Estimates and models of the statistical dependence between local earthquakes and flank eruptions at Mt. Etna volcano (Italy): an old topic revised through new historical data

Andrea Bevilacqua(1), Raffaele Azzaro(2), Stefano Branca(2), Augusto Neri(1), Franco Flandoli(3), Salvatore D'Amico(2), Emanuela De Beni(2)

(1) Istituto Nazionale di Geofisica e Vulcanologia, Pisa, Italy
(2) Istituto Nazionale di Geofisica e Vulcanologia, Catania, Italy
(3) Scuola Normale Superiore, Pisa, Italy.
Our target is the **statistical modeling** of the correlation between **major earthquakes** and **flank eruptions** at Mt. Etna volcano.

We target to quantify:

- **how much is the increment** of the probabilistic rate of major earthquakes in the days or months after a flank eruption onset or end,

- **for how much time** this hypothetical increase of the probability can last for.

In the following we are detailing **three different topics** which are **strongly linked** together:

- The analysis of the historical time series of the **earthquakes** (EQs)

- The analysis of the historical time series of the **flank eruptions** (either onset or end)

- The time series of the EQs observed **from the point of view** of flank eruptions.

In particular, we consider two updated datasets: the **Macroseismic Catalog of Etnean Earthquakes** (CMTE), and the **Flank Eruptions catalog** over the time interval 1800-2018.

**CRUCIAL QUESTION**

How much and how long the Civil Protection authorities should expect to deal with **damaging shocks** during and after a flank eruption emergency?

**KEY IDEA**

We look at the time series of the EQ from a family of **frames of reference** which are left-side anchored to the onset or the end of flank eruptions.

We do this for **every flank eruption** and we make statistics of what we see.
### Previous statistical studies

<table>
<thead>
<tr>
<th>Study</th>
<th>Data</th>
<th>Test/Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sharp, Lombardo, Davis (1981)</strong></td>
<td>Earthquakes in the time interval 1600 - 1978 with $I_0 \geq V$ (620 events - 146 main-shocks)</td>
<td>Statistical test of independence between Poisson processes, based on <em><a href="https://doi.org/10.2307/2344634">Cox (1955)</a></em>, generalized to a case with rate changes. Conclusion – (i) Poisson distribution of flank eruptions and main-shocks. (ii) Abnormal number of flank eruptions after summit eruptions and after main-shocks earthquakes.</td>
</tr>
<tr>
<td>Eruptions in the time interval 1600 - 1978</td>
<td>(132 events – of which 49 flank)</td>
<td></td>
</tr>
<tr>
<td><strong>Nercessian, Him, Sapin (1991)</strong></td>
<td>Modification of the empirical method of aftershock removal (620 events – of which 180 main-shocks)</td>
<td>Test of <em><a href="https://doi.org/10.2307/2344634">Cox (1955)</a></em> assuming the eruptions as precursors of the earthquakes. Conclusion – Abnormal number of earthquakes after the onset of flank eruptions and after the end of flank eruptions.</td>
</tr>
<tr>
<td><strong>Gasperini, Gresta, Mulargia (1990)</strong></td>
<td>Earthquakes in the time interval 1978 - 1987 magnitude &gt; 2.8 (1458 events)</td>
<td>Earthquake clusters recognition and modeling. Conclusion – correlation not calculated because ‘insufficient data’, and not qualitatively apparent to the authors.</td>
</tr>
<tr>
<td>Eruptions (18 events - 9 flank)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flank eruptions (11 events)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gresta, Marzocchi, Mulargia (1994)</strong></td>
<td>Earthquakes in the time interval 1600 - 1989. with $I_0 \geq IX$ (7 events)</td>
<td>Correlation test according to the Spearman ranking coefficient. Conclusion – correlation between the end of major eruptions and the major earthquakes, and not with the eruption onsets.</td>
</tr>
<tr>
<td>Eruptions with volume $\geq 10^7$ m$^3$ (40 events)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Mulargia, Tinti, Boschi (1985)** Kolmogorov-Smirnov test confirms Poisson distribution of the flank eruptions.}

