Statistical theory of probabilistic hazard maps:

A probability distribution for the hazard boundary location

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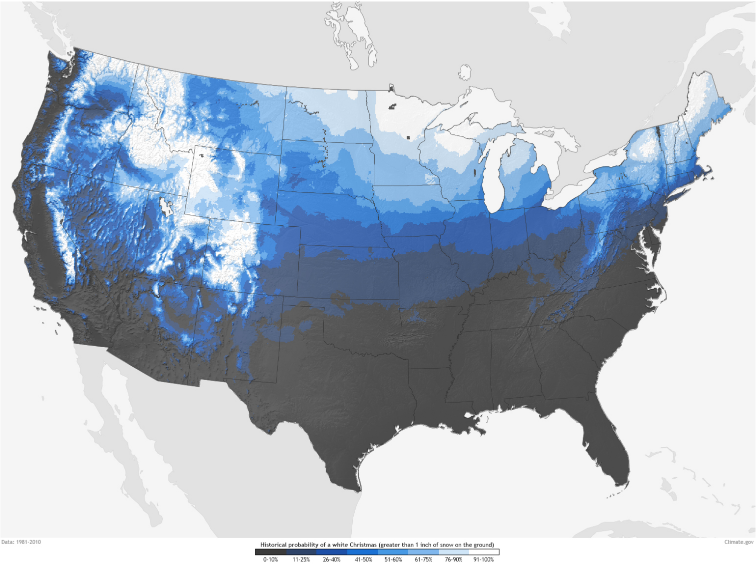
Probability Maps

A Festive Example: Prob. White Christmas



Probability Maps

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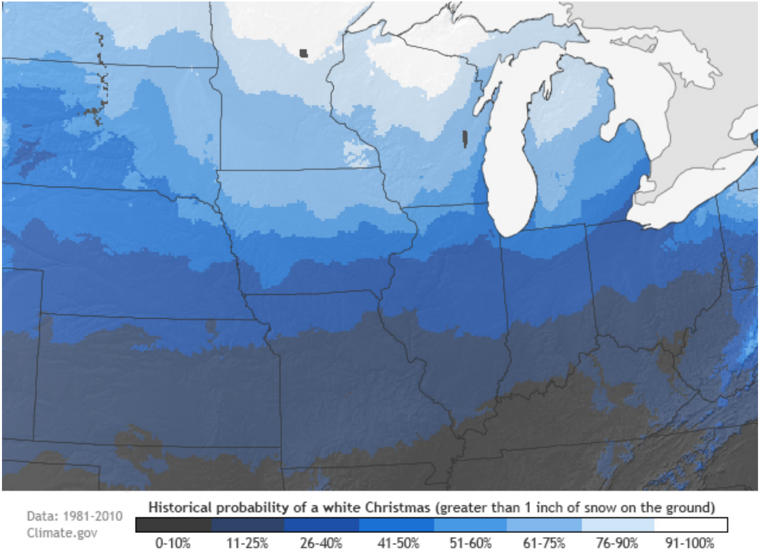
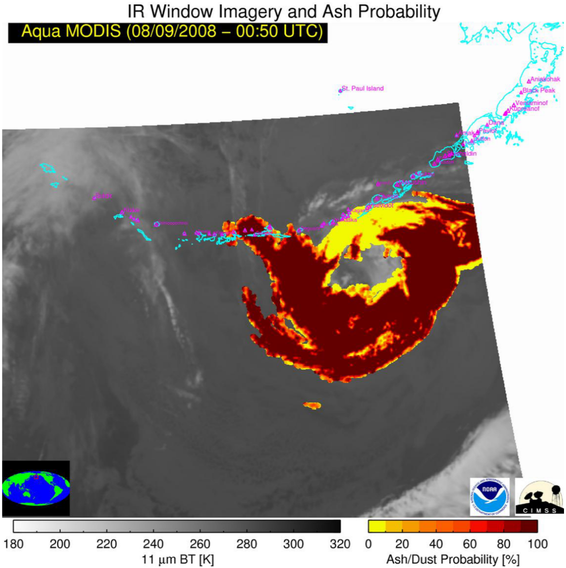


Probability Maps

A Volcanic Example: Ash from Kasatochi Volcano (2008)

> Where is the most likely boundary of the detectable ash?



