Space and time probability modeling of eruptive events in the Long-Valley volcanic region (CA)

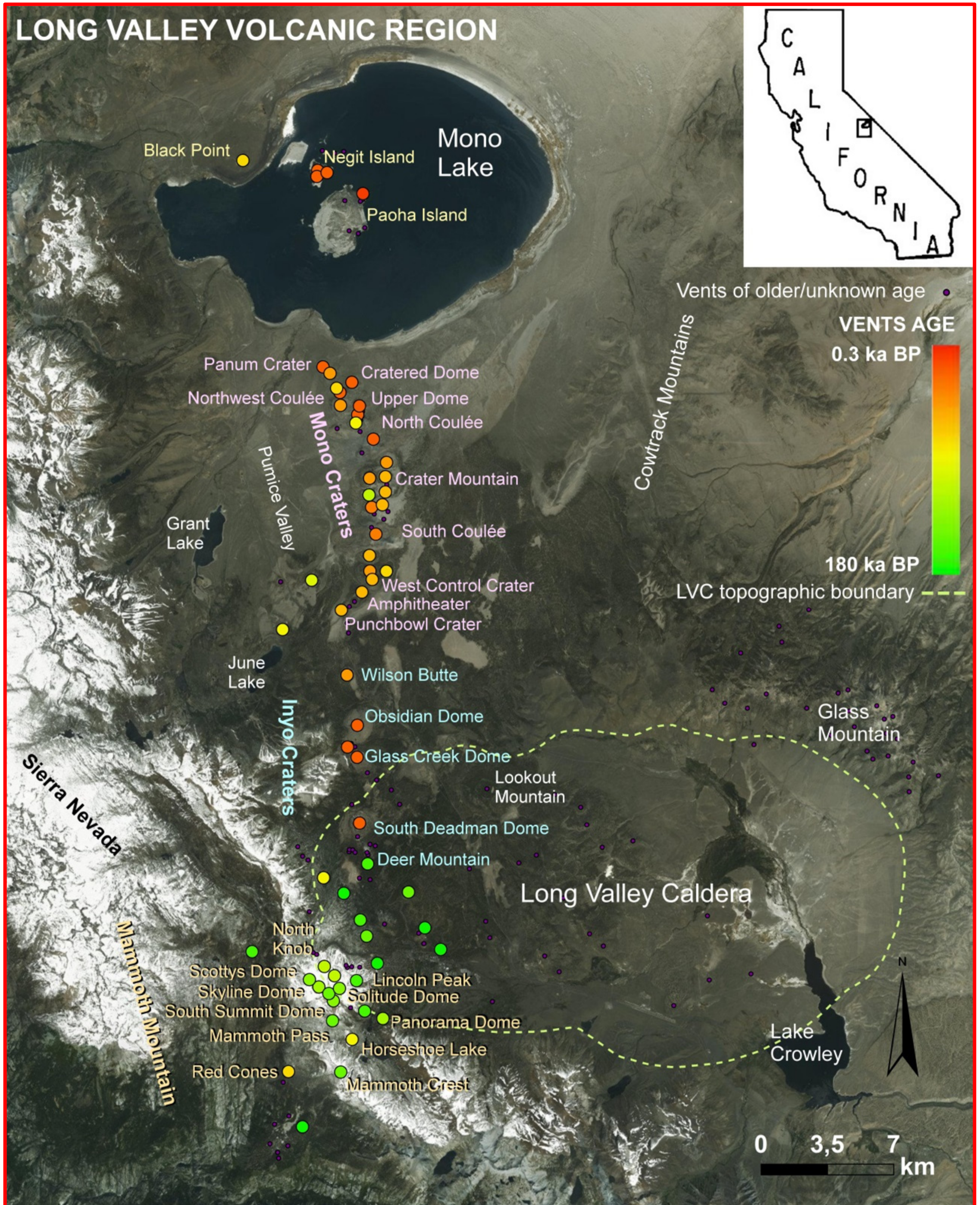
A research study in collaboration with:
Marcus Bursik[(1)], Abani K. Patra[(2)], E. Bruce Pitman[(3)]

(1) University at Buffalo, Department of Geology
(2) University at Buffalo, Department of Mechanical and Aerospace Engineering
(3) University at Buffalo, Department of Materials Design and Innovation

Project Hazard SEES: Persistent volcanic crises resilience in the face of prolonged and uncertain risk,
National Science Foundation, 2015 - 2018.