**DATASET 479 YRS**

$\sim 10^2$ vs $10^2$ events

**DATASET 10 -17 YRS**

$\sim 10^3$ vs $10^1$ events

**DATASET 490 YRS**

$\sim 10^1$ vs $10^2$ events
Overview of CMTE and of the Flank Eruptions catalog

Figure. (a) cumulative number and (b) annual rate of EQs. Years with >45 events are marked.

(c) histograms of magnitude ($M_L$). The main thresholds adopted are marked.

(d) cumulative and (e) annual number of flank eruptions.

(f) histograms of inter-event times. The 20th, 50th and 90th percentiles are marked.

We use a macroseismic estimate of the magnitude $M_L$ in the pre-instrumental part of the catalog (Azzaro, D'Amico, Tuvè, 2011).

The annual rate is calculated with first-order, left-side finite differences:

$$\dot{\lambda}(t) = \frac{N(t) - N(t - T)}{T}$$
We considered the events after 1875 or 1850, depending on the magnitude considered.

**Figure.** Cumulative number and annual rate of the EQs.

(a,b) $M_L \geq 3.0$, years with >8 events are labeled.  
(c,d) $M_L \geq 3.5$, years with >1 event are labeled.  
(e,f) $M_L \geq 4.0$, all events are labeled.

In (e), the purple colored events were less than 120 days after a flank eruption.

**Underlined** events occurred during a flank eruption.

In (e,f) a relation with flank eruptions of $M_L \geq 4.0$ is observed in 9/12 events.
The sumative variable $\xi(\Delta t)$

In literature, to test if the events occurred at the times $(B_j)_{j=1,..,m}$ are precursors to those occurred in $(A_i)_{i=1,..,n}$ a sumative variable is evaluated. First we repeat this approach on our new datasets.

Let be $\forall (t_1, t_2): N([t_1, t_2]) := |(A_i)_{i=1,..,n} \cap [t_1, t_2]|$.

Then $\forall \Delta t > 0$, let $\xi(t)$ be the variabile defined as:

$$\xi(\Delta t) = \sum_{j=1}^{m} N((B_j, B_j + \Delta t]).$$

For example, $\xi(t)$ is the number of EQs happened less than $t$ days after a flank eruption onset or end.

Then:

$\xi(\Delta t)$ is approximated by the sum of $m$ independent Poisson variables, and so, by a new Poisson random variable. Moreover, we have $E[\xi(\Delta t)] = m \lambda \Delta t \approx \frac{nm\Delta t}{T}$.

The calculation is generalized to the case of $N$ being a nonhomogeneous Poisson process, assuming $\lambda$ constant over appropriate subintervals of $[0, T]$. In the following we adopt subintervals of 25 yrs duration.

$H_0$ - null hypothesis

$\xi(\Delta t)$ is a Poisson random variable and $E[\xi(t)] = \frac{nm\Delta t}{T}$.

$H_1$ - alternative hypothesis

$\xi(\Delta t)$ is not a Poisson random variable with $E[\xi(t)] = \frac{nm\Delta t}{T}$, and so the events in $(A_i)_{i=1,..,n}$ are not independent of those in $(B_j)_{j=1,..,m}$.

The test works under the hypothesis that $(A_i)_{i=1,..,n}$ is Poisson, and $(B_j)_{j=1,..,m}$ is not clustered at the scale of $\Delta t$.

The result of the test as a function of $t$ is a step graph, marking the times $\Delta t$ at which $H_0$ is rejected with a level of confidence $\alpha = 90\%$.

The test is performed by comparing $\xi(\Delta t)$ to the 5th and 95th percentiles of a Poisson random variable of intensity $\frac{nm\Delta t}{T}$.

