Using multi-model and coupled model based simulations and UQ for analysis of volcanic hazards

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Introdution

Geohazard Assessment

In geohazard assessment, acceptably accurate numerical simulation of the complex geologic activities such as as **debris** & **pyroclastic flows**, **snow avalanches** and **landslides** is of crucial importance.

Continuum Model

Considering the geophysical mass flow as an **incompressible continuum**, the conservation of mass and momentum equations are:

$$\begin{aligned} \nabla \cdot \underset{\sim}{\mathbf{u}} &= 0 \\ \frac{\partial}{\partial t}(\rho \, \underset{\sim}{\mathbf{u}}) + \nabla \cdot (\rho \, \underset{\sim}{\mathbf{u}} \otimes \underset{\sim}{\mathbf{u}}) &= \nabla \cdot \underset{\sim}{\sigma} + \rho \, \underset{\sim}{\mathbf{g}} \end{aligned} \tag{1}$$

Introdution

Rheology of flow

Mechanical behavior of the flowing material or the rheology of the flow is a mathematical model appearing in the **Cauchy stress tensor**, $\underset{\sim}{\sigma}$.

S-W Assumptions

Geophysical mass flows exhibit a shallow flow geometry. Using **Shallow-Water** approximations, the **shallowness parameter** is assumed to be very small, $\epsilon \triangleq h/L \ll 1$.

Depth-Averaging

Shallow-Water assuption allows to perform a Depth-averaging of conservation variables (integrating conservation equations along the flow thickness).

Mohr-Coulomb Model

Depth-Averaged conservation Equations

$$
\begin{aligned}
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) &= 0, \\
\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}\left(h\bar{u}^2 + \frac{1}{2}k_{ap}g_z h^2\right) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) &= S_x, \\
\frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}\left(h\bar{v}^2 + \frac{1}{2}k_{ap}g_z h^2\right) &= S_y
\end{aligned}
\tag{2}
$$

Active-Passive coefficient

$$
k_{ap} = \begin{cases} 2\frac{1-\sqrt{1-\cos^2(\phi_{int})(1+\tan^2(\phi_{bed}))}}{\cos^2(\phi_{int})} \mp 1, & \nabla\cdot\underset{\sim}{\bar{\mathbf{u}}} > 0 \ (\nabla\cdot\underset{\sim}{\bar{\mathbf{u}}} < 0), \\ 1, & \nabla\cdot\underset{\sim}{\bar{\mathbf{u}}} = 0, \end{cases} \tag{3}
$$

Mohr-Coulomb Model

The Source terms S_x and S_y are:

$$\begin{aligned} S_x = g_x h - \frac{\bar{u}}{\|\underset{\sim}{\bar{\mathbf{u}}}\|}\left[h\tan(\phi_{bed})\left(g_z + \frac{\bar{u}^2}{r_x}\right)\right] \\ -hk_{ap}\,\mathrm{sgn}\left(\frac{\partial \bar{u}}{\partial y}\right)\frac{\partial(g_z h)}{\partial y}\sin(\phi_{int}), \\ S_y = g_y h - \frac{\bar{v}}{\|\underset{\sim}{\bar{\mathbf{u}}}\|}\left[h\tan(\phi_{bed})\left(g_z + \frac{\bar{v}^2}{r_y}\right)\right] \\ -hk_{ap}\,\mathrm{sgn}\left(\frac{\partial \bar{v}}{\partial x}\right)\frac{\partial(g_z h)}{\partial x}\sin(\phi_{int}) \end{aligned} \tag{4}$$

Bed friction angle, ϕ_{bed}, and **internal friction** angle, ϕ_{int}, are model parameters.

Pouliquen-Forterre Model

The knowledge of two functions is sufficient to define the empirical friction law,$\mu_b(\|\underset{\sim}{\bar{\mathbf{u}}}\|, h)$, in the whole range of velocity and thickness:

$$\begin{aligned} \mu_{start}(h) &= \tan(\phi_{start}(h)) \\ \mu_{stop}(h) &= \tan(\phi_{stop}(h)) \end{aligned} \tag{5}$$

As a result, for the basal friction coefficient in the dynamic friction regime where $Fr \geq \beta$:

$$\mu(h, Fr) = \mu_{stop}(h\beta/Fr) \tag{6}$$

In the intermediate friction regime when $0 < Fr < \beta$, the friction coefficient is given by a power law extrapolation between the friction laws in the static and dynamic friction regimes as:

$$\mu(h, Fr) = \left(\frac{Fr}{\beta}\right)^{\gamma} [\mu_{stop}(h) - \mu_{start}(h)] + \mu_{start}(h) \tag{7}$$

