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Estimates and models of the statistical dependence between local earthquakes and flank eruptions at Mt. Etna volcano (Italy): an old topic revised through new historical data

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Overview

Our target is the **statistical modeling** of the correlation between **major earthquakes** and **flank eruptions** at Mt. Etna volcano.

We target to quantify:

- **how much is the increment** of the probabilistic rate of major earthquakes in the days or months after a flank eruption onset or end,
- **for how much time** this hypothetical increase of the probability can last for.

CRUCIAL QUESTION

How much and **how long** the Civil Protection authorities should expect to deal with **damaging shocks** during and after a flank eruption emergency?

In the following we are detailing **three different topics** which are **strongly linked** together:

- The analysis of the historical time series of the **earthquakes** (EQs)
- The analysis of the historical time series of the **flank eruptions** (either onset or end)
- The time series of the EQs observed **from the point of view** of flank eruptions.

KEY IDEA

We look at the time series of the EQ from a family of **frames of reference** which are left-side anchored to the onset or the end of flank eruptions.

We do this for **every flank eruption** and we make statistics of what we see.

In particular, we consider two updated datasets: the **Macroseismic Catalog of Etnean Earthquakes** (CMTE), and the **Flank Eruptions catalog** over the time interval 1800-2018.

Previous statistical studies

Sharp, Lombardo, Davis (1981) Earthquakes in the time interval 1600 - 1978 with $I_0 \geq V$ (620 events - 146 main-shocks) Eruptions in the time interval 1600 - 1978 (132 events – of which 49 flank)	Statistical test of independence between Poisson processes, based on *Cox (1955)*, generalized to a case with rate changes. Conclusion – (i) Poisson distribution of flank eruptions and main-shocks. (ii) Abnormal number of flank eruptions after summit eruptions and after main-shocks earthquakes.
Nercessian, Hirn, Sapin (1991) Modification of the empirical method of aftershock removal (620 events – of which 180 main-shocks)	Test of *Cox (1955)* assuming the eruptions as precursors of the earthquakes. Conclusion – Abnormal number of earthquakes after the onset of flank eruptions and after the end of flank eruptions.
Gasperini, Gresta, Mulargia (1990) Earthquakes in the time interval 1978 - 1987 magnitude > 2.8 (1458 events) Eruptions (18 events - 9 flank)	Earthquake clusters recognition and modeling. Conclusion – correlation not calculated because ‘insufficient data’, and not qualitatively apparent to the authors.
Mulargia (1992) Seismic sequences in 1974-1991 (12 events) Flank eruptions (11 events)	Statistical test of Poisson independence from *Brillinger (1976)*. Conclusion – flank eruptions are precursors of seismic sequences and not the contrary.
Gresta, Marzocchi, Mulargia (1994) Earthquakes in the time interval 1600 - 1989. with $I_0 \geq IX$ (7 events) Eruptions with volume ≥ 10^7 m^3 (40 events)	Correlation test according to the Spearman ranking coefficient. Conclusion – correlation between the end of major eruptions and the major earthquakes, and not with the eruption onsets.

Mulargia, Tinti, Boschi (1985)
Kolmogorov-Smirnov test confirms Poisson distribution of the flank eruptions.

DATASET 479 YRS
~ 10^2 vs 10^2 events

~10^3 vs 10^1 events

DATASET 10 -17 YRS

~10^1 vs 10^1 events

DATASET 490 YRS
~ 10^1 vs 10^2 events

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Overview of CMTE and of the Flank Eruptions catalog

Figure. (a) cumulative number and (b) annual rate of EQs. Years with >45 events are marked.

(c) histograms of magnitude (M_L). The main tresholds adopted are marked.

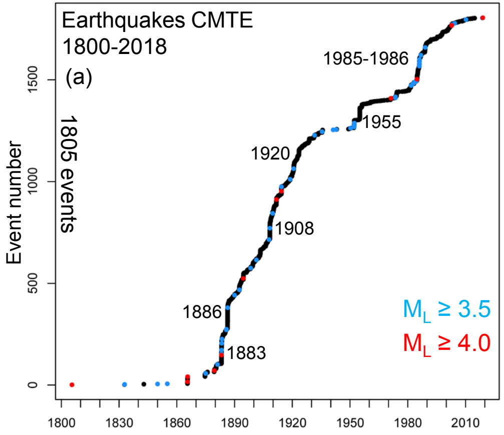
(d) cumulative number and (e) annual rate of flank eruptions.

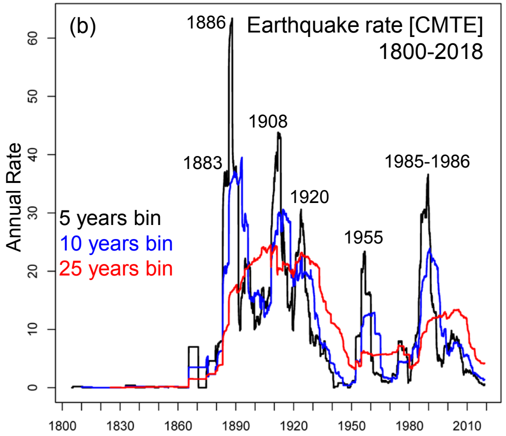
(f) histograms of inter-event times. The 20th, 50th and 90th percentiles are marked.

