A probability tomography approach to the analysis of potential field data in the Campi Flegrei caldera (Italy)

Teresa Iuliano (1), Paolo Mauriello (2) and Domenico Patella (3)
(1) Dipartimento di Scienze Fisiche, Università «Federico II», Napoli, Italy
(2) Istituto per le Tecnologie Applicate ai Beni Culturali, CNR, Roma, Italy

Abstract
The results of the application of the 3D probability tomography imaging approach to the study of the Campi Flegrei (CF) caldera are presented and discussed. The tomography approach has been applied to gravity, magnetic and ground deformation data already available in literature. The analysis of the 3D tomographic images is preceded by a brief qualitative interpretation of the original survey maps and by an outline of the probability tomography approach for each geophysical prospecting method. The results derived from the 3D tomographic images are the high occurrence probabilities of both gravity and ground deformation source centres in the CF caldera under the town of Pozzuoli. A Bouguer negative anomaly source centre is highlighted in the depth range 1.6-2 km b.s.l., whereas a positive ground deformation point source, responsible for the bradyseismic crisis of 1982-1984, is estimated at a mean depth of 3-4 km b.s.l. These inferences, combined with the results of a previous analysis of magnetotelluric, dipolar geoelectrical and self-potential data, corroborate the hypothesis that the bradyseismic events in the CF area may be explained by hot fluids vertical advection and subsequent lateral diffusion within a trapped reservoir overlying a magma chamber.

Key words applied geophysics – potential fields – probability tomography – Campi Flegrei caldera

1. Introduction
The Campi Flegrei (CF) volcanic area is located in South Italy around the western border of Naples. The CF volcanic history is characterized by many explosive eruptions from different vents up to the last event in 1538 A.D. with the eruption of Monte Nuovo (Rosi and Sbrana, 1987). Tectonically, the CF area is cut by a complex system of faults and fractures due to regional and local stress fields with NW-SE and NE-SW strike. Figure 1 shows a structural and volcanological sketch map of the CF area.

Geological investigations (Orsi et al., 1996, 1999a) suggest that the CF area is a nested structure resulting from two major caldera-forming events. The first event was the Campanian Ignimbrite eruption 370,000 years B.P. followed by the formation of a caldera with a diameter of 12-14 km. The second event was the Yellow Tuff eruption which occurred 130,000 years B.P., also accompanied by a caldera collapse which began during the same eruption. The northeastern sector of this younger caldera was then affected by both widespread volcanism and block resurgence due to a simple shearing mechanism (Di Vito et al., 1999). The most uplifted struc-
ture is the La Starza marine terrace along the Pozzuoli coastline. No evidence of eruptions appears offshore in the Bay of Pozzuoli, that is in the central and southern parts of the younger CF caldera.

Kinematics in the CF area is very complex, as shown by the two unrest episodes of 1969-1972 and 1982-1984. The result of these two episodes was a maximum vertical uplift of 3.5 m in the area of Pozzuoli (Berrino et al., 1984).

The CF area has been little investigated with geophysical methods (Hunsche et al., 1981; Monaco et al., 1986; Cassano and La Torre, 1987; Barberi et al., 1991; Orsi et al., 1999b). Di Maio et al. (2000) recently performed a detailed analysis of dipole geoelectrical (DG), magnetotelluric (MT) and self-potential (SP) data sets collected over the emerged portion of the CF caldera floor. The main results are summarised as follows.

A structural outline of the CF depression was deduced from a 2D combined interpretation of MT and DG soundings along a nearly E-W profile from east of Agnano to about Cuma (see fig. 1). A 4000 Ω m resistive body in the shallower 2-3 km, about 4 km wide in the E-W direction from Solfatara to southwest of Gauro, was associated with the dry volcanics of the La Starza block. It overlies a 10 Ω m conductive layer of 1.5-2 km of maximum thickness, which was attributed to a trapped hot fluids reservoir, separated from the resistive overburden by an impermeable layer of similar high conductivity. Resistivity dispersion, detected only at the margins of the resistive body, was in fact ascribed to

Fig. 1. A structural and volcanological sketch map of the Campi Flegrei area (after Di Vito et al., 1999).
hydrothermally altered zones sealing the fracture systems bordering the resurgent La Starza block. Finally, an active pressure-temperature source at a depth not less than 5 km under the Bay of Pozzuoli was inferred from the analysis of SP data using the 3D probability tomography imaging method proposed by Patella (1997a,b).

Altogether, these results corroborate the hypothesis that the CF area must still be considered a highly hazardous volcanic system. Therefore, in order to assess a modern risk management strategy, the knowledge of its structure and dynamics needs to be further implemented, mainly within the shallowest 10 km. In this paper, we extend the geophysical investigation of the CF area by applying for the first time to gravity (GR), magnetic (MG