Volcanology Seminar
24 February, SUNY at Buffalo

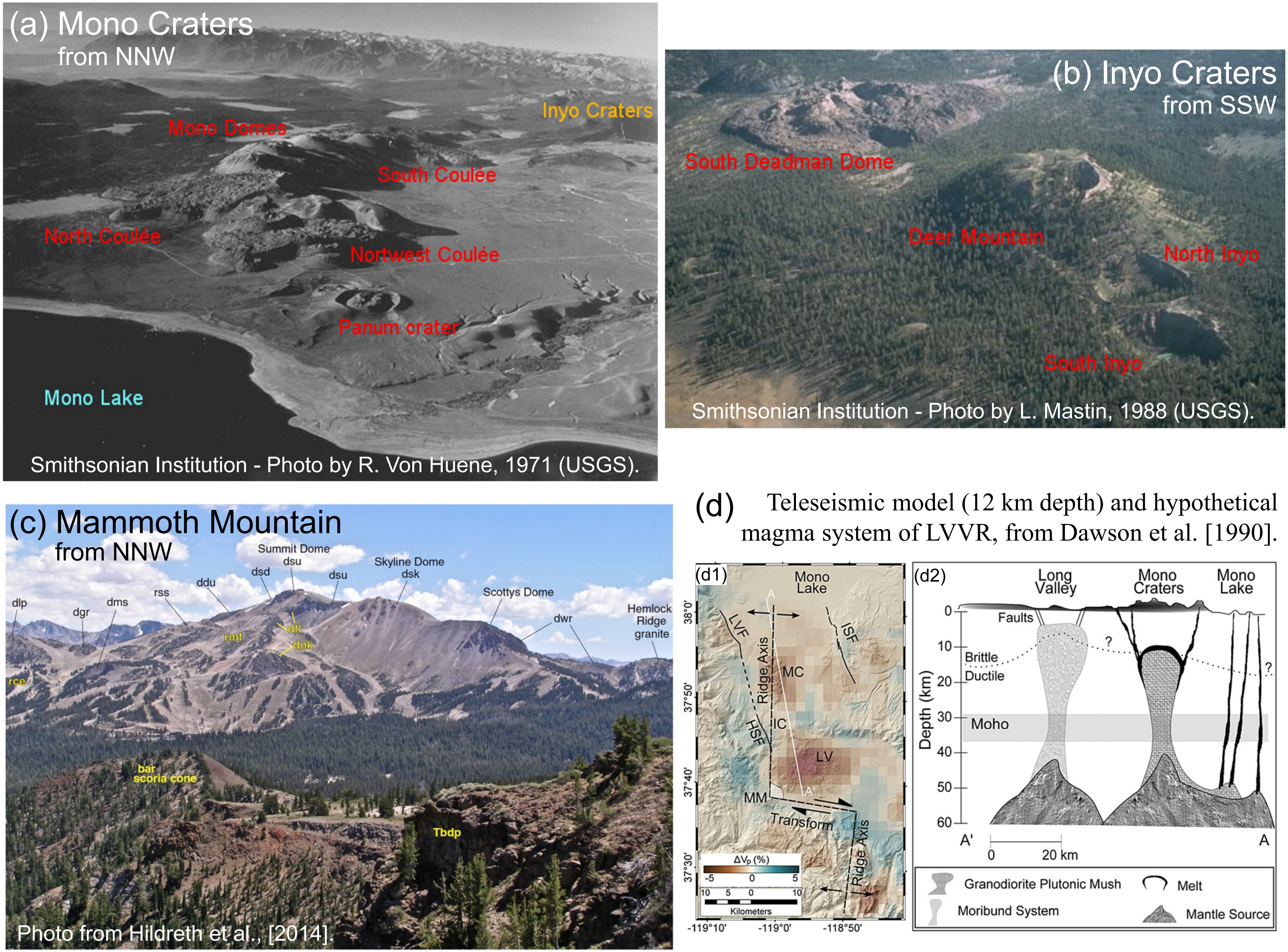
- The Long Valley volcanic region -

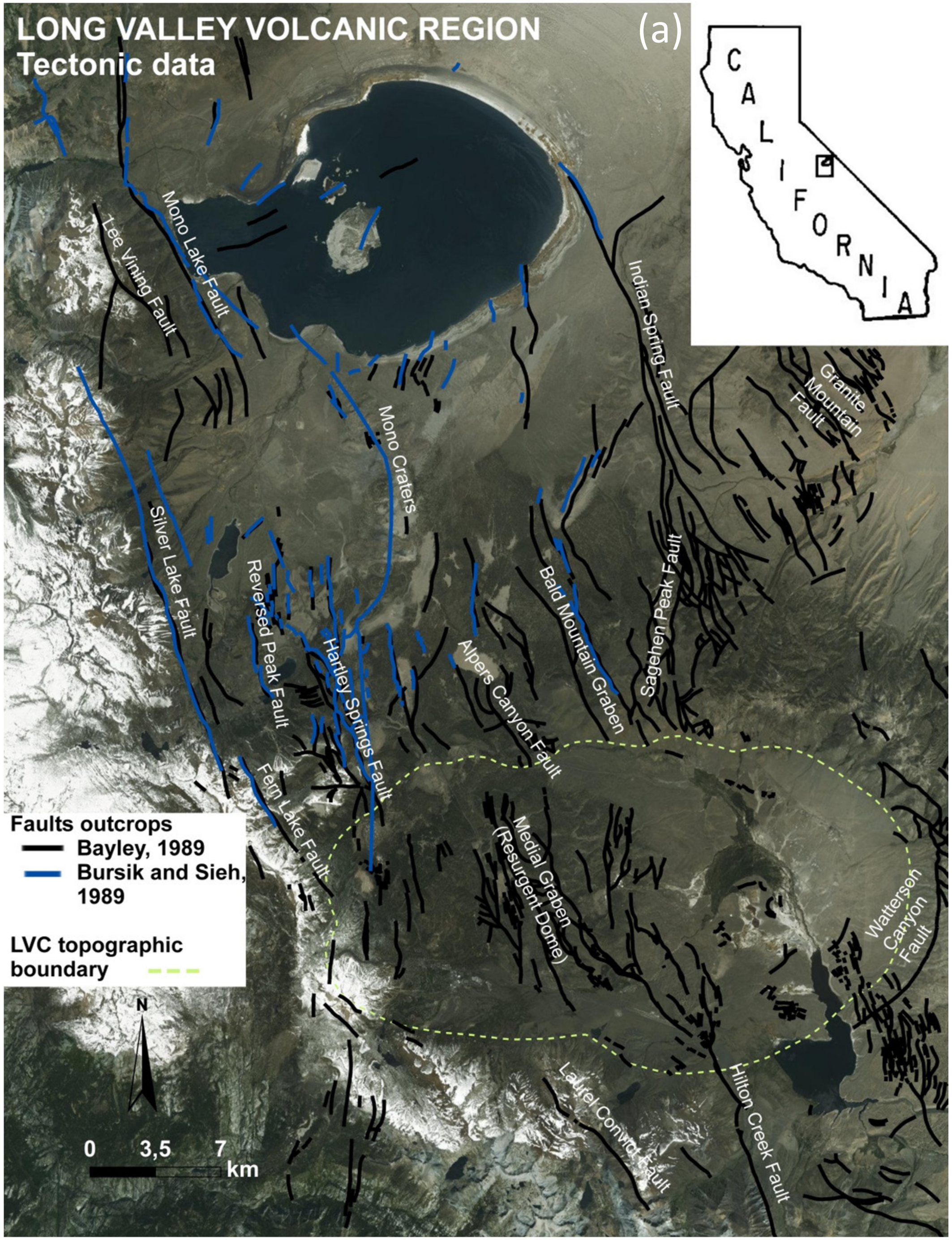


Long Valley caldera (LVC), was created by the eruption of >200km^3 tephra ~ 760ka BP (Bishop tuff).

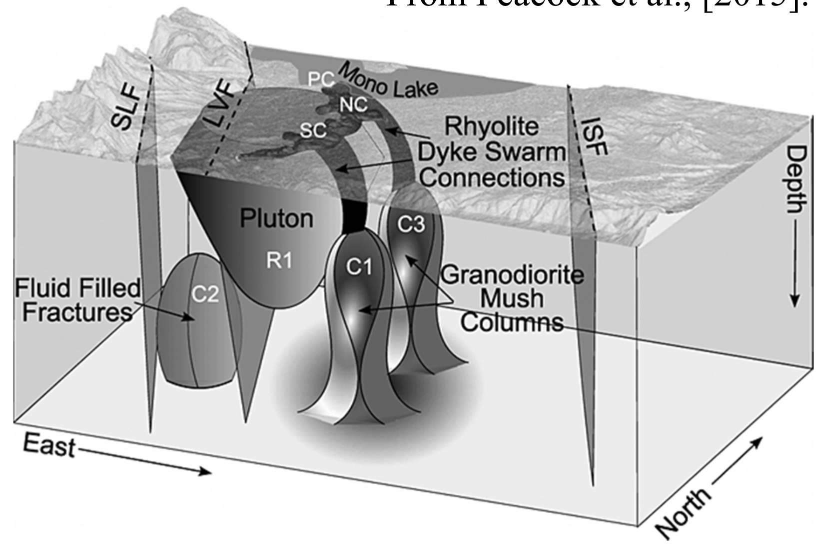
Over the last 180ka the eruptions have been mostly localized at **Mammoth Mountain,** on the western rim of LVC and along the **Mono-Inyo Craters** volcanic chain, stretching ~45km North towards Mono lake.

The most recent period of **unrest started in 1978** - several seismic swarms in LVC and below Mammoth mountain, and diffuse volcanic CO_2 emissions.

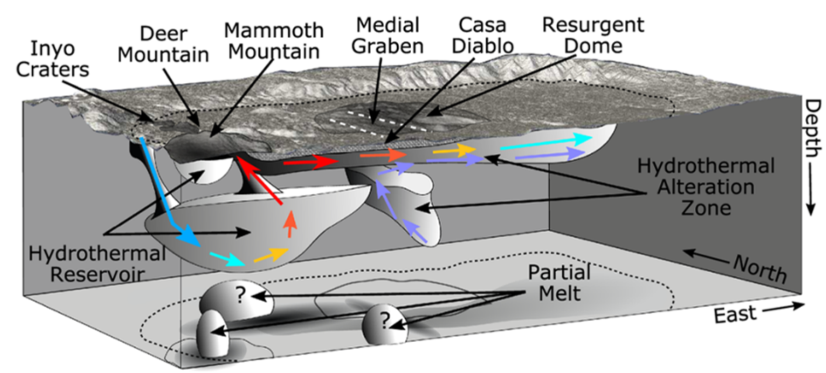

(a) Mono Craters
from NNW
Inyo Craters
Mono Domes
South Coulée
North Coulée
Northwest Coulée
Panum crater
Mono Lake
Smithsonian Institution - Photo by R. Von Huene, 1971 (USGS).
(b) Inyo Craters
from SSW
South Deadman Dome
Deer Mountain
North Inyo
South Inyo
Smithsonian Institution - Photo by L. Mastin, 1988 (USGS).
(c) Mammoth Mountain
from NNW
Summit Dome
dsu
dsd
dsu
Skyline Dome
dsk
ddu
rss
dms
dgr
dlp
Scottys Dome
dwr
Hemlock
Ridge
granite
rmf
dfl
dnk
rce
bar
scoria cone
Tbdp
Photo from Hildreth et al., [2014].
(d) Teleseismic model (12 km depth) and hypothetical magma system of LVVR, from Dawson et al. [1990].
(d1)
Mono
Lake
A
LVF
Ridge Axis
ISF
MC
HSF
IC
LV
A'
MM
Transform
Ridge Axis
ΔVp (%)
-5
0
5
10
5
0
10
Kilometers
38°0'
37°50'
37°40'
37°30'
-119°10'
-119°0'
-118°50'
(d2)
Long
Valley
Mono
Craters
Mono
Lake
Faults
?
Brittle
Ductile
?
Moho
Depth (km)
0
10
20
30
40
50
60
A'
A
0
20 km
Granodiorite Plutonic Mush
Melt
Moribund System
Mantle Source



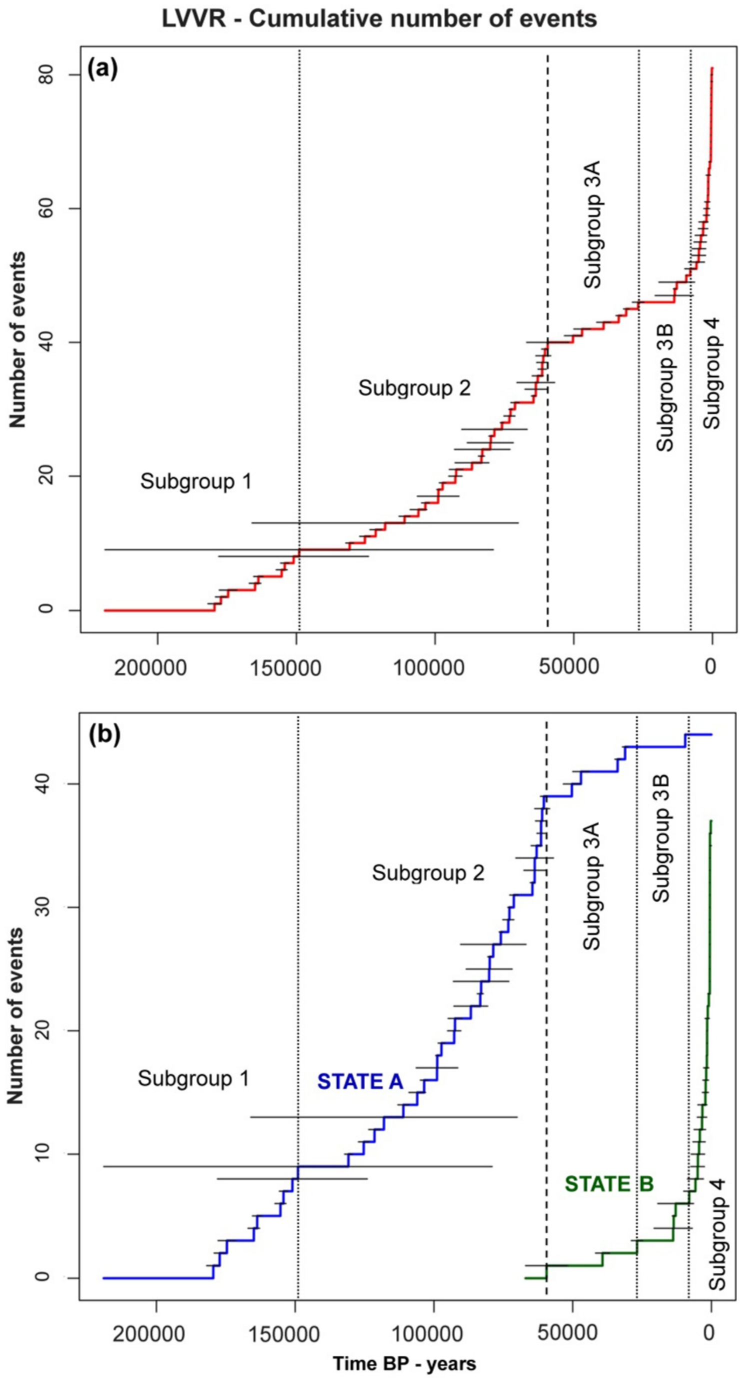
(b) Conceptual geologic model based on the electric resistivity features of Mono region. From Peacock et al., [2015].



(c) Schematic of the electric resistivity model and hydrothermal flow of LVC region. Arrow colors represent temperature, in purple is the paleohydrothermal flow. From Peacock et al., [2016]



- Eruptive record description -



TEMPORAL RECORD

Past record was divided into five subgroups.

A

1) 180/149 ka BP - 9 events. average return time ~3,400 yrs, concluded with a ~18 ka period of quiescence.

2) 131/60 ka BP - 30 events, average return time ~2,350 yrs, concluded by a ~ 20 km location shift to North.

3A) 59/27 ka BP - 7 events, average return time ~4,650 yrs, concluded by a ~13 ka period of quiescence.