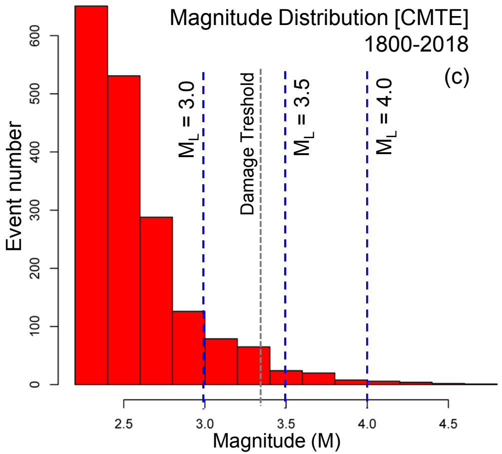
We use a macroseismic estimate of the magnitude M_L in the **pre-instrumental** part of the catalog *(Azzaro, D'Amico, Tuvè, 2011).*

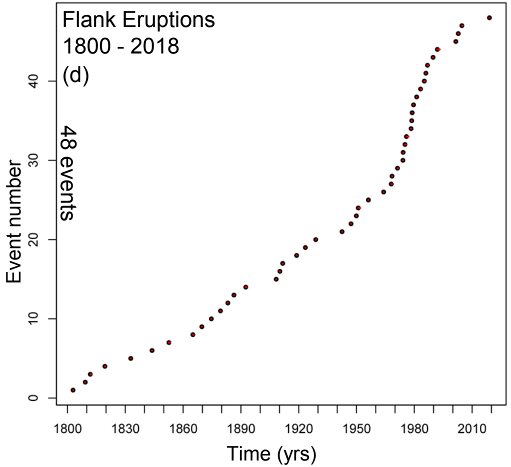
The annual rate is calculated with first-order, left-side finite differences:

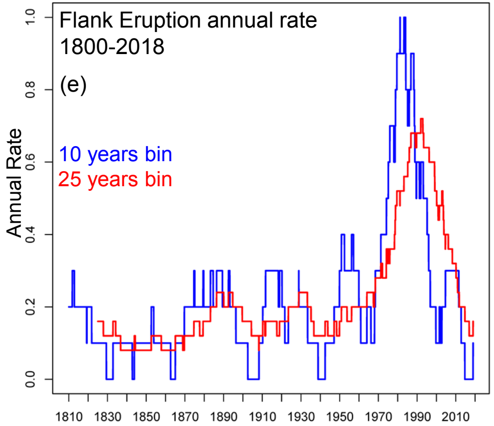
$$\lambda(t) = \frac{N(t) - N(t-T)}{T}$$

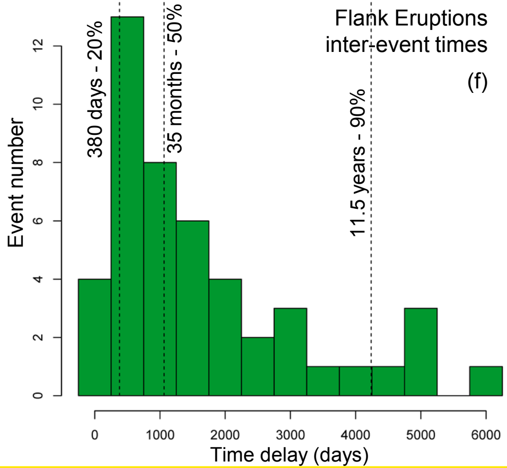












Annual rates of the earthquakes

We considered the events after 1875 or 1850, depending on the magnitude considered.

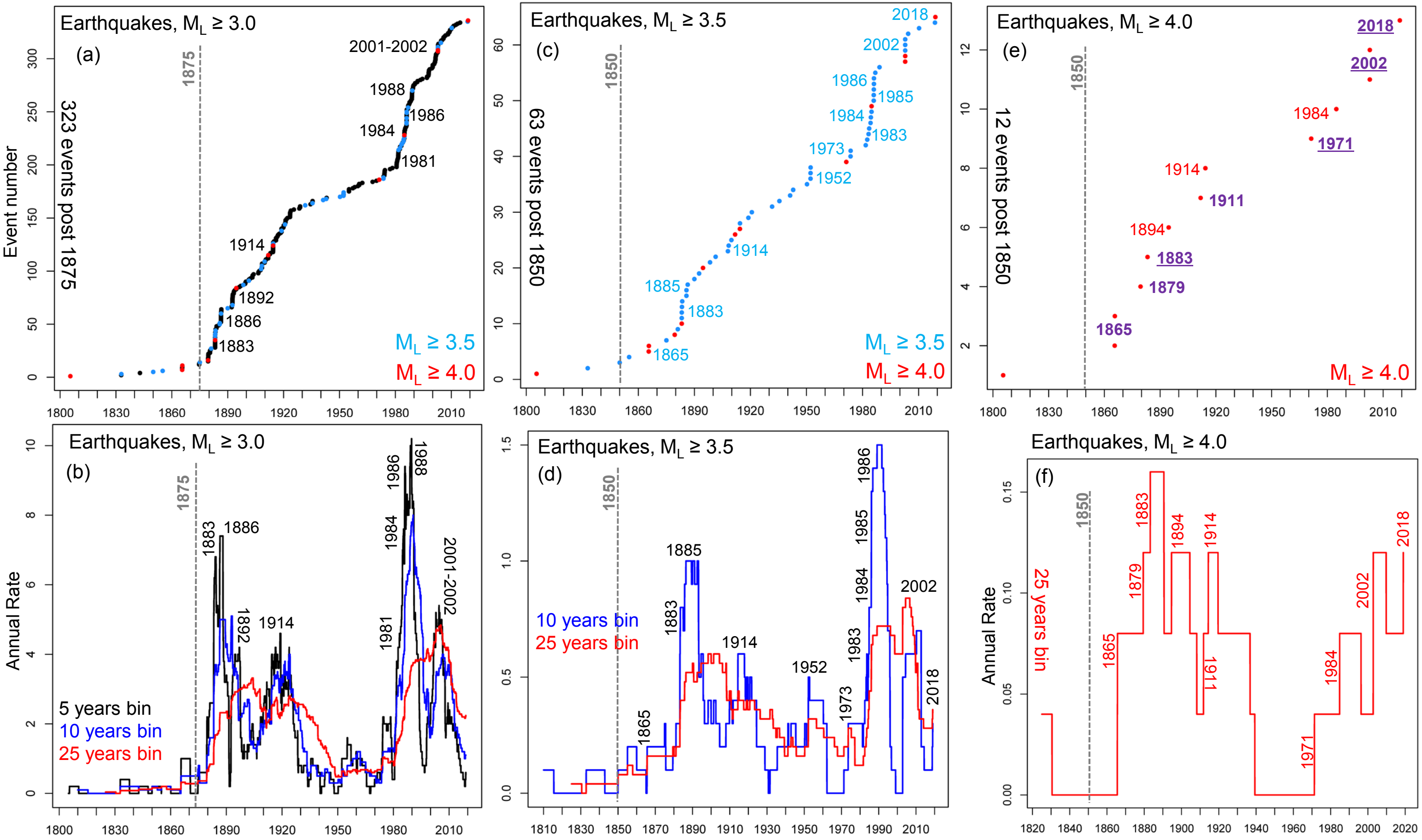
Figure. Cumulative number and annual rate of the EQs.

