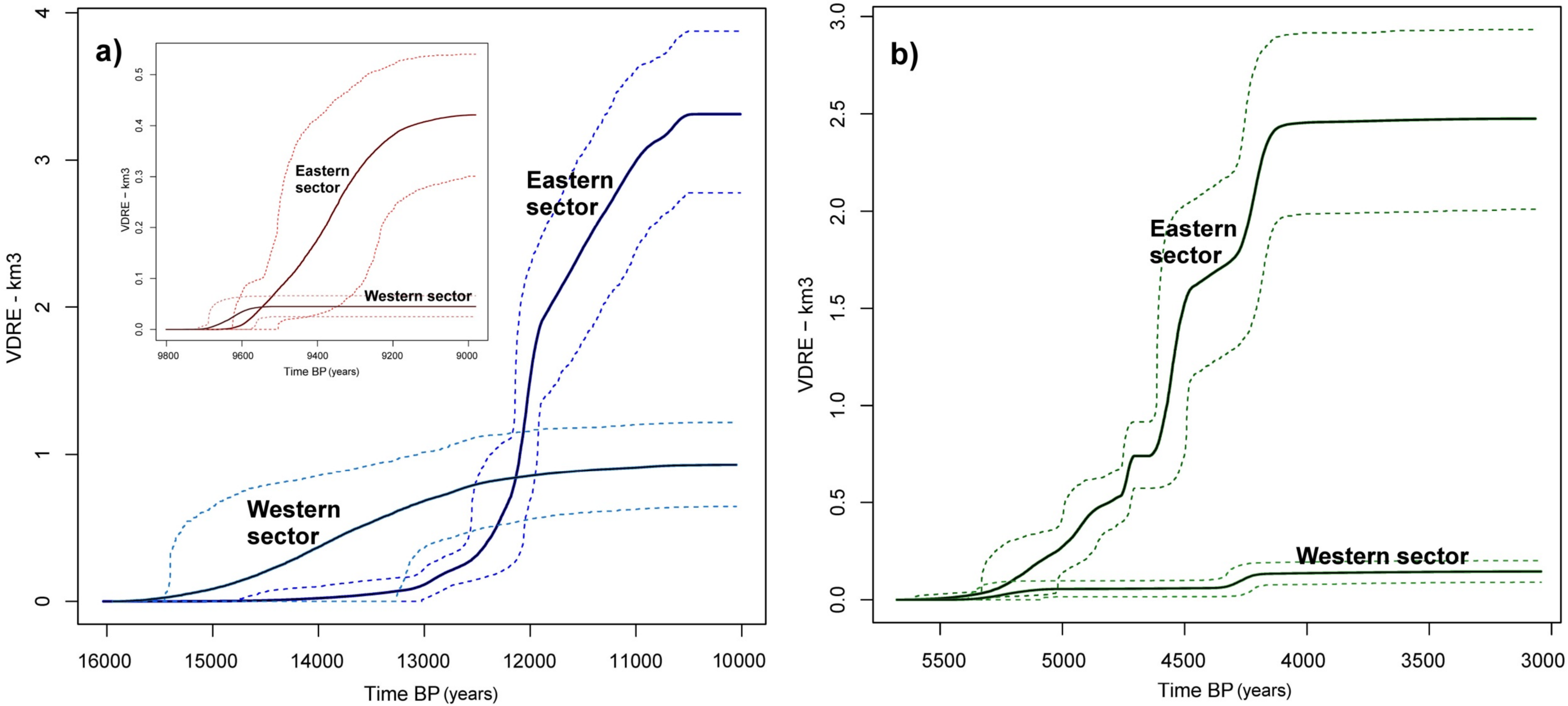



**Figure 7.** Cumulative volume erupted as a function of time during Epoch I (including Epoch II in a small box) (a), and Epoch III (b), with a separation between the eastern and the western sectors of the caldera.
The bold line is the mean value and the dashed lines are 5th and 95th percentiles of the uncertainty.

# Cox-Hawkes counting processes

The model adopted for representing the number of eruptive events that occur in each zone $l$ of the caldera as a function of time relies on a Cox-Hawkes process.

A Cox process is a non-homogeneous Poisson process, in which the model parameters are assumed affected by uncertainty (e.g. Jacquet et al. [2000]).

A Hawkes process is a non-homogeneous Poisson process in which the intensity rate increases whenever an event occurs and instead decreases as time passes without any event occurring (e.g. Bebbington and Cronin [2010]).

The intensity function λ has the meaning of the average density of eruptive events. Indeed the integral ∫λdt gives the average number of events in the selected time interval.

In the case of a Hawkes process we have:

$$\lambda^l(t,\omega) = \lambda_0^l(e) + \sum_{t_i^l(w)<t} [\varphi(e)](t - t_i^l(\omega))$$

It is the sum of a constant term $\lambda_0$ (base rate) and of a time dependent random term that represents an additional intensity produced by each previous event for a prescribed time range after its occurrence (self-excitement duration).