B

3B) 14/8 ka BP - 5 events, average return time ~1,150 yrs, concluded by a great increase of activity rate.

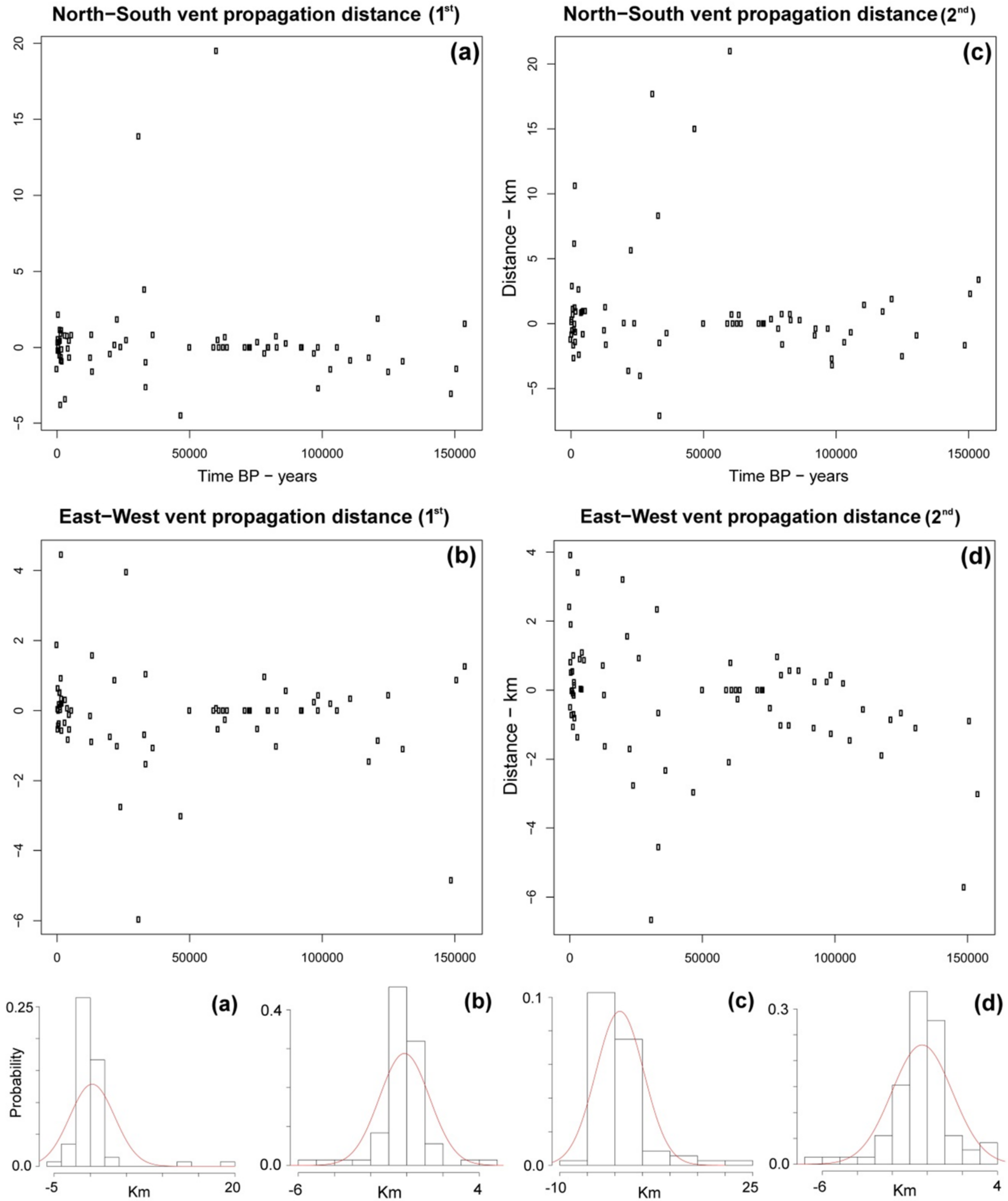
4) < 6 ka BP - 30 events, average return time ~200 yrs, 11 vents active together at 625-600 yr BP.

Two states of volcanic activity, A and B, are considered, with a probability p_1 of changing state when a new event will occur.

The state A concerns the **Mammoth Mountain** area, whereas the state B mostly the **Mono basin**, with Inyo Craters lying in the middle.

The separation after 60ka BP corresponded with the **activation of the northern part** of the region.

DISTANCES OF PROPAGATION

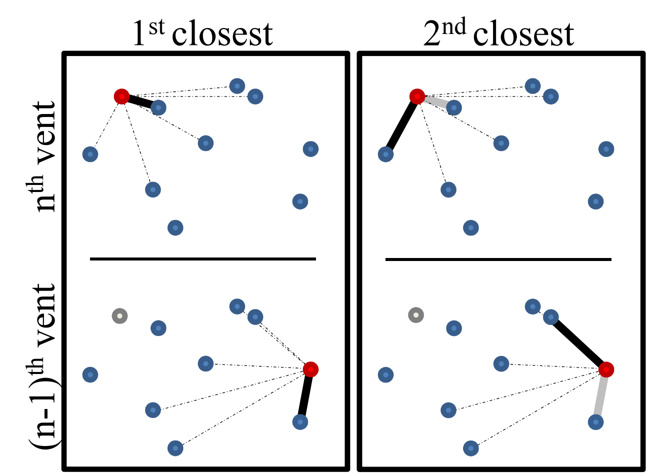


Alternative distances are easily defined: the spatially closest previously active vent, the second closest vent, etc.

A **20km jump** in the N/S distance of propagation is noticed around 60ka BP.

The vent propagation process is likely Gaussian, but not isotropic: the N/S scale is ~5 times larger than the E/W scale

There is a significant increase of N/S propagation distances after such jump.



Examples of 1st and 2nd spatially closest previously active vents.

- Spatial modeling: vent opening probability maps -

Doubly stochastic vent opening maps

A "map of vent opening" is the **spatial estimate** of the probability of vent opening per km^2 in each point of the region of interest.

That probability is **conditional** on the occurrence of a new eruption, without temporal window.

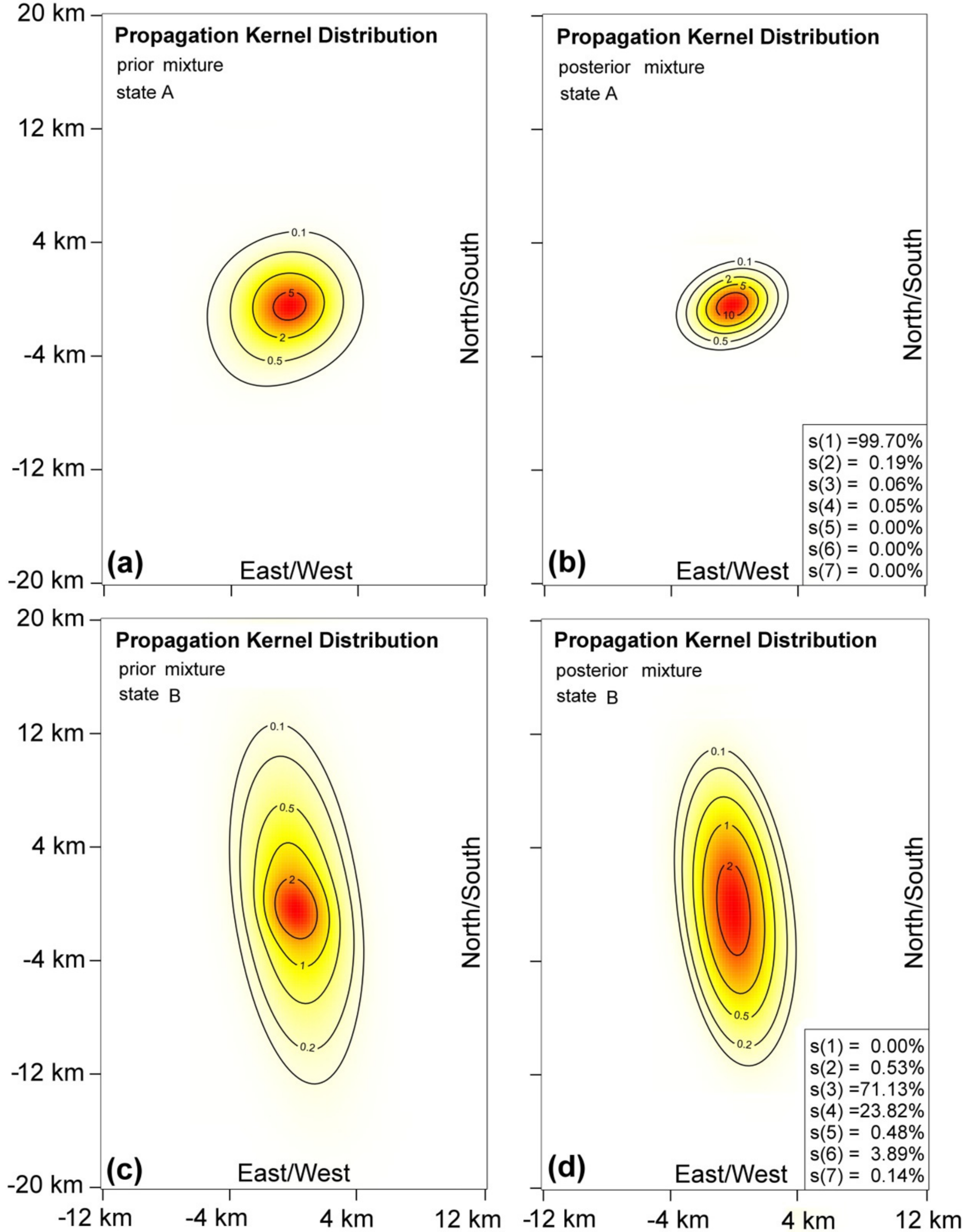
The volcano is presented as a **random system** that must be assessed with **uncertain information**.

Adopting a **doubly stochastic approach,** some ill-constrained parts of the long-term probability models will be randomly changed, reporting the effect on the probability estimates.

As a consequence of this approach, some probability estimates will have their own confidence intervals.

Even the final probability maps will be affected by uncertainty: we calculated the **mean, 5th and 95th percentile** values for the vent opening probability density functions.