(a,b) **$M_L \geq 3.0$**, years with >8 events are labeled.
(c,d) **$M_L \geq 3.5$**, years with >1 event are labeled.
(e,f) **$M_L \geq 4.0$**, all events are labeled.

In (e), the **purple colored** events were less than 120 days after a flank eruption.

Underlined events occurred during a flank eruption.

In (e,f) a relation with flank eruptions of $M_L \geq 4.0$ is observed in 9/12 events.



The summative variable $\xi(\Delta t)$

Cox, (1955)
Sharp et al., (1981)
Nercessian et al., (1991)

In literature, to test if the events occurred at the times $(B_j)_{j=1\ldots,m}$ are precursors to those occurred in $(A_i)_{i=1\ldots,n}$ a summative variable is evaluated. First we repeat this approach on our new datasets.

Let be $\forall\ (t_1, t_2)$: $N(\,[t_1, t_2]\,) := |\ (A_i)_{i=1\ldots,n} \cap [t_1, t_2]\ |$.

Then $\forall\ \Delta t > 0$, let $\xi(t)$ be the variabile defined as:

$$\xi(\Delta t) = \sum_{j=1}^{m} N(\,(B_j, B_j + \Delta t]\,).$$

For example, $\xi(t)$ is the number of EQs happened less than t days after a flank eruption onset or end.

then:
$\xi(\Delta t)$ is approximated by the sum of *m* independent Poisson variables, and so, by a new Poisson random variable. Moreover, we have $E[\xi(\Delta t)] = m\lambda\Delta t \approx \frac{nm\Delta t}{T}$.

The calculation is generalized to the case of N being a nonhomogeneous Poisson process, assuming λ constant over appropriate subintervals of [0, T]. In the following we adopt subintervals of 25 yrs duration.

H0 - null hypothesis

$\xi(\Delta t)$ is a Poisson random variable and $E[\xi(t)] = \frac{nm\Delta t}{T}$.

H1 - alternative hypothesis

$\xi(\Delta t)$ is not a Poisson random variable with $E[\xi(t)] = \frac{nm\Delta t}{T}$, and so the events in $(A_i)_{i=1\ldots,n}$ are not independent of those in $(B_j)_{j=1\ldots,n}$.

The result of the test as a function of t is a step graph, marking the times Δt at which H0 is rejected with a level of confidence α = 90%.

The test is performed by comparing $\xi(\Delta t)$ to the 5th and 95th percentiles of a Poisson random variable of intensity $\frac{nm\Delta t}{T}$.

The test works under the hypothesis that $(A_i)_{i=1\ldots,n}$ is Poisson, and $(B_j)_{j=1\ldots,m}$ is not clustered at the scale of Δt.

In the special case that:
$(A_i)_{i=1\ldots,n-1} = (B_j)_{j=1\ldots,m}$,

the test verifies instead the **total randomness,**
i.e. the Poisson hypothesis.

This test does not reject the Poisson hypothesis on the **flank eruptions** onsets or ends.

The same test **rejects H0** for every EQ dataset, because of the clusters.

But if we empirically remove the **aftershocks**, the test does not reject the Poisson hypothesis on the **main shocks** .

This enables us to test if the flank eruption onsets or ends **increase the probabilistic rate** of the main-shocks, and hence the rate of all the EQs.