In the special case that: $(A_i)_{i=1,..,n-1} = (B_j)_{j=1,..,m}$, the test verifies instead the total randomness, i.e. the Poisson hypothesis.

This test does not reject the Poisson hypothesis on the flank eruptions onsets or ends.

This test does not reject the Poisson hypothesis on the main shocks.

But if we empirically remove the aftershocks, the test does not reject the Poisson hypothesis on the main shocks.

The same test rejects $H_0$ for every EQ dataset, because of the clusters.

This enables us to test if the flank eruption onsets or ends increase the probabilistic rate of the main-shocks, and hence the rate of all the EQs.

For example, $\xi(t)$ is the number of EQs happened less than $t$ days after a flank eruption onset or end.
**Main-shocks and aftershocks**

**Empirical algorithm**

*Sharp et al., (1981)*

∀\(t_i\) earthquake, \(t_j\) aftershock if:

\[ t_i - t_j < f(M_L(t_j)) \]

- \(f(3.0) = 38\) days
- \(f(3.5) = 71\) days
- \(f(4.0) = 135\) days

+Nercissian et al. (1991)

some deleted \(t_i\) are re-inserted as a main-shock if:

\[ M_L(t_i) \geq M_L(t_j) \text{ and } M_L(t_i) = \max\{M_L(t_k): t_i - t_j < f(M_L(t_j))\} \]

The mean offspring of a main-shock is:

\[ \mu = (E[\text{size}] - 1) / E[\text{size}] \]

because \(E[\text{size}]\) is the sum of a geometric series of factor \(\mu\).

---

**Figure.** (a-c) Histograms of the cluster size, EQ post 1800.

(d-f) Cumulative number of empirical Main-shocks.

<table>
<thead>
<tr>
<th>Event number</th>
<th>Main-shocks, (M_L \geq 3.0)</th>
<th>Main-shocks, (M_L \geq 3.5)</th>
<th>Main-shocks, (M_L \geq 4.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1875</td>
<td></td>
<td>1865</td>
<td>2018*</td>
</tr>
<tr>
<td>1985</td>
<td>169 clusters (M_L \geq 3.0)</td>
<td>47 clusters (M_L \geq 3.5)</td>
<td>11 clusters (M_L \geq 4.0)</td>
</tr>
<tr>
<td>1879</td>
<td>1985</td>
<td>1914</td>
<td>2002</td>
</tr>
<tr>
<td>1894</td>
<td></td>
<td>1911</td>
<td></td>
</tr>
<tr>
<td>1883</td>
<td></td>
<td>1894</td>
<td></td>
</tr>
<tr>
<td>1865</td>
<td></td>
<td></td>
<td>1865</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cluster size</th>
<th>Number of clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>10</td>
</tr>
<tr>
<td>3-4</td>
<td>20</td>
</tr>
<tr>
<td>5-6</td>
<td>30</td>
</tr>
<tr>
<td>7-8</td>
<td>40</td>
</tr>
<tr>
<td>9-10</td>
<td>50</td>
</tr>
<tr>
<td>11-12</td>
<td>60</td>
</tr>
<tr>
<td>13-14</td>
<td>70</td>
</tr>
<tr>
<td>15-16</td>
<td>80</td>
</tr>
<tr>
<td>17-18</td>
<td>90</td>
</tr>
<tr>
<td>19-20</td>
<td>100</td>
</tr>
</tbody>
</table>

\[ E[\text{size}] = 1.99 \quad \mu = 0.50 \]

\[ E[\text{size}] = 1.38 \quad \mu = 0.28 \]

\[ E[\text{size}] = 1.18 \quad \mu = 0.15 \]

---

San Francisco (CA), 11 December 2019
Rejection of the independence hypothesis of Main-shocks after Flank eruptions

Figure. Step graphs related to the test of independence.

$\xi(\Delta t)$ sums the main-shocks occurred less than $\Delta t$ days after the (a-c) onset or (d-f) end of any flank eruption.