Key Questions

- Where is the most likely hazard/ no hazard boundary?

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- How can we apply traditional statistical measures to probabilistic hazard maps?
- What is the connection between underlying parameter space, a physical model, and a PHM for the process?

Table of Contents

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  $f : \mathcal{B} \to \mathbb{R}^+$ which measures the parameter space $\mathcal{B}$.

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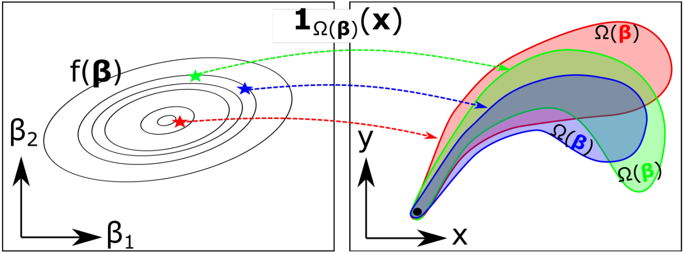
- Physics model output variable:
  $h = h(\mathbf{x}; \beta)$ (flow height, ash concentration, etc.)

Constructing a prob. hazard map (PHM)

Ensemble Physical Modelling

- $\beta \in \mathcal{B} \subset \mathbb{R}^n$ (model input parameters)
  $f : \mathcal{B} \to \mathbb{R}^+$ which measures the parameter space $\mathcal{B}$.
- Physics model output variable:
  $h = h(\mathbf{x}; \beta)$ (flow height, ash concentration, etc.)
- Indicator Function: $\mathbf{1}_{\Omega(\beta)}(\mathbf{x}) := \begin{cases} 1 & \{\mathbf{x} \mid h(\mathbf{x};\beta) > h_0\} \\ 0 & \{\mathbf{x} \mid h(\mathbf{x};\beta) \leq h_0\} \end{cases}$



PHM Definition

With a parameter PDF $f(\beta)$.

PHM Definition

$$\phi(\mathbf{x}) := \int_{\mathcal{B}} f(\boldsymbol{\beta}) \mathbf{1}_{\Omega(\boldsymbol{\beta})}(\mathbf{x}) \, \mathrm{d}\boldsymbol{\beta} \tag{1}$$

PHM Definition

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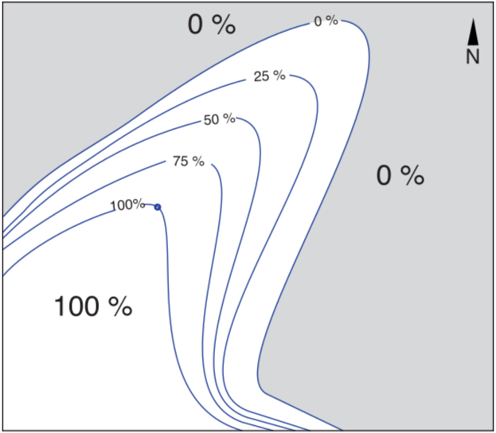
PHM Definition

$$\phi(\mathbf{x}) := \int_{\mathcal{B}} f(\beta)\mathbf{1}_{\Omega(\beta)}(\mathbf{x})\,\mathrm{d}\beta \qquad (1)$$