- Spatial Model I: Gaussian kernel density estimator -



KERNEL FUNCTIONS

Given the locations $(x_i, y_i)_{i=1,\dots,N}$ of the past N events, a new event propagates from one event location **randomly chosen** from the previous, to a **random distance**:

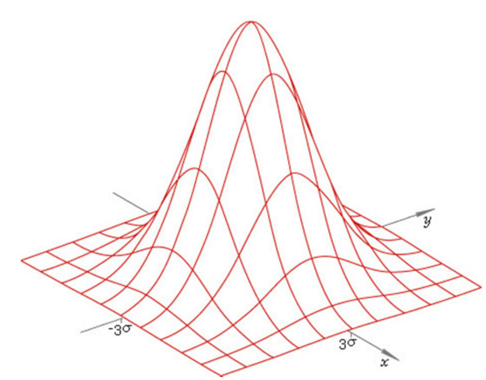
$$X = (x_k + d_1, y_k + d_2),$$

where X is the spatial location of the next vent,

k is a discrete random variable in $\{1,\dots,N\}$ sampling one of the previous vents,

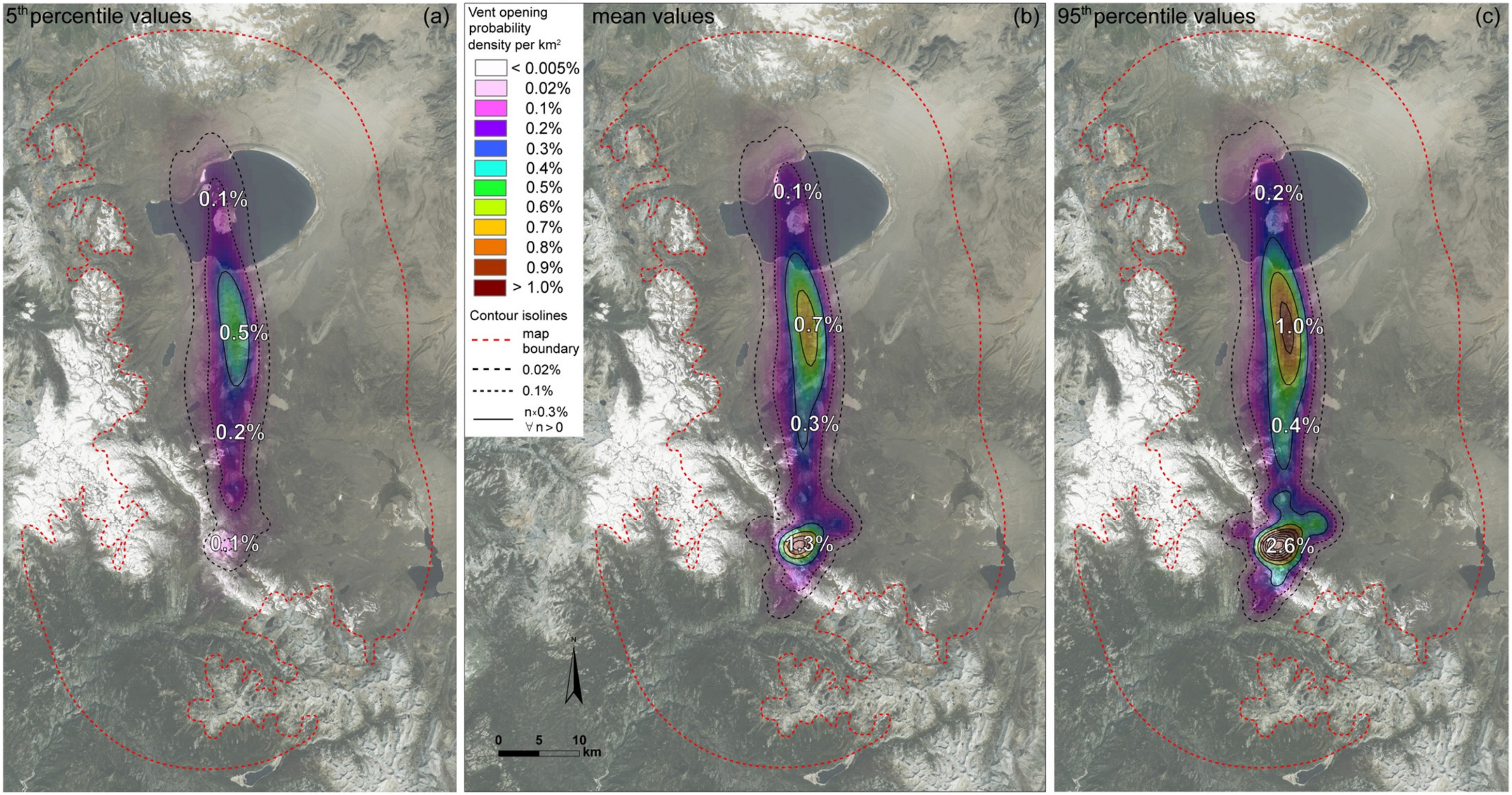
$d = (d_1, d_2)$ is a two dimensional Gaussian random vector with mean μ and covariance matrix Σ.

The random variables k, d change according with the state - in the figure are shown d_a and d_b based on the statistics of the past propagation distances.



Example of Gaussian density plot.

Vent opening probability maps - method 1 (kernel density estimator)



The density f of $\boldsymbol{X}$ is obtained by convolving the anisotropic **probability kernel** describing $\boldsymbol{d}$ with the past vent location disks $(D_i)_{i=1,\ldots,N}$, each weighted relying on the distribution of $\boldsymbol{k}$.

A **Monte Carlo sampling** varies the uncertainty parameter p_1.
[i.e. Mammoth Mountain unknown relevance].

- Spatial Model II: Bayesian update of IFO map -

We defined an additional spatial parameter $\zeta = (\zeta_1, \zeta_2)$ called "interacting fault outcrop" (IFO).

The model assumes that the new vents open **near a random IFO** location, at a random distance ***d***.

The IFO *a priori* distribution is the linear combination of **log-extension data** and uniform distributions.

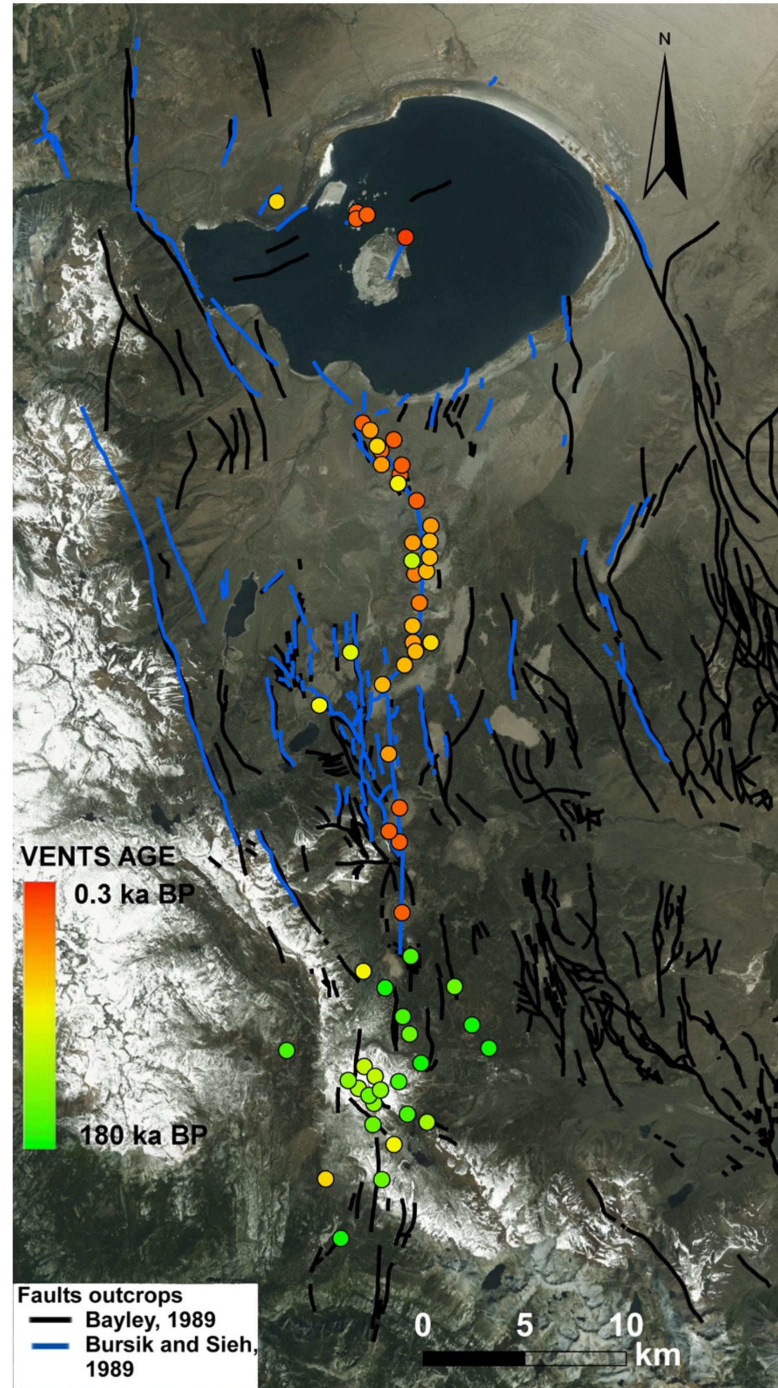
Conditioned on an IFO location $\zeta = (x, y)$, the likelihood for vent location is a symmetric **Gaussian function** of mean ζ and covariance matrix $\sigma^2\mathbf{I}$.

Two alternative models are used to constrain the standard deviation σ.