Main-shocks and aftershocks

Empirical algorithm

Sharp et al., (1981)
$\forall t_j$ earthquake, t_i aftershock if:
$t_i - t_j < f[M_L(t_j)]$
f(3.0) = 38 days
f(3.5) = 71 days
f(4.0) = 135 days
+
Nercessian et al. (1991)
some deleted t_i are re-inserted as a main-shock if:
$M_L(t_i) \geq M_L(t_j)$ and
$M_L(t_i) = \max\{M_M(t_k) : t_i - t_j < f[M_L(t_j)]\}$

The mean offspring of a main-shock is:

μ = (E[size] – 1) / E[size]

because E[size] is the sum of a geometric series of factor μ.

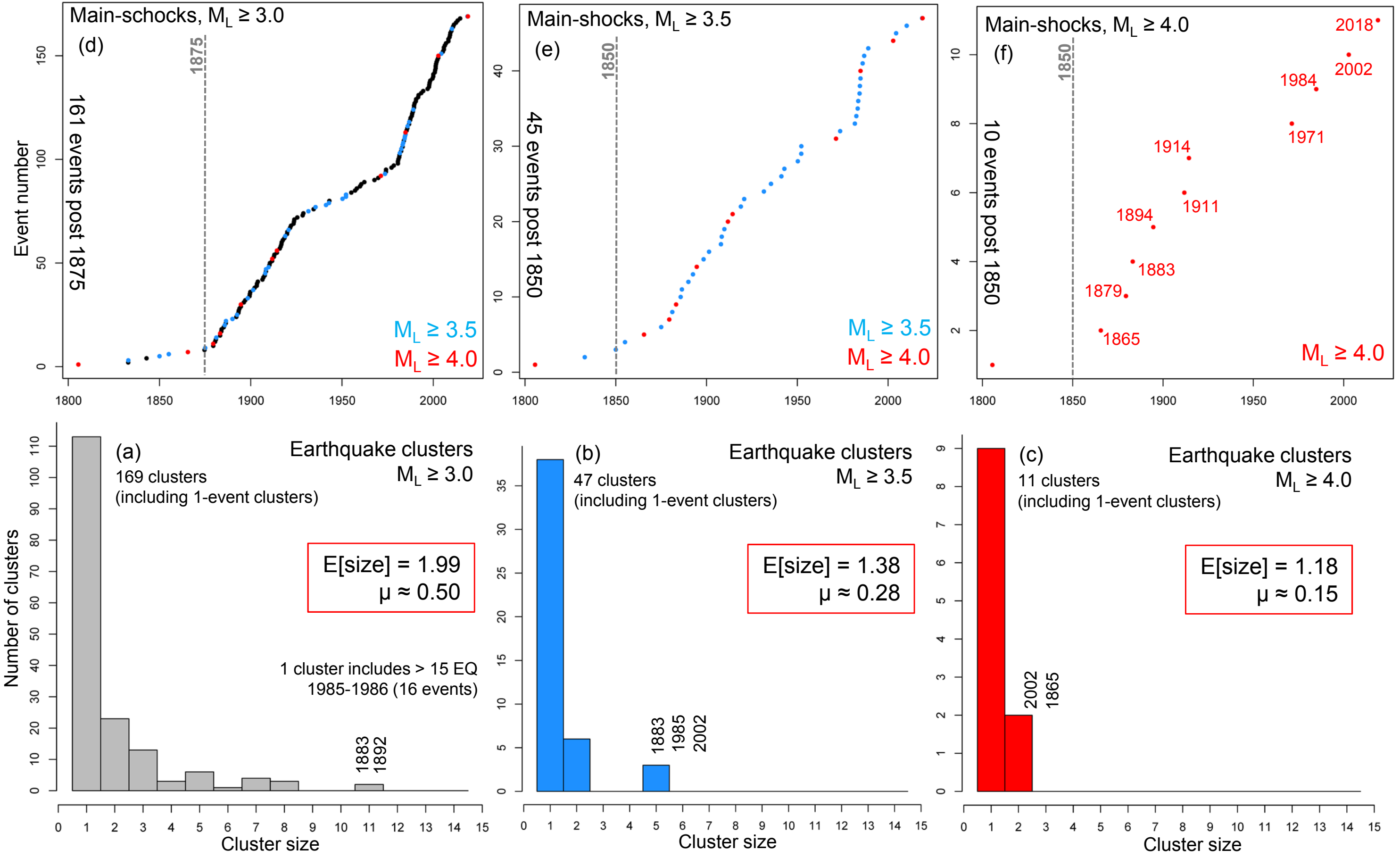


Figure. (a-c) Histograms of the cluster size, EQ post 1800.

(d-f) Cumulative number of empirical Main-shocks.

Rejection of the independence hypothesis of Main-shocks after Flank eruptions

Figure. Step graphs related to the test of independence.

ξ(Δt) sums the main-shocks occurred less than Δt days after the (a-c) **onset** or (d-f) **end** of any flank eruption.

ERUPTION ONSET

(a) Main-shocks, $M_L \geq 3.0$.
H0 rejected at Δt ≲ 25 days.

(b) Main-shocks, $M_L \geq 3.5$.
H0 rejected at Δt ≲ 50 days.