If $\beta \sim \mathcal{U}$:

$$\phi(\mathbf{x}) := ⨍_{\mathcal{B}} \mathbf{1}_{\Omega(\beta)}(\mathbf{x})\,\mathrm{d}\beta \qquad (2)$$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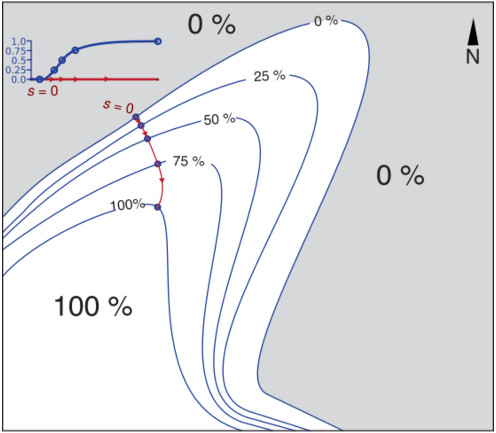
Integral Curves:

$$\frac{d\mathbf{x_n}}{ds} = \nabla\phi$$

$$\phi(s) := \phi(\mathbf{x_n}(s))$$

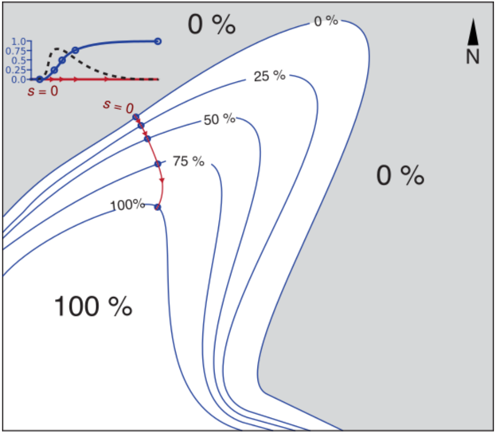
$$= \mathbb{P}(s_{bdy} \leq s)$$

> CDF for the hazard boundary along the integral curves



> CDF for the hazard boundary along the integral curves

> Can define associated PDF with derivative along curve

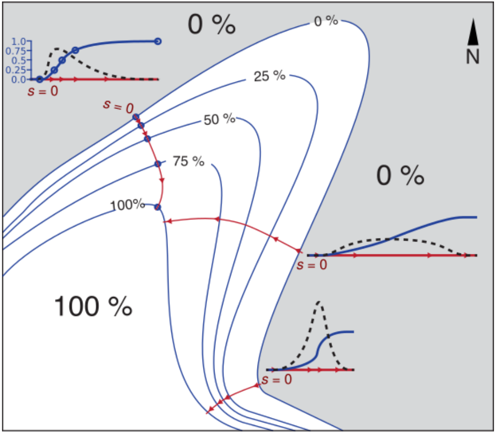


> CDF for the hazard boundary along the integral curves

> Can define associated PDF with derivative along curve

> Apply to all curves

> Q: Global definition for the PDF?



Connect the PDFs on all the integral curves:

$$\frac{d\phi(s)}{ds} = \nabla\phi(\mathbf{x_n}(s)) \cdot \frac{d\mathbf{x_n}}{ds} = |\nabla\phi(\mathbf{x_n}(s))|^2 \tag{3}$$

Prob. Hazard Density Map (PHDM)

Flow boundary PDF

Connect the PDFs on all the integral curves:

$$\frac{d\phi(s)}{ds} = \nabla\phi(\mathbf{x_n}(s)) \cdot \frac{d\mathbf{x_n}}{ds} = |\nabla\phi(\mathbf{x_n}(s))|^2 \qquad (3)$$

$$\int_0^\infty \frac{d\phi}{ds}\,\mathrm{d}s = \int_0^\infty |\nabla\phi(\mathbf{x_n}(s))|^2\,\mathrm{d}s = 1 \qquad (4)$$

Connect the PDFs on all the integral curves:

$$\frac{d\phi(s)}{ds} = \nabla\phi(\mathbf{x_n}(s)) \cdot \frac{d\mathbf{x_n}}{ds} = |\nabla\phi(\mathbf{x_n}(s))|^2 \tag{3}$$

$$\int_0^\infty \frac{d\phi}{ds}\,\mathrm{d}s = \int_0^\infty |\nabla\phi(\mathbf{x_n}(s))|^2\,\mathrm{d}s = 1 \tag{4}$$