Pouliquen-Forterre Model

The functions μ_{stop} and μ_{start} are given by fits to experimental measurements as transitions between the relevant critical angles. Therefore:

$$\mu_{stop}(h) = \tan\phi_1 + \frac{\tan\phi_2 - \tan\phi_1}{1 + h/\mathcal{L}} \tag{8}$$

and

$$\mu_{start}(h) = \tan\phi_3 + \frac{\tan\phi_2 - \tan\phi_1}{1 + h/\mathcal{L}} \tag{9}$$

The **critical angles** ϕ_1, ϕ_2 and ϕ_3 and the parameter $\mathcal{L}$ (the characteristic depth of the flow over which a transition between the angles ϕ_1 and ϕ_2 occurs) in addition to the β are the material properties.

$$
\begin{aligned}
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) &= 0, \\
\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}\left(h\bar{u}^2 + \frac{1}{2}k_{ap}g_z h^2\right) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) &= S_x, \\
\frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}\left(h\bar{v}^2 + \frac{1}{2}k_{ap}g_z h^2\right) &= S_y
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
S_x &= g_x h - g_z h \left(\frac{\bar{u}}{\|\underset{\sim}{\bar{\mathbf{u}}}\|} \; \mu_b(\|\underset{\sim}{\bar{\mathbf{u}}}\|, h) \; + \frac{\partial h}{\partial x} \right) \\
S_y &= g_y h - g_z h \left(\frac{\bar{v}}{\|\underset{\sim}{\bar{\mathbf{u}}}\|} \; \mu_b(\|\underset{\sim}{\bar{\mathbf{u}}}\|, h) \; + \frac{\partial h}{\partial y} \right)
\end{aligned}
\tag{11}
$$

Voellmy-Salm Model

The principlal relation between shear and normal stresses are:

$$\tau = \mu\sigma + \frac{\rho\|\underset{\sim}{\mathbf{g}}\|}{\xi}\|\underset{\sim}{\bar{\mathbf{u}}}\|^2 \tag{12}$$

σ denotes the normal stress at the bottom of the fluid layer and $\underset{\sim}{\mathbf{g}} = (g_x, g_y, g_z)$ represents the gravity vector.

The total basal friction splits into:

- **Velocity independent dry-Coulomb term** which is proportional to the normal stress at the flow bottom (coefficient μ).
- **Velocity dependent viscous or turbulent term** (coefficient ξ).

Voellmy-Salm Model

Depth-Averaged conservation Equations & Source terms

$$
\begin{aligned}
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) &= 0, \\
\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}\left(h\bar{u}^2 + \frac{1}{2}k_{ap}g_z h^2\right) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) &= S_x, \\
\frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}\left(h\bar{v}^2 + \frac{1}{2}k_{ap}g_z h^2\right) &= S_y
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
S_x &= g_x h - \frac{\bar{u}}{\|\underset{\sim}{\bar{\mathbf{u}}}\|}\left(\mu h g_z + \frac{\|\underset{\sim}{\mathbf{g}}\|}{\xi}\|\underset{\sim}{\bar{\mathbf{u}}}\|^2\right), \\
S_y &= g_y h - \frac{\bar{v}}{\|\underset{\sim}{\bar{\mathbf{u}}}\|}\left(\mu h g_z + \frac{\|\underset{\sim}{\mathbf{g}}\|}{\xi}\|\underset{\sim}{\bar{\mathbf{u}}}\|^2\right)
\end{aligned}
\tag{14}
$$

Block-and-ash flow example

April 16, 1991, Volcán de Colima, Mexico

Property	Value
Pile location (UTM East)	644956.0 m
Pile location (UTM North)	2157970.0 m
Material Volume	$1.4 \times 10^5\ m^3$

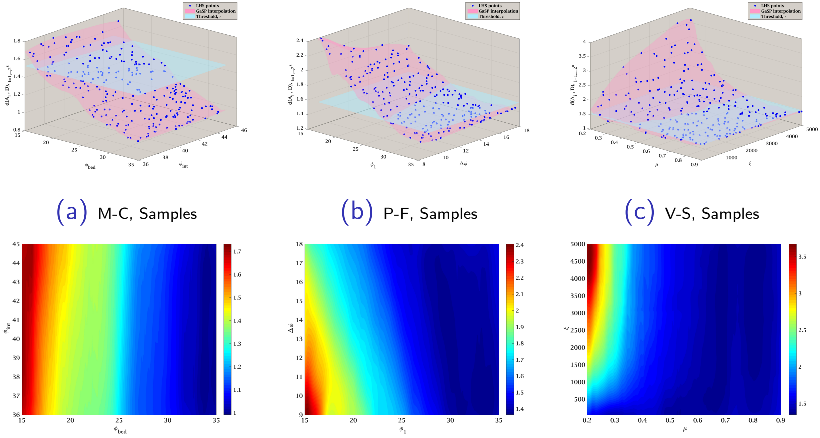
Table : Slumping pile properties

Suppose that A_i is a random simulated estimate for D; therefore, the distance function is defined as:

$$d(A_i, D) := \frac{A_i \Delta D}{D} = \frac{(A_i \cup D) \backslash (A_i \cap D)}{D} \tag{15}$$

Block-and-ash flow example

Distance function distributions



(a) M-C, Samples (b) P-F, Samples (c) V-S, Samples

(d) M-C, GaSP (e) P-F, GaSP (f) V-S, GaSP

Figure : Distance function distributions, $d(A_i, D)$, for the rheology models.