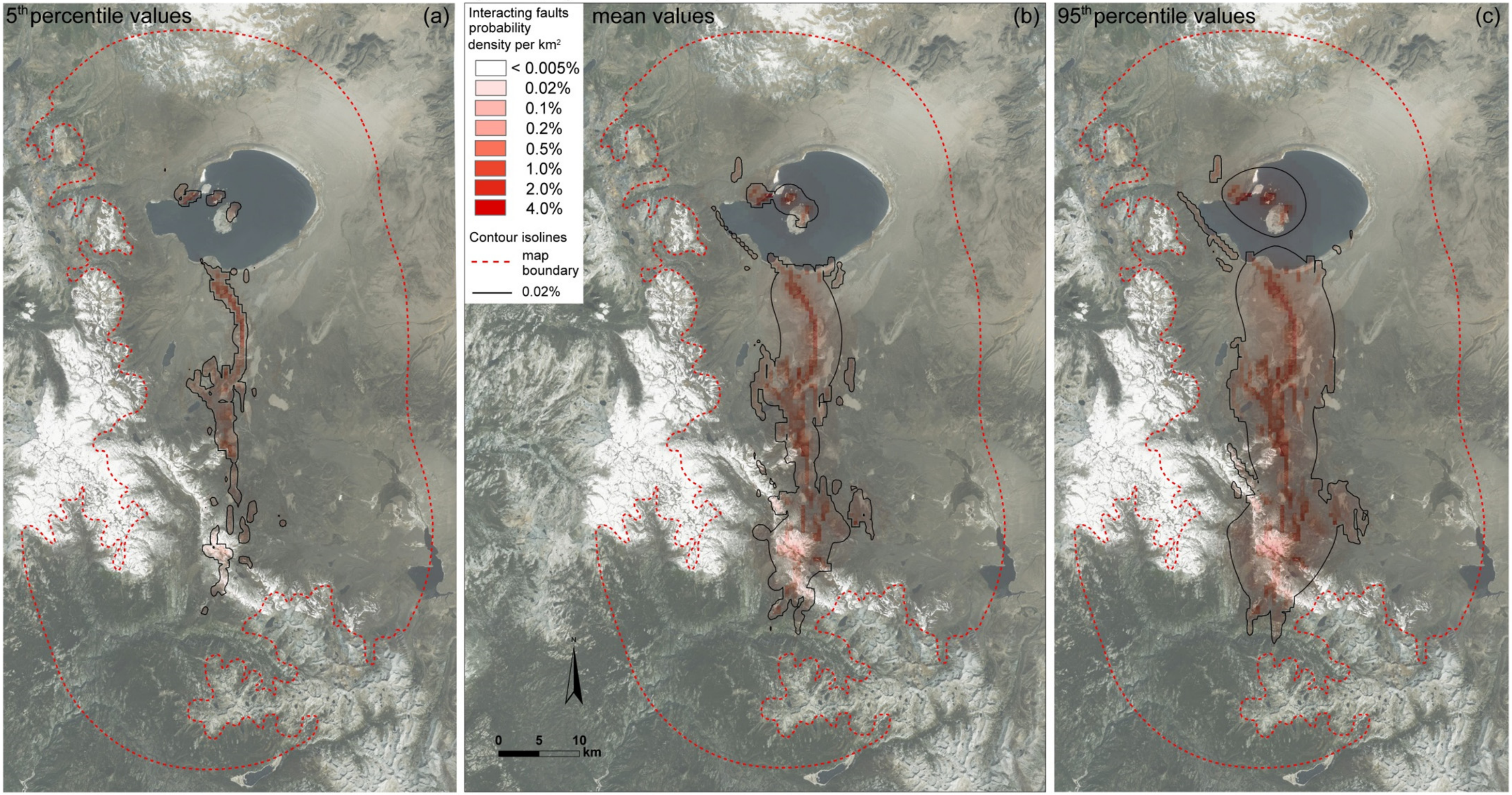
Indeed σ relates to the expected distance from the fault outcrops to the vent openings:

- 10 km depending on the **brittle/ductile transition** depth, or

- 2 km depending on **numerical models** for dike propagation.



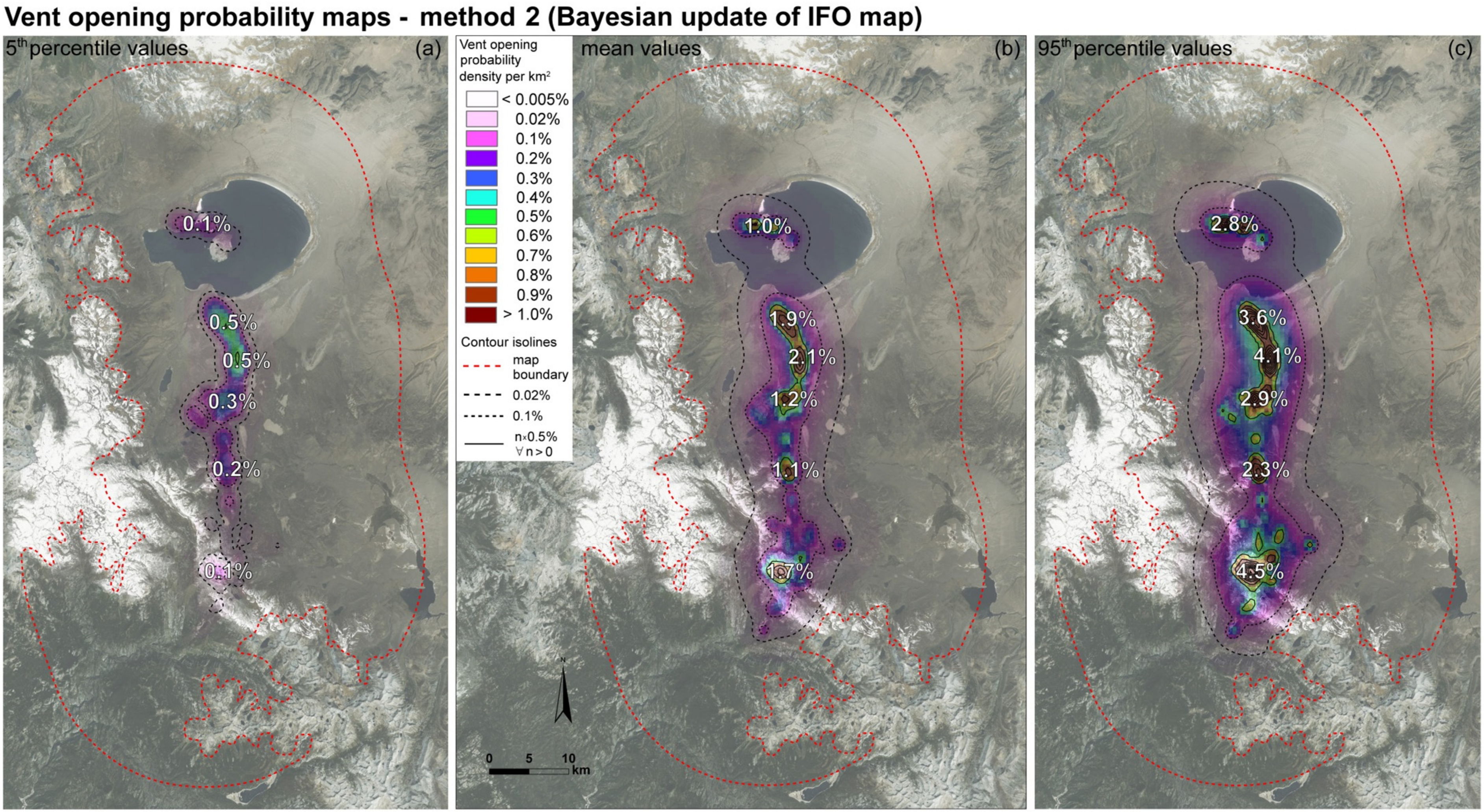
Interacting faults outcrops probability map (*a posteriori*)



The Bayes Theorem is used to calculate the *a posteriori* IFO probability density as the **product** of the *a priori* IFO and the likelihood functions of past vents locations.

It describes the fault locations that lie **closer to past events** as the most probable to drive a dike path again.

That map includes the **uncertainty sources** concerning tectonic data, and Mammoth Mountain unknown relevance.



The new density *f* of ***X*** is obtained by convolving the **likelihood kernel** describing ***d*** with the *a posteriori* IFO map.

Everything is done inside a **Monte Carlo sampling** that varies the uncertainty parameters p_1-p_4.

- Models integration: Bayesian Model Averaging -

Bayesian model averaging (BMA)

Let $(M_i)_{i=1,\dots,n}$ be different probability models for a random quantitative output δ, for example the vent opening probability map, such that we do not know which is the correct one.

The main step of BMA is to define some **scores** $[s(i)]_{i=1,\dots,n}$ for the models based on the available observations D about the quantity δ.

The scores are re-scaled to sum one, hence to define a discrete probability distribution on the models set $(M_i)_{i=1,\dots,n}$. These scores represent the probability of the models to be correct.

Equal *a priori* scores $[s(i)]_{i=1,\dots,n}$ were assumed for the models, such that $s_i = 1/n$ for all i.
The Bayes Theorem states that, for each i=1,…,n, the *a posteriori* weights are:

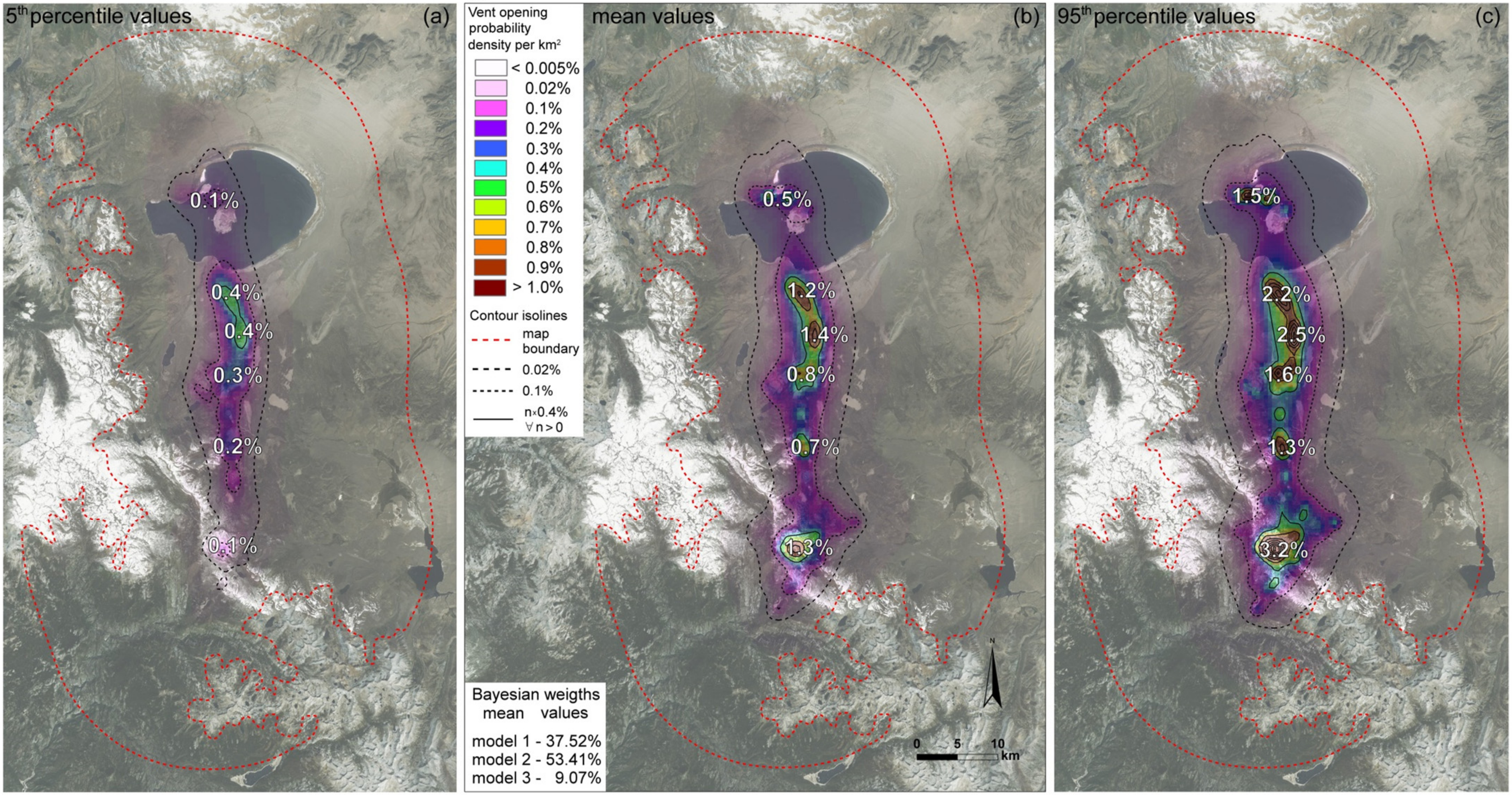
$$s(i|D) = \frac{L_i(D) \cdot s(i)}{C},$$

where L_i is the likelihood associated to model i, and C is a normalizing constant such that the new weights still sum to one.

The *a posteriori* scores will be proportional to the **likelihood** that the models give to the observed data.

An integrated model will defined by the **linear combination** of the outputs given by the different models, with the scores $[s(i|D)]_{i=1,\dots,n}$ as weighting coefficients.

Vent opening probability maps - Integrated method



Through the BMA we obtain the weights $(q_j)_{j=1,2,3}$ for a linear combination of the alternative models.
The uncertainty comes from the Monte Carlo sampling of $(p_i)_{i=1,\ldots,4}$.

q_1 = [35.02%, **37.52%**, 40.62%] – for the kernel density estimator
q_2 = [50.16%, **53.41%**, 56.59%] – for the Bayesian update of IFO map
q_3 = [7.27%, **9.07%**, 11.19%] – for a uniform distribution.

- Temporal modeling: Poisson-type processes -

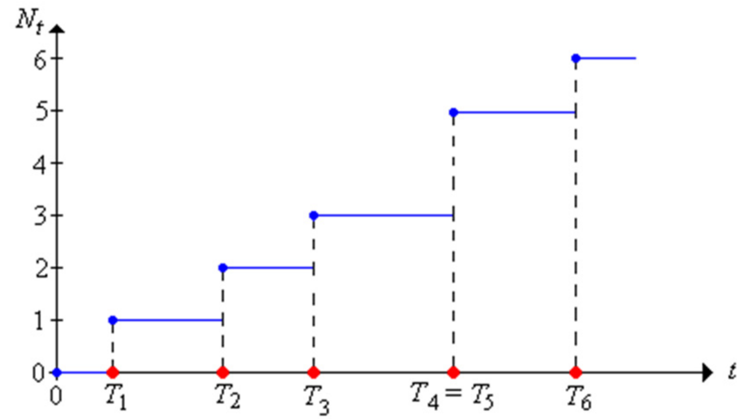
If Z is a **counting process,** Z(t) counts the number of events occurred before time t.

Its random paths are a non-decreasing step-functions.

Poisson processes sample the waiting times between events as i.i.d. exponential random variables.

Waiting times have mean 1/λ, where λ is called **intensity** of the process.

These are "memory-less", and there is no correlation between the points.



Example of counting process graph.

Nonhomogeneous Poisson processes (NHPP) enable the intensity λ(t) to change as a function of time.

Locally, the intensity function λ has still the meaning of the **average density** of new events occurring.

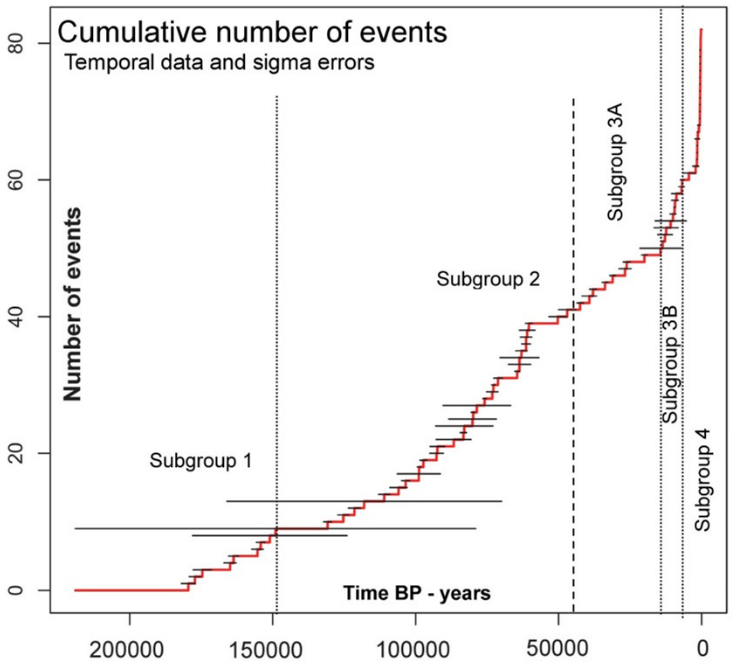
The integral ∫λ(t)dt gives the average number of events in the selected time interval.

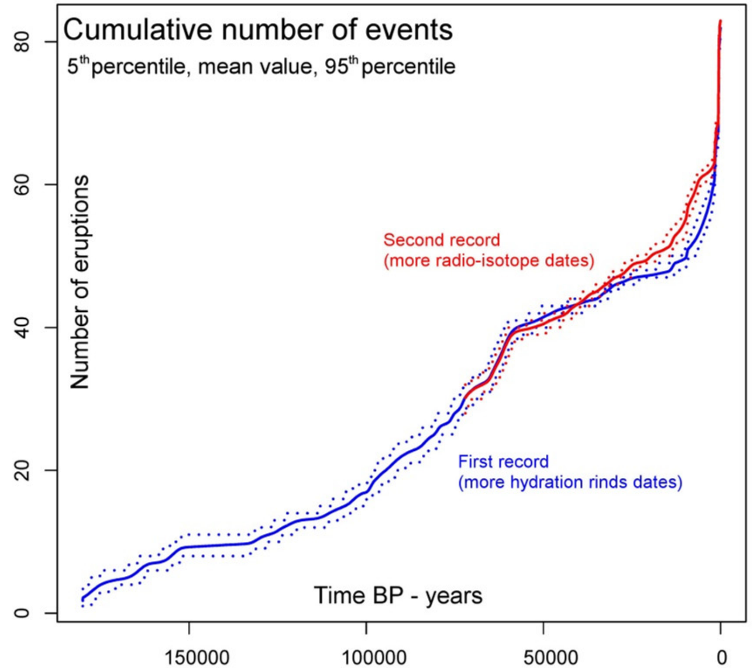
λ is the derivative of the *compensator function* - which morally is the average path of the counting process.

The **Cox processes** assume the intensity function λ affected by uncertainty.
They are also called doubly stochastic NHPP.

We will describe **three different Cox-type processes** for modeling the eruption record of LVVR, also including various self-excitement features.

- Temporal uncertainty quantification -

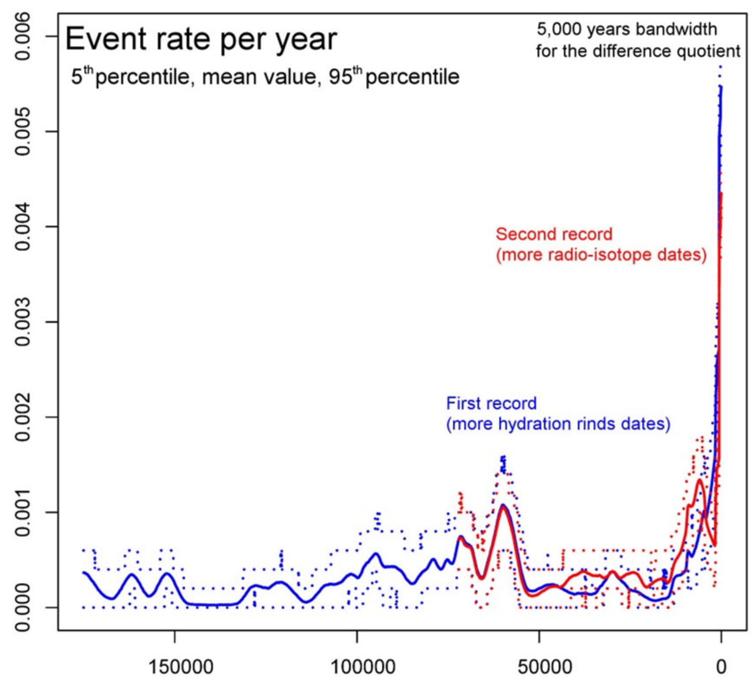




Uncertainty #1 - the model assumes Gaussian errors on the dates.

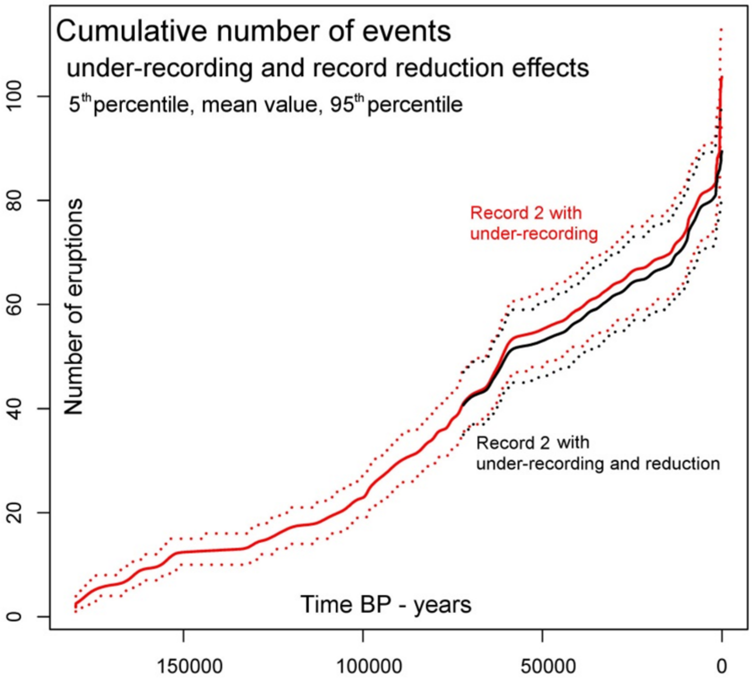
The record adopted here included some updates on [70 - 2] ka BP.

The intensity function λ is approximated by **difference quotients** on finite intervals.



Larger intervals smooth the variability of λ tending to a **constant value**.

Shorter intervals tend to a sequence of **local spikes**, localized at the time of past events.



Uncertainty #2 - **under-recording of eruptions**

it is likely that a number of lost events does exist.

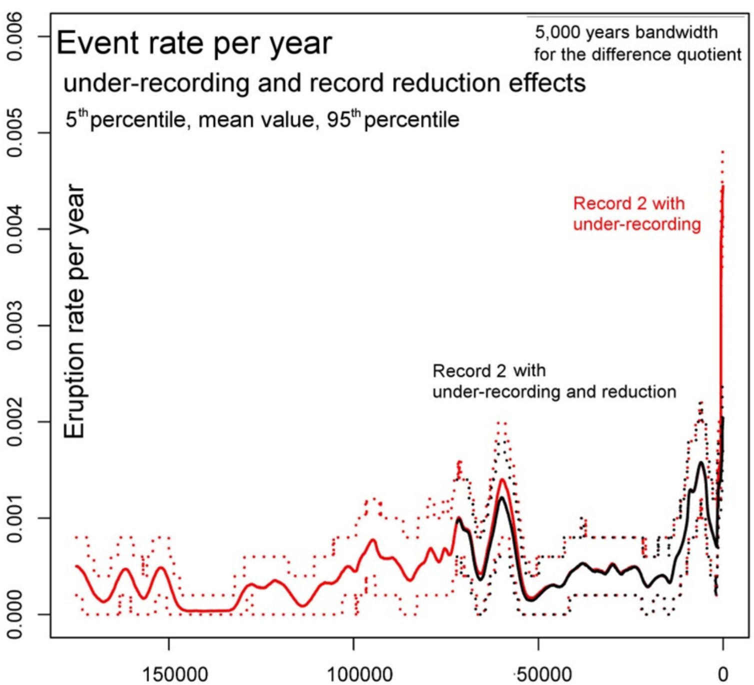
We assumed that past events can have a "**shadow** companion", i.e. sampled with the same age uncertainty, with chance *p(t)*.

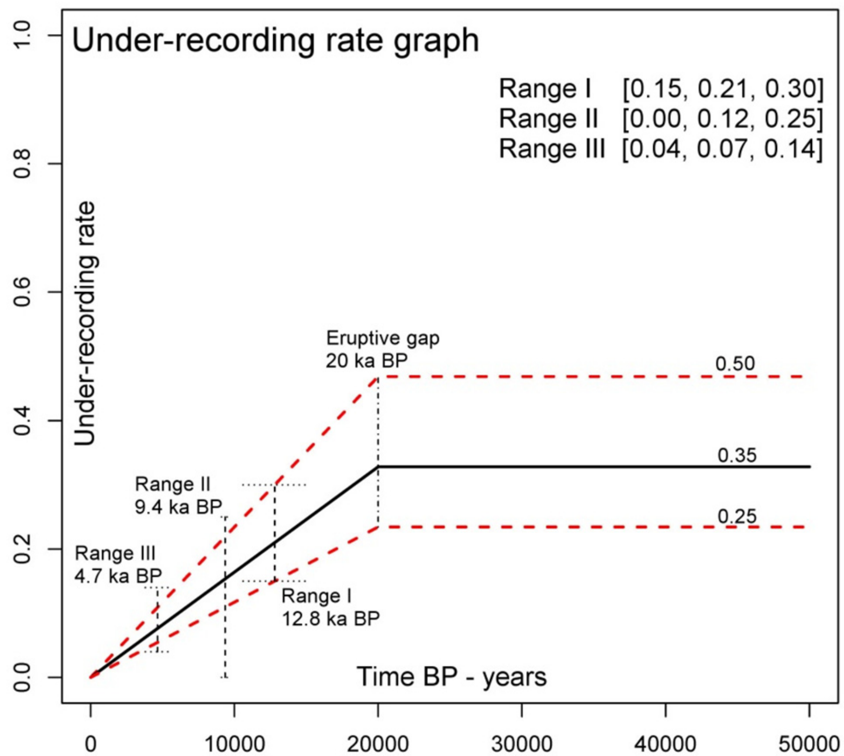
More lost events are sampled when the eruption frequency was higher. Including new events uniformly in time is also possible.

Uncertainty #3 - **over-counting of vents**

some of the recent single eruptions include several vents.

Even if spatially distinct, same-time events was counted as one eruption.



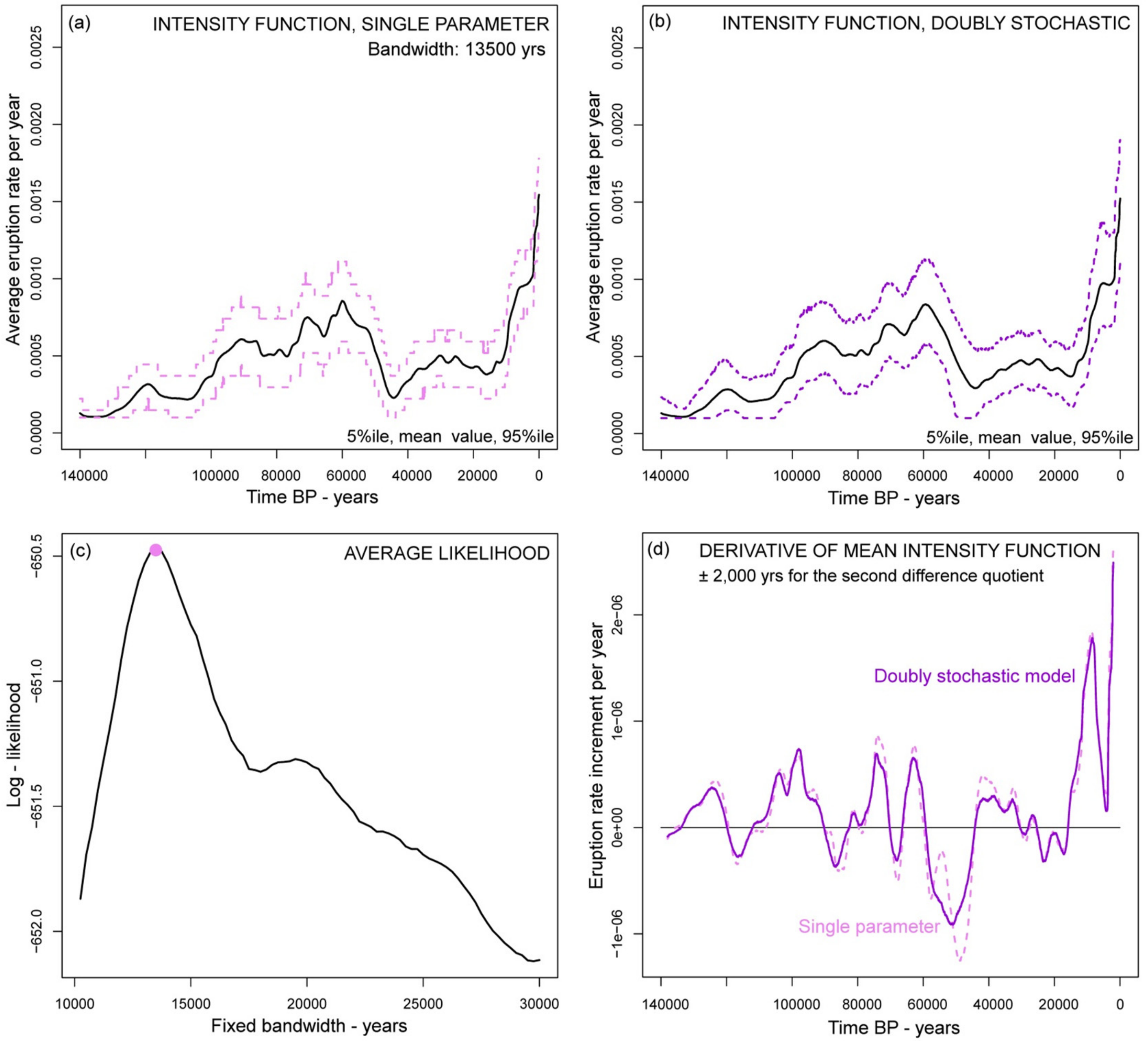


Preliminary lost events rate *p(t)*, relying on Campi Flegrei **expert judgement** data and uncertainty estimates.

A more specific model for LVVR under-recording rates is under development.

- Three temporal models -

MODEL I - NHPP WITH TIME WINDOW FIXED



The intensity function $\lambda(t)$ is the ratio

#events / T

on the left window [t-T, t].

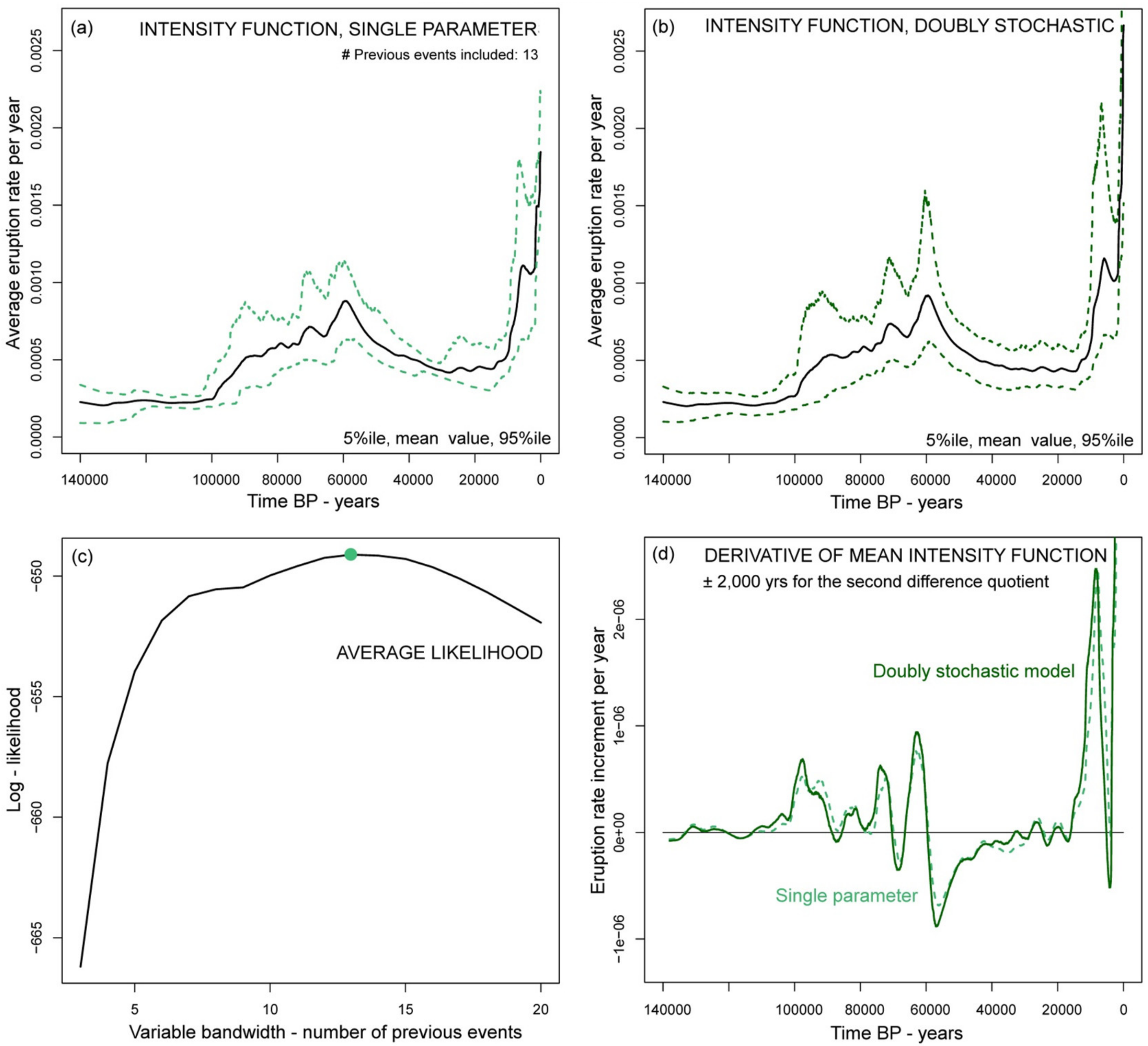
A minimum rate of $1/10^4$ avoids null intensity in the past.

The bandwidth T is selected to **maximize likelihood** of past events based on the preceding sequence.

This can be done in average, or **sample-wise** inside a Monte Carlo simulation.

The **second derivative** expresses the **evolution** of the intensity function.

MODEL II - NHPP WITH TIME WINDOW VARIABLE



The intensity function $\lambda(t)$ is the ratio

$k / (t_n - t_{n-k})$

where t_n is the time of last event before t.

Again minimum rate of $1/10^4$ is imposed.

The number k is selected to **maximize likelihood**, in average or sample-wise.

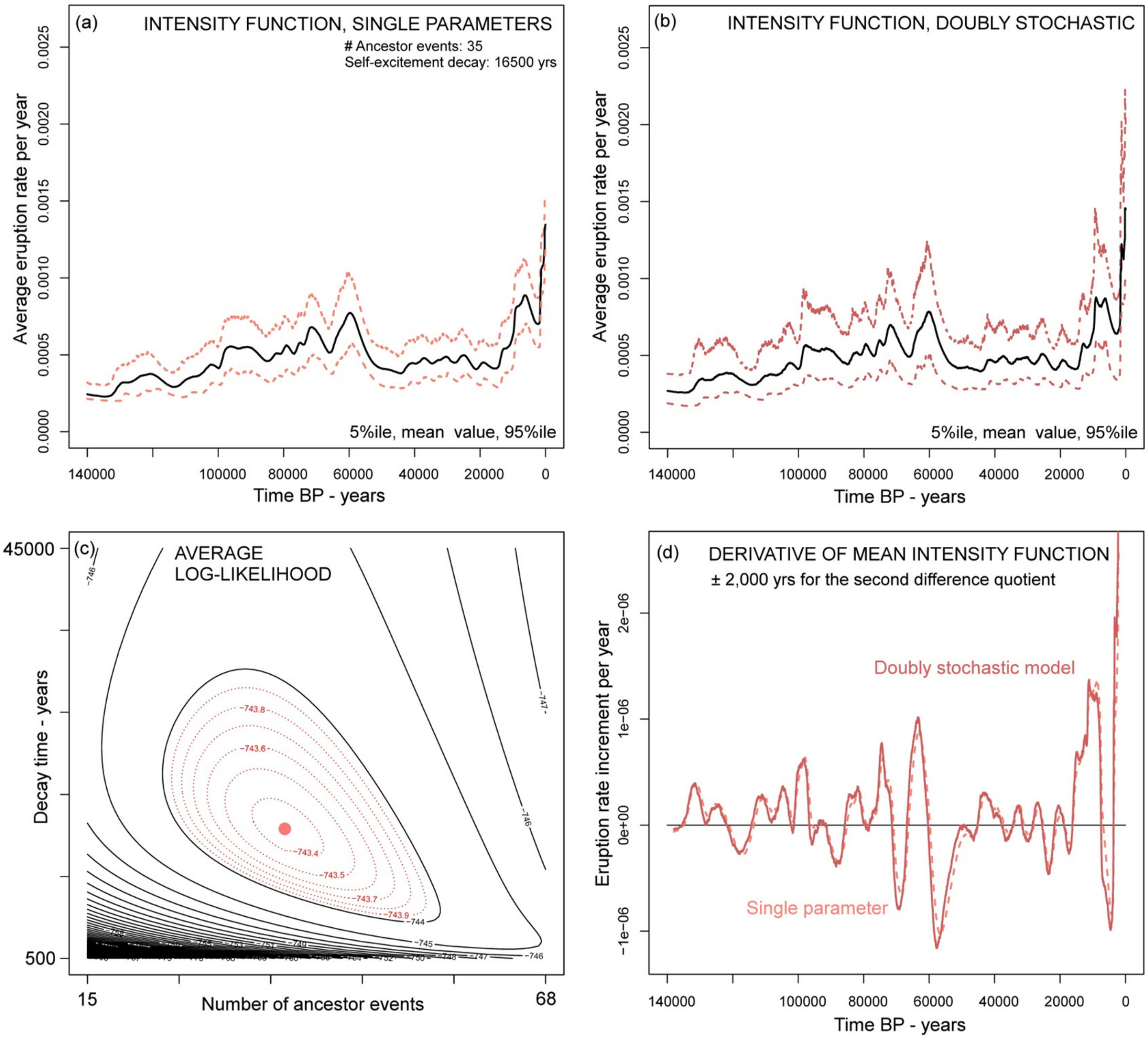
After event n^{th} the intensity decreases as a **rational function**

$k / (C + \Delta t)$

of the additional time Δt.

This gives the potential for **higher spikes** of intensity, and **longer tails**.

MODEL III - COX HAWKES PROCESS (EXPONENTIAL DECAY)



The intensity function $\lambda(t)$ is the sum

$$\lambda_0 + f(t)$$

where f is given by

$$\sum_{i=1}^{n} h \exp[-k\,(t - t_i)]$$

and t_n is the time of last event before t.

Each event produces additional intensity, which then decreases **exponentially**.

The likelihood function is now two dimensional:

λ_0 is equivalent to the **#ancestor** events,

f is selected by the choice of self-**excitement decay** duration.

- Preliminary temporal probability forecasts-

Summary of the models parameters: maxima of average likelihood, and doubly stochastic values.

Table 1a. Model Parameters

	TYPE	PARAMETERS		PARAMETERS VALUES							
				MAXIMIZING AVERAGE		MAXIMIZING LIKELIHOOD					
						5%ile		mean		95%ile	
MODEL I	FIXED BANDWIDTH	Bandwidth (yrs)		13500 yrs		8750		14020		21750	
MODEL II	VARIABLE BANDWIDTH	# Previous events		13		7		11.76		16	
MODEL III	COX-HAWKES (exponential decay)	# Ancestor events	Self-excite. duration (yrs)	35	16500 yrs	27	2000 yrs	41	14692 yrs	64	31000 yrs

Extrapolating the current intensity function, we obtained **probability forecasts** for future eruptions in the **next 10 and 50 yrs**.

Table 1b. Temporal Forecasts

	TYPE	MEAN RETURN TIME (currently) - yrs -			ERUPTION PROBABILITY FORECAST (exponential distribution)					
					10 years			50 years		
		5%ile	mean	95%ile	5%ile	mean	95%ile	5%ile	mean	95%ile
MODEL I	FIXED BANDWIDTH	525	657	906	1.10%	1.51%	1.89%	5.37%	7.33%	9.08%
MODEL II	VARIABLE BANDWIDTH	186	375	660	1.50%	2.63%	5.23%	7.30%	12.48%	23.56%
MODEL III	COX-HAWKES (exponential decay)	500	690	975	1.02%	1.44%	1.98%	5.00%	6.99%	9.52%

Model II gives the higher probabilities - its intensity is very sensitive by the recent **many close-timed** events. It focuses on them forgetting the previous record.