Def PHDM

$$\psi(\mathbf{x}) = \frac{1}{Q}|\nabla\phi|^2 \tag{5}$$

where

$$Q = \int_{\Omega^0} |\nabla\phi|^2\,\mathrm{d}A$$

On each integral curve:

- $$\mu = \int_0^\infty 1 - \phi(\mathbf{x_n}(s))\,\mathrm{d}s \qquad (6)$$

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- $$\mu = \int_0^\infty 1 - \phi(\mathbf{x_n}(s))\,\mathrm{d}s \qquad (6)$$
- $$\sigma^2 = \int_0^\infty s^2 \frac{d\phi}{ds}\,\mathrm{d}s - \mu^2 \qquad (7)$$

On each integral curve:

- $$\mu = \int_0^\infty 1 - \phi(\mathbf{x_n}(s))\,\mathrm{d}s \tag{6}$$
- $$\sigma^2 = \int_0^\infty s^2 \frac{d\phi}{ds}\,\mathrm{d}s - \mu^2 \tag{7}$$
- Spatial Mean: $\mathbf{x}_\mu = \mathbf{x_n}(\mu)$
  Region Encompassing
  1σ Uncertainty: $\mathbf{x}_{\mu\pm\sigma} = \mathbf{x_n}(\mu \pm \sigma)$

On each integral curve:

$$\mathbf{x}_{median} = 50\% \text{ Prob. Contour} \tag{8}$$

$$\mathbf{x}_{mode} = \underset{\mathbf{x_n}(s)}{\arg\max}\, \psi \tag{9}$$

Table of Contents

Granular Flow on an Inclined Plane

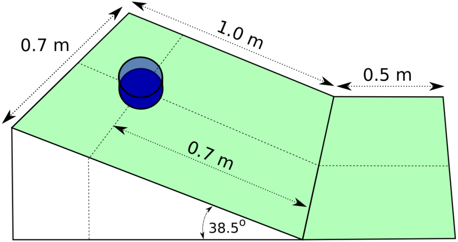
Setup

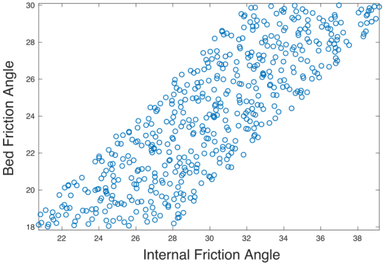
> Initial condition:
5 cm radius pile

> Titan2D geophysical mass flow code

> Mohr-Coulomb rheology

> More info at:
Patra et al., 2018.
arxiv.org/abs/1805.12104



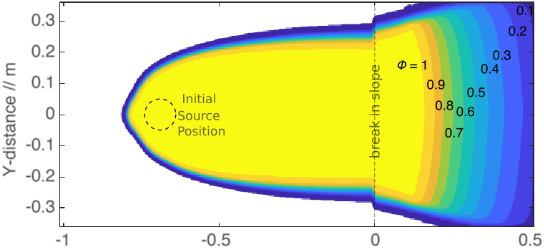


Granular Flow on an Inclined Plane
PHM

> Flow constrained on the slope

> High prob. dispersion in the runout

> Sudden spread at the break in slope

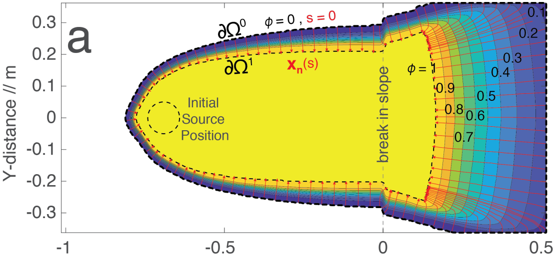


> Flow constrained on the slope

> High prob. dispersion in the runout

> Sudden spread at the break in slope

> Euler's method to calculate integral curves $\mathbf{x_n}(s_k)$



Granular Flow on an Inclined Plane

Mean, Median Comparison

> Numerical integration to find μ, σ

> Mean $\neq$ Median $\implies$ Distribution is asymmetric

> Region within 1σ $\neq$ middle 68%

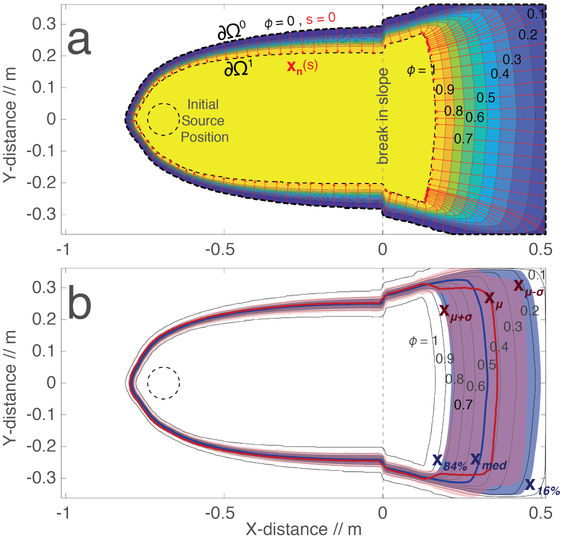


Table of Contents

Natural Debris Flow

Atenquique, Mex., 1955

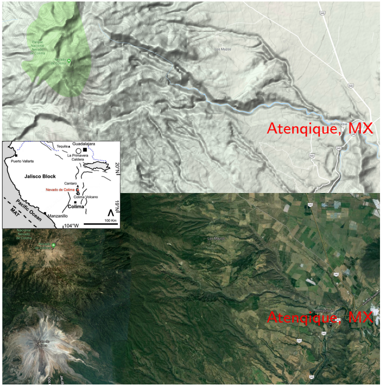
> Deadly debris flow triggered by multiple landslides on the flanks of Nevado de Colima (México)

> Voellmy-Salm rheology: variation of friction parameters

> More info at:

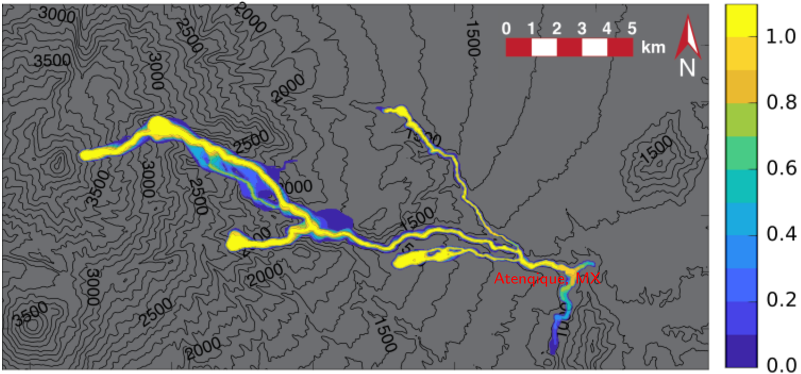
Bevilacqua et al.,
Nat. Haz. Earth Syst. Sci. Discuss,
doi.org/10.5194/nhess-2018-294,

in review, 2018.