ERUPTION ONSET
(a) Main-shocks, $M_s \geq 3.0$. H0 rejected at $\Delta t \leq 25$ days.
(b) Main-shocks, $M_s \geq 3.5$. H0 rejected at $\Delta t \leq 50$ days.
(c) Main-shocks, $M_s \geq 4.0$. H0 rejected at $\Delta t \leq 60$ days.

ERUPTION END
(d) Main-shocks, $M_s \geq 3.0$. H0 rejected at $\Delta t \leq 15$ days.
(e) Main-shocks, $M_s \geq 3.5$. H0 rejected at $\Delta t \leq 30$ days.
(f) Main-shocks, $M_s \geq 4.0$. H0 rejected at $\Delta t \leq 30$ days.
Figure. Histograms of the difference $\Delta t = (t_i - e_j)$ between earthquakes times $t_i$ and flank eruption onset times $e_j$.

We consider all the possible pairs with a time delay lower than 120 days:

$$C = \{ \Delta t : \Delta t < 120 \text{ days} \}$$

(a) $M_L \geq 3.0$,  
(b) $M_L \geq 3.5$,  
(c) $M_L \geq 4.0$.

### Table.

<table>
<thead>
<tr>
<th></th>
<th>$M_L \geq 3.0$</th>
<th>$M_L \geq 3.5$</th>
<th>$M_L \geq 4.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQ in the days before the eruption onset</td>
<td>31%</td>
<td>28%</td>
<td>0%</td>
</tr>
<tr>
<td>EQ in the same day of the eruption onset</td>
<td>9%</td>
<td>10%</td>
<td>29%</td>
</tr>
<tr>
<td>EQ in the days after the eruption onset</td>
<td>60%</td>
<td>62%</td>
<td>71%</td>
</tr>
<tr>
<td>If excluding differences &lt; 5 days:</td>
<td>28%</td>
<td>20%</td>
<td>0%</td>
</tr>
<tr>
<td>Of those EQs after the eruption onset:</td>
<td>69%</td>
<td>89%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Time difference percentile values of the EQ and the flank eruption onsets (if $\Delta t < 120$ days).

EQ before the eruption onset

EQ after the eruption onset

EQ within 0 to 45 days from the eruption

EQ within 45 to 120 days from the eruption
**Figure.** Histograms of the difference $\Delta t = (t_i - \hat{t}_j)$ between earthquake times $t_i$ and flank eruption end times $\hat{t}_j$.

We consider all the possible pairs with a time delay lower than 120 days:

$$C = \{\Delta t : \Delta t < 120 \text{ days}\}$$

(a) $M_L \geq 3.0$,  
(b) $M_L \geq 3.5$,  
(c) $M_L \geq 4.0$.

<table>
<thead>
<tr>
<th>Time difference percentile values of the EQs and the flank eruption ends, (if $\Delta t &lt; 120$ days).</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_L \geq 3.0$</td>
</tr>
<tr>
<td>EQ in the days before the eruption end</td>
</tr>
<tr>
<td>EQ in the same day of the eruption end</td>
</tr>
<tr>
<td>EQ in the days after the eruption end</td>
</tr>
<tr>
<td>Of those EQs after the eruption end:</td>
</tr>
<tr>
<td>EQ within 0 to 45 days from the eruption</td>
</tr>
<tr>
<td>EQ within 45 to 120 days from the eruption</td>
</tr>
</tbody>
</table>
The annual rate is calculated with first-order, finite differences over a time window \([0, \Delta t]\) with respect to the onset or the end of any flank eruption.

A negative \(\Delta t\) means a time window \([-\Delta t, 0]\) before the onset or the end.

In (a-c) the rate increase is asymmetric and skewed towards positive \(\Delta t\).

In (d-f) the annual rates are significantly larger than the average rates both before and after the flank eruption ends.