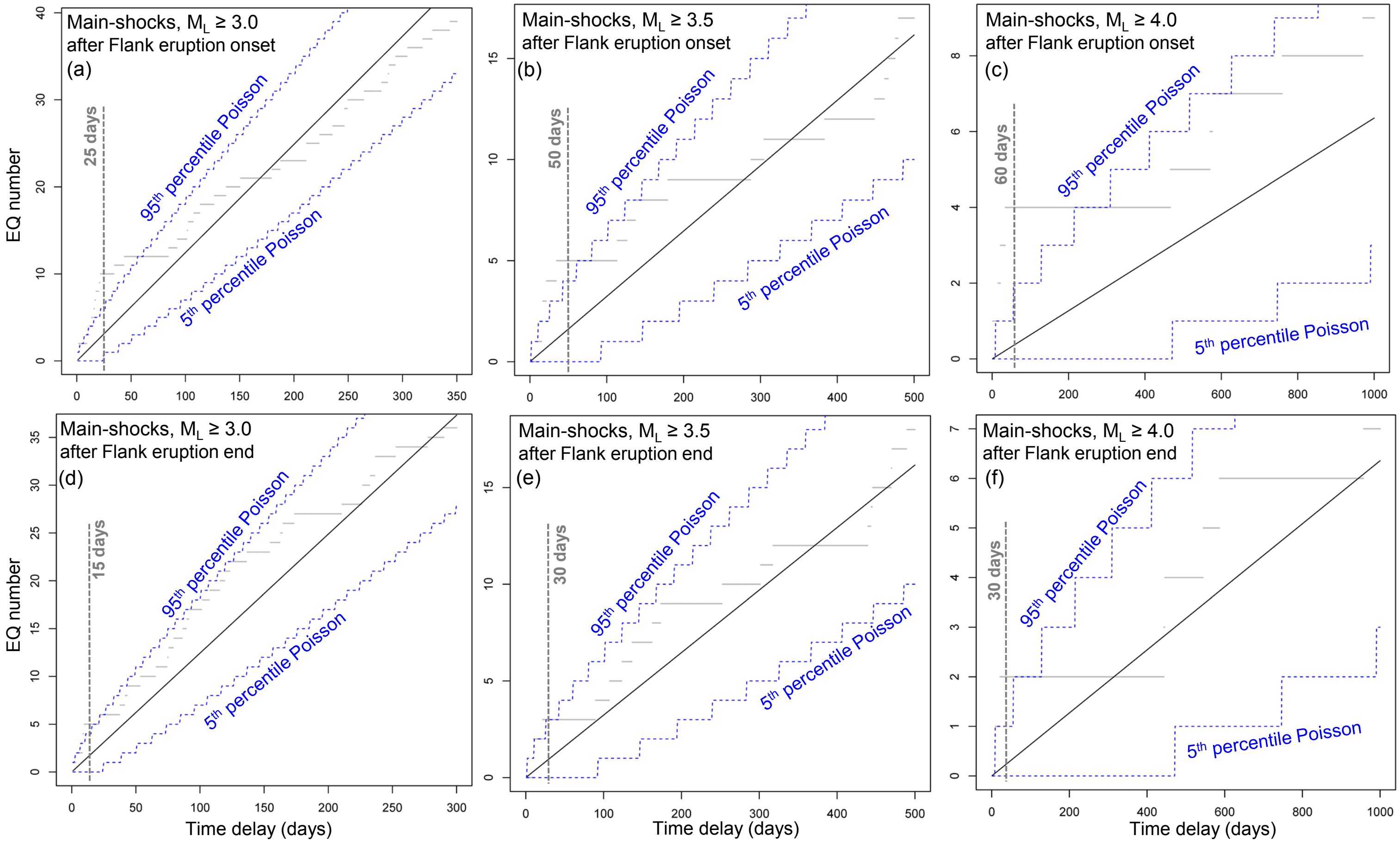
(c) Main-shocks, $M_L \geq 4.0$.
H0 rejected at Δt ≲ 60 days.

ERUPTION END

(d) Main-shocks, $M_L \geq 3.0$.
H0 rejected at Δt ≲ 15 days.

(e) Main-shocks, $M_L \geq 3.5$.
H0 rejected at Δt ≲ 30 days.

(f) Main-shocks, $M_L \geq 4.0$.
H0 rejected at Δt ≲ 30 days.



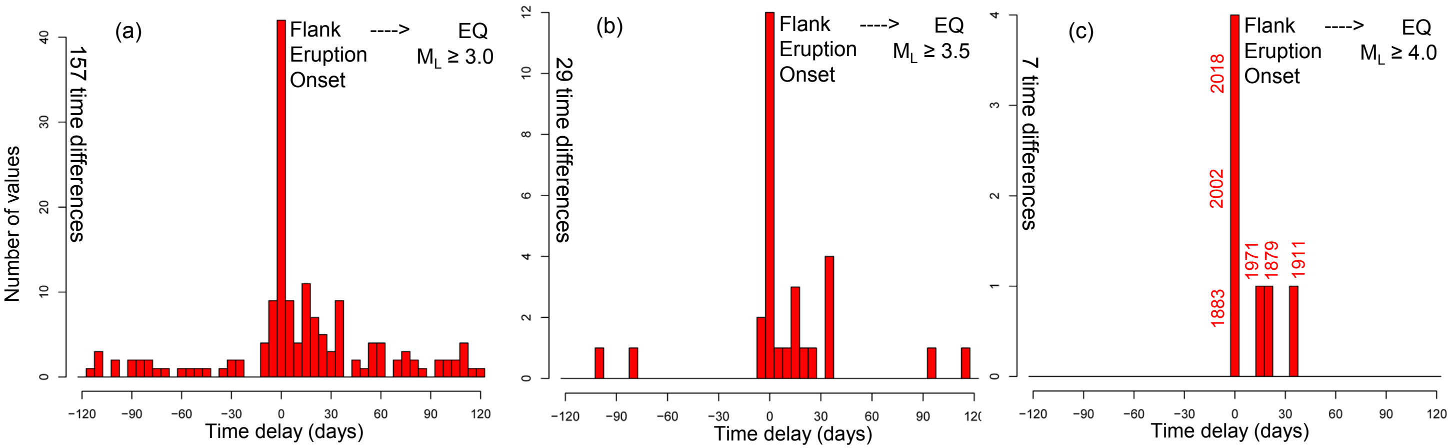
Time difference between Earthquakes and Flank eruption onsets

Figure. Histograms of the difference $\Delta t = (t_i - e_j)$ between earthquakes times t_i and flank eruption onset times e_j.