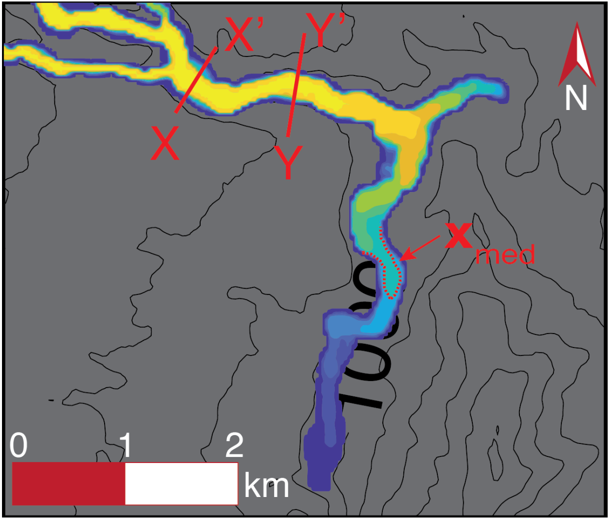
Natural Debris Flow

PHM: plausible parameter sampling

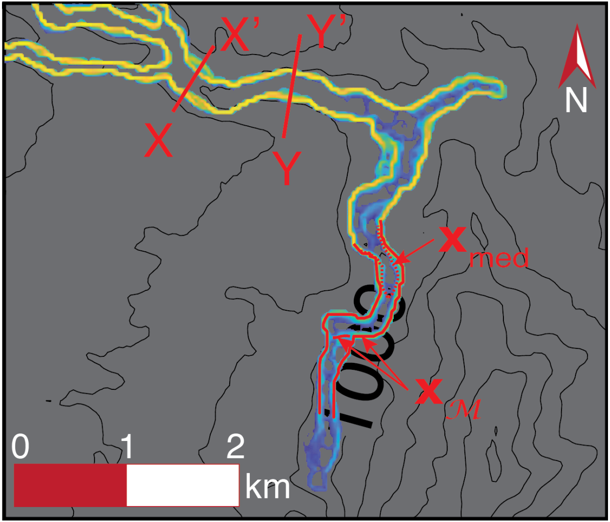


Natural Debris Flow

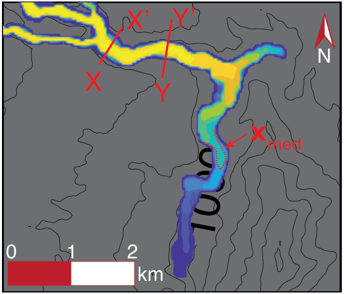
PHM: A closer look

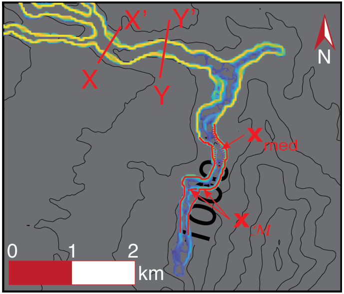


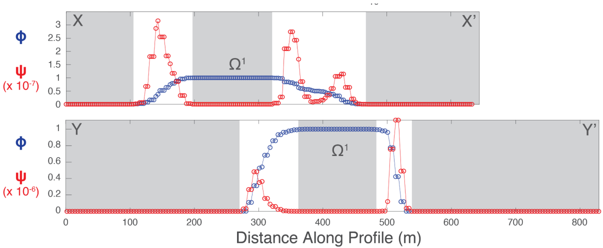
Natural Debris Flow
PHDM



Natural Debris Flow: Atenquique, 1955







Conclusions

> PHM propagate uncertainty in parameter space into uncertainty in geographic space and represent a distribution for the location of the likely boundary of the hazard (not just point-wise probability).

> Can define a hazard density function (PHDM - square gradient of PHM) which gives a spatially varying PDF for boundary location

> Can calculate the full suite of typical statistical quantities from the PHM (moments, mode(s), likelihood, etc.)

> Distribution of hazard boundary location is typically non-normal (even worse - asymmetric, spatially varying, multimodal)

> See full text at:

Hyman, D. M., Bevilacqua, A., and Bursik, M. I. , Nat. Hazards Earth Syst. Sci. Discuss., doi.org/10.5194/nhess-2018-344, in review, 2018.

{ Thank You}