The base rate represents the average density of new clusters (even of one point), while their additional intensity generates the offspring points.

# Conditional likelihood expression

For each zone j of the caldera, the self-interaction decay function φ is assumed exponential.

$$[\varphi(e)](s) = h(e)\exp(-k(e)s)$$

The parameters $\lambda_0(e)$, $k(e)$ and $h(e)$ are conditional on the epistemic assumptions, represented by $e$. We estimated them by a maximum likelihood procedure.

Indeed it is possible to calculate an expression for the likelihood of an eruption sequence in zone $l$, before time $t$.

$$L_l\left((t_i^l)_{i=1,\dots,n^l}, t\right) = \left(\prod_{i=1}^{n^l} \lambda^l(t_i^l)\right)\exp\left(-\int_0^t \lambda^l(s)ds\right)$$

This procedure was implemented in a nested Monte Carlo simulation and repeated for each sample of the uncertainty $e$.

# Maximum likelihood results

Conditionally fitting the parameters on the three eruptive epochs results in quite different values.

In all cases, the likelihood of 477 years without activity after Monte Nuovo eruption is very low.

Calculating separate likelihoods for the three epochs assumed as independent samples permits to obtain global results that are consistent with the Epoch III activity except for a potentially longer duration of each cluster.

**COMPLETE ERUPTIVE EPOCHS**

| Statistics/Eruption record | Epoch I | | | Epoch II | | | Epoch III | | | Epochs I x II x III | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5th %ile | mean | 95th %ile | 5th %ile | mean | 95th %ile | 5th %ile | mean | 95th %ile | 5th %ile | mean | 95th %ile |
| 1/base rate (1/λ0) [a] | 98 | **148** | 237 | 43 | **63** | 94 | 80 | **106** | 142 | 82 | **105** | 140 |
| self excitement duration (T) [a] | 60 | **658** | 1320 | 3 | **101** | 304 | 11 | **96** | 196 | 48 | **189** | 435 |
| mean offspring (μ) | 0.14 | **0.30** | 0.43 | 0.13 | **0.23** | 0.36 | 0.30 | **0.42** | 0.50 | 0.26 | **0.41** | 0.59 |
| cluster generation probability | 13% | **26%** | 35% | 12% | **21%** | 30% | 26% | **34%** | 39% | 23% | **34%** | 44% |
| number of clusters | 2.4 | **6.0** | 9.9 | 0.6 | **1.3** | 2.1 | 3.6 | **5.5** | 7.6 | - | **-** | - |
| mean cluster size | 2.2 | **2.7** | 3.2 | 2.2 | **2.4** | 2.9 | 2.6 | **3.2** | 3.6 | 2.6 | **3.1** | 4.3 |
| Probability of having 477 years with no events after MN | 0.6% | **3.8%** | 10.5% | 0.0% | **0.1%** | 0.5% | 0.2% | **0.9%** | 2.3% | 0.2% | **0.8%** | 2.2% |

**Table 2.** Maximizing the likelihood of the different eruption records we report: the renewal time of the new clusters $1/\lambda_0$, the duration of the clusters T, the mean offspring of each event μ (first three rows) and several other results.

Mean values, $5^{th}$ and $95^{th}$ percentiles are reported, as a function of the sources of uncertainty considered.

# Sensitivity to different volcanological assumptions

**1)** Considering only the events occurred in the first parts of the epochs before the climatic eruptions, gives longer durations of the clusters, and slightly smaller base rates, confirming the qualitatively observed 'slow starts' of the epochs of activity.

**2)** Separating the eastern record from the western, the fewer events occurred in the western sector produce much different results. The clustering behavior is weakened and the base rate drops down.

This assumption tends to increase consistently the likelihood of having observed 477 years without activity after Monte Nuovo eruption.

**3)** Rejecting the separation in epochs and fitting the model on the whole eruptive record of the last 15 ka (including even the periods of quiescence) produces lower base rates and longer duration of the clusters.

Anyways the non-homogeneity between the eruptive epochs and the periods of quiescence seems too strong to be captured by such a simple model without losing accuracy on the re-production of the pattern inside epochs.

# Summary and conclusions

- A new uncertainty probability model was developed for quantifying the effect of epistemic uncertainties affecting past eruption record.

- An innovative time-space doubly stochastic self-exciting counting process was implemented for re-producing the eruptive activity of Campi Flegrei, including the effect of uncertainty.

- The model was tested against different volcanological assumptions for assessing the sensitivity of the outcomes on them.

- The most likely possibility is that Monte Nuovo belongs to a new epoch of activity. Under this assumption, the likelihood to have observed 477 years without eruptions is significantly low.

- This study is part of the work aimed at the construction of a robust temporal model capable of producing a background probability distribution for the time of the next explosive eruption at CF, including the clustering behaviour and the uncertainty effects.

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