**Figure.** Annual rate of earthquakes temporally close to flank eruption (a-c) onsets, or (d-f) ends

Average annual rate post (a,b) 1875 or (c,d) 1850 is displayed in grey.
The annual rate is calculated with first-order, finite differences over a **fixed-size** time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

A *negative* $t$ means a time window before the onset: $[t, \min(0, \Delta t + 10 \text{ days})]$. The annual rate is calculated with first-order, finite differences over a fixed-size time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

A negative $t$ means a time window before the onset: $[t, \min(0, \Delta t + 10 \text{ days})]$. The annual rate is calculated with first-order, finite differences over a fixed-size time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

A negative $t$ means a time window before the onset: $[t, \min(0, \Delta t + 10 \text{ days})]$. The annual rate is calculated with first-order, finite differences over a fixed-size time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

A negative $t$ means a time window before the onset: $[t, \min(0, \Delta t + 10 \text{ days})]$. The annual rate is calculated with first-order, finite differences over a fixed-size time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

A negative $t$ means a time window before the onset: $[t, \min(0, \Delta t + 10 \text{ days})]$. The annual rate is calculated with first-order, finite differences over a fixed-size time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

A negative $t$ means a time window before the onset: $[t, \min(0, \Delta t + 10 \text{ days})]$. The annual rate is calculated with first-order, finite differences over a fixed-size time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

A negative $t$ means a time window before the onset: $[t, \min(0, \Delta t + 10 \text{ days})]$. The annual rate is calculated with first-order, finite differences over a fixed-size time window: $[\max(0, \Delta t - 10 \text{ days}), \Delta t]$ with respect to the onset or the end of any flank eruption.

Figure. Annual rate of earthquakes temporally close to flank eruption (a-c) onsets and (d-f) ends. 10 days-long time windows. Average annual rate post (a,b) 1875 or (c,d) 1850 is displayed in grey. Time delays producing higher-than average rates are marked in black.
Conclusions and Future Work

- A statistical test rejects at 90% the independence hypothesis of flank eruptions onsets or ends before the main-shocks. This was tested for the first time on the state-of-the-art seismic and eruption databases of Mt. Etna.

- The time difference percentile values between EQs and flank eruption onsets indicate that, if $\Delta t < 120$ days:
  - 60% to 71% of the major EQs occurred in the days after the eruption onset.
  - none of the EQs with ML $\geq 4.0$ occurred before the flank eruption onset.
  - if excluding $|\Delta t| < 5$ days, 72% to 100% of the EQs occurred after the eruption onset.
  - 69% to 100% of the EQs after the eruption onset occurred in the first 45 days.

- The time difference percentile values between EQs and flank eruption ends indicate that, if $\Delta t < 120$ days:
  - 50% to 56% of the major EQs occurred in the days after the eruption onset.
  - none of the EQs occurred in the same day of the flank eruption end.
  - 55% to 75% of the EQs after the eruption onset occurred in the first 45 days.

- The rate increase of the EQs is asymmetric and skewed towards positive $\Delta t$ with respect to the flank eruption onsets, while it is symmetrical with respect to the flank eruption ends.

- The probabilistic rate of the EQ is more than 5 times higher than the average rate, after the flank eruption onsets and ends. It can be 10 times higher.

- An increase of the probability can last for $\Delta t \in [-16, 45]$ days with respect to the flank eruption onsets, and $\Delta t \in [-30, 42]$ days with respect to the flank eruption ends. The exact duration depends on the ML threshold considered.

FUTURE WORK

1) We are going to perform further analysis based on the quantification of the $M_w$.

The seismic moments can provide the energy released by the EQs (Azzaro et al., 2019). We will investigate for a link to the energy release of the flank eruptions.

2) The datasets include spatial information as well. Many EQs were related to specific fault systems (Azzaro et al., 2017). We will investigate if the correlation between flank eruptions and EQs is a property of the volcano-tectonic system as a whole, or of local sectors.