Block-and-ash flow example

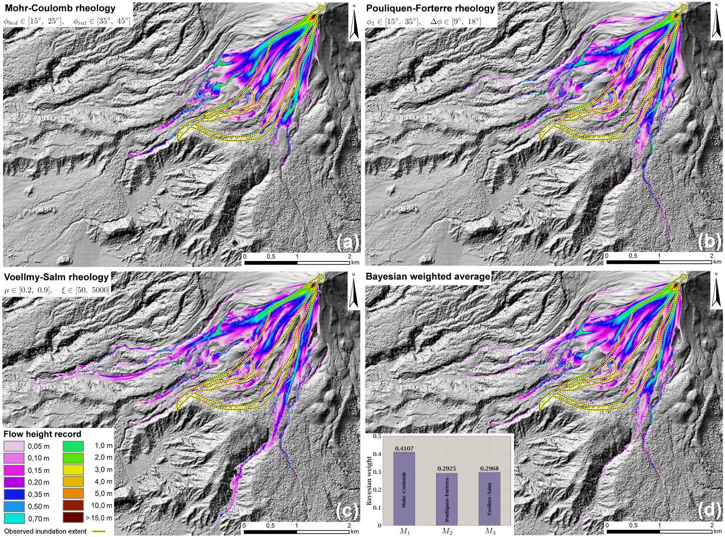


Figure : Mean value of max flow height record over each model's parameter space, (a)-(c), and their Bayesian weighted average, (d).

Debris flow example

October 16, 1955, Atenquique, Mexico

Pile number	Center location (UTM East)	Center location (UTM North)	Material volume
1	647077.0 m	2163900.0 m	$1.57 \times 10^6\ m^3$
2	649512.0 m	2165360.0 m	$1.57 \times 10^6\ m^3$
3	652228.0 m	2160730.0 m	$1.57 \times 10^6\ m^3$

Table : Slumping piles properties

Debris flow example

October 16, 1955, Atenquique, Mexico

Location number	Deposit thickness
1	2.2 m
2	2.2 m
3	4.6 m
4	4.3 m

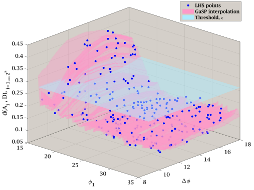
Table : Deposit thicknesses measured at the field, $D = \{H_1, H_2, H_3, H_4\}$

Since here the available observed data is the flow thickness at four particular locations, we can use the following metric definition:

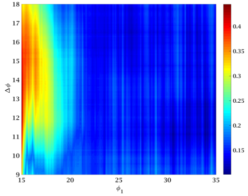
$$d(A_i, D) := \frac{1}{4} \sum_{j=1}^{4} \frac{|D_j - A_i|}{D_j} \tag{16}$$

Debris flow example

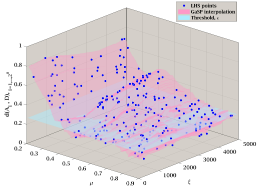
Distance function distributions



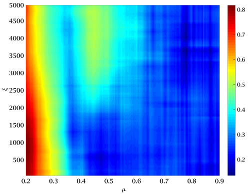
(a) P-F, Samples.



(b) P-F, GaSP.



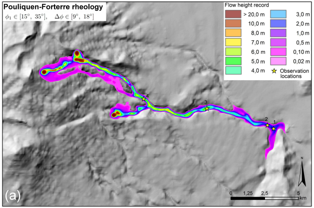
(c) V-S, Samples.



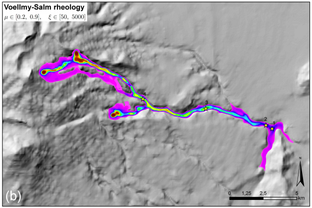
(d) V-S, GaSP.

Figure : Distance function distributions, $d(A_i, D)$, for the rheology models.

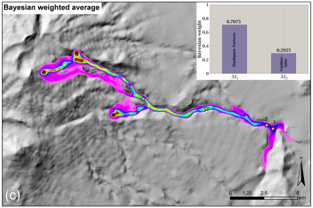
Debris flow example



(a)



(b)



(c)

Figure : Mean value of max flow height record over each model's parameter space, (a) & (b), and their Bayesian weighted average, (c).

The End!