We consider all the possible pairs with a time delay lower than 120 days:

$$C = \{\Delta t : \Delta t < 120 \text{ days}\}$$

(a) **$M_L \geq 3.0$**,
(b) **$M_L \geq 3.5$**,
(c) **$M_L \geq 4.0$**.



	$M_L \geq 3.0$	$M_L \geq 3.5$	$M_L \geq 4.0$	
EQ in the days before the eruption onset	31%	28%	0%	
EQ in the same day of the eruption onset	9%	10%	29%	
EQ in the days after the eruption onset	**60%**	**62%**	**71%**	
If excluding differences < 5 days:	28%	20%	0%	**EQ before the eruption onset**
	72%	**80%**	**100%**	**EQ after the eruption onset**
Of those EQs after the eruption onset:	**69%**	**89%**	**100%**	**EQ within 0 to 45 days from the eruption**
	30%	11%	0%	**EQ within 45 to 120 days from the eruption**

Table.
Time difference percentile values of the EQ and the flank eruption onsets (if $\Delta t < 120$ days).

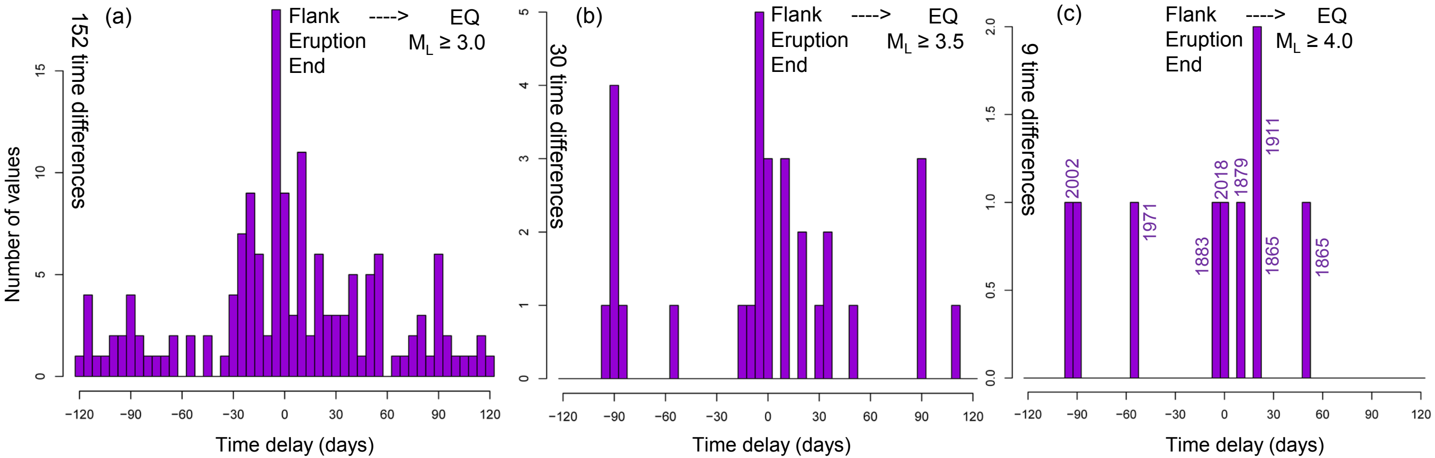
Time difference of Earthquakes and Flank eruption ends

Figure. Histograms of the difference $\Delta t = (t_i - \hat{e}_j)$ between earthquakes times t_i and flank eruption end times $\hat{e}_j$.

We consider all the possible pairs with a time delay lower than 120 days:

$$C = \{\Delta t : \Delta t < 120 \text{ days}\}$$

(a) **$M_L \geq 3.0$**,
(b) **$M_L \geq 3.5$**,
(c) **$M_L \geq 4.0$**.



	$M_L \geq 3.0$	$M_L \geq 3.5$	$M_L \geq 4.0$	
EQ in the days before the eruption end	**51%**	**50%**	**56%**	
EQ in the same day of the eruption end	0%	0%	0%	
EQ in the days after the eruption end	**49%**	**50%**	**44%**	
Of those EQs after the eruption end:	**55%**	**67%**	**75%**	**EQ within 0 to 45 days from the eruption**
	45%	33%	25%	**EQ within 45 to 120 days from the eruption**

Table.
Time difference percentile values of the EQs and the flank eruption ends, (if $\Delta t < 120$ days).

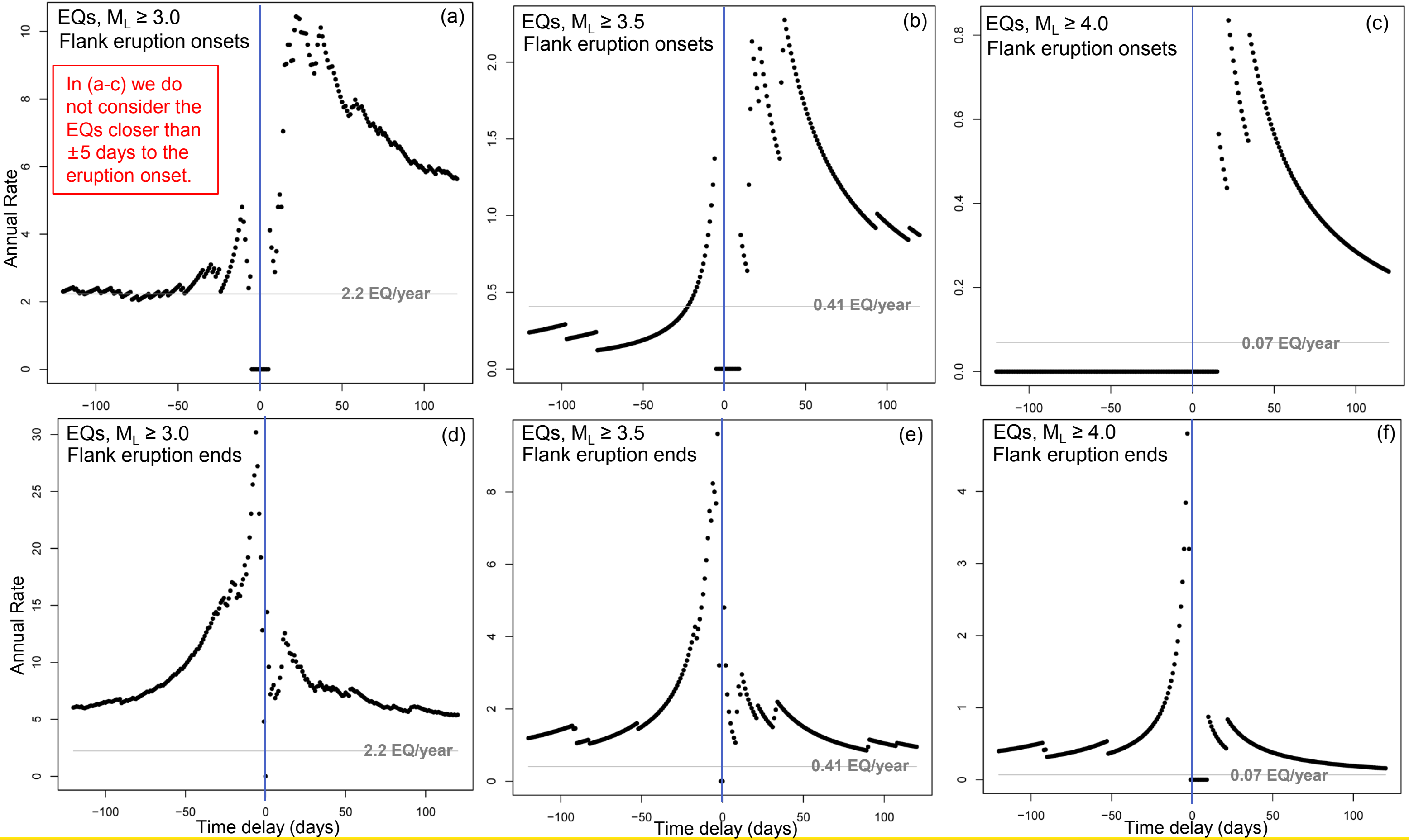
Annual rates of the EQs temporally close to flank eruptions on the time windows [e_j, $e_j + \Delta t$]

The annual rate is calculated with first-order, finite differences over a time window [0, Δt] with respect to the **onset** or the **end** of any flank eruption.

A **negative** Δt means a time window [Δt, 0] before the onset or the end.

In (a-c) the rate increase is **asymmetric and skewed** towards positive Δt.

In (d-f) the annual rates are **significantly larger** than the average rates both before and after the flank eruption ends.



In (a-c) we do not consider the EQs closer than ±5 days to the eruption onset.

Figure. Annual rate of earthquakes temporally close to flank eruption (a-c) onsets, or (d-f) ends

Average annual rate post (a,b) 1875 or (c,d) 1850 is displayed in grey.

Annual rates of the EQs temporally close to flank eruptions on the time windows $[e_j + \Delta t - 10, e_j + \Delta t]$

The annual rate is calculated with first-order, finite differences over a **fixed-size** time window:

$[\max(0, \Delta t - 10 \text{ days}), \Delta t]$

with respect to the **onset** or the **end** of any flank eruption.

A **negative** t means a time window before the onset:

$[t, \min(0, \Delta t + 10 \text{ days})]$.

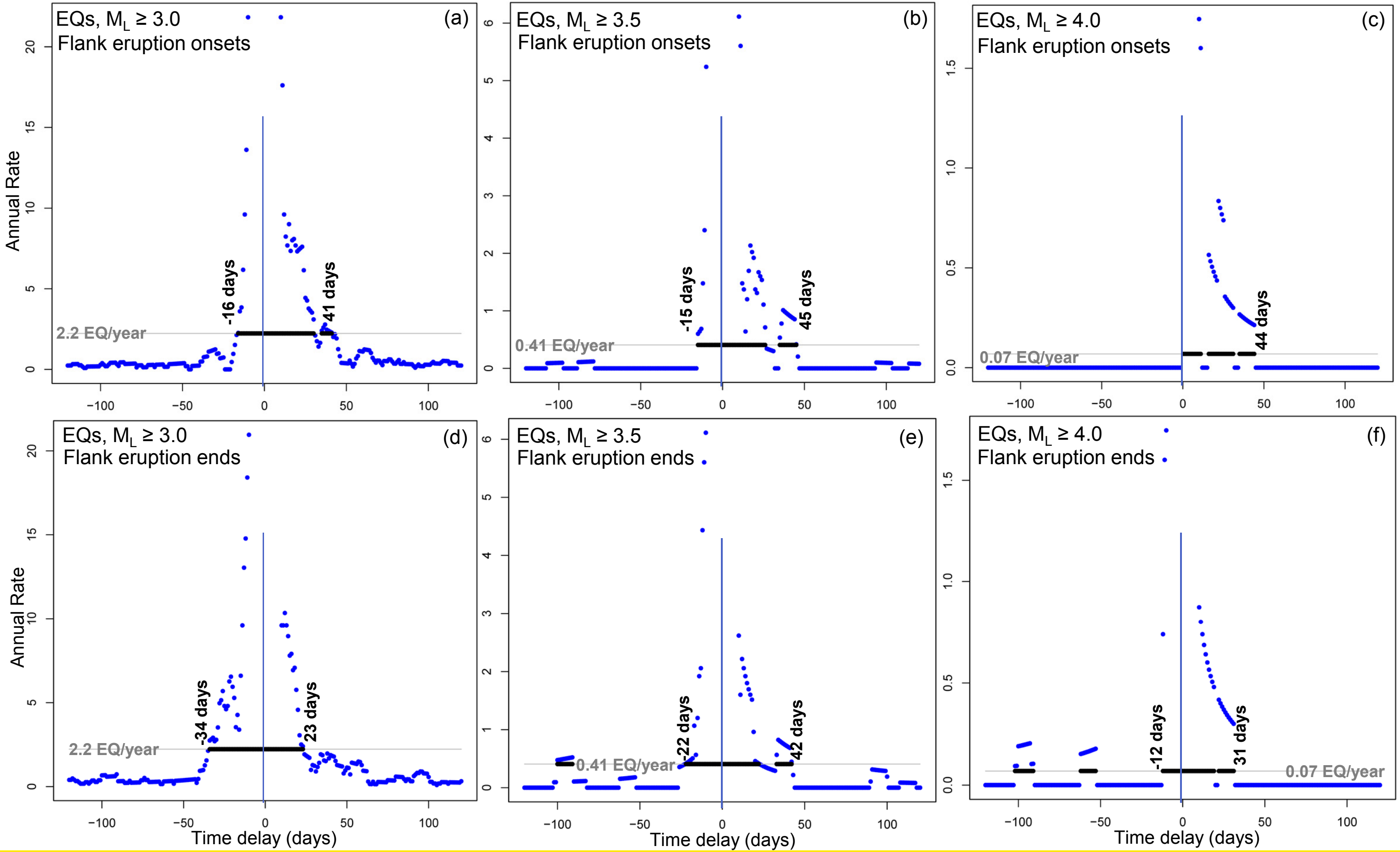


Figure. Annual rate of earthquakes temporally close to flank eruption (a-c) onsets and (d-f) ends.

10 days-long time windows.

Average annual rate post (a,b) 1875 or (c,d) 1850 is displayed in grey.

Time delays producing higher-than average rates are marked in **black**.

Conclusions and Future Work

- A statistical test **rejects at 90% the independence hypothesis** of flank eruptions onsets or ends before the main-shocks. This was tested for the first time on the state-of-the-art seismic and eruption databases of Mt. Etna.

- The **time difference percentile values** between EQs and flank eruption onsets indicate that, if $\Delta t < 120$ days:
  - **60% to 71%** of the major EQs occurred in the days after the eruption onset.
  - **none** of the EQs with $M_L \geq 4.0$ occurred before the flank eruption onset.
  - if excluding $|\Delta t| < 5$ days, **72% to 100%** of the EQs occurred after the eruption onset.
  - **69% to 100%** of the EQs after the eruption onset occurred in the first 45 days.

- The **time difference percentile values** between EQs and flank eruption ends indicate that, if $\Delta t < 120$ days:
  - 50% to 56% of the major EQs occurred in the days after the eruption onset.
  - none of the EQs occurred in the same day of the flank eruption end.
  - 55% to 75% of the EQs after the eruption onset occurred in the first 45 days.

- The rate increase of the EQs is **asymmetric and skewed** towards positive Δt with respect to the flank eruption onsets, while it is symmetrical with respect to the flank eruption ends.

- The probabilistic rate of the EQ is **more than 5 times higher** than the average rate, after the flank eruption onsets and ends. It can be 10 times higher.

- An increase of the probability **can last for** $\Delta t \in [-16, 45]$ days with respect to the flank eruption onsets, and $\Delta t \in [-30, 42]$ days with respect to the flank eruption ends.
  The exact duration depends on the M_L threshold considered.

FUTURE WORK

1) We are going to perform further analysis based on the quantification of the M_w.

The **seismic moments** can provide the energy released by the EQs (*Azzaro et al., 2019*).
We will investigate for a link to the energy release of the flank eruptions.

2) The datasets include spatial information as well. Many EQs were related to **specific fault systems** (*Azzaro et al., 2017*).

We will investigate if the correlation between flank eruptions and EQs is a property of the volcano-tectonic system **as a whole**, or of local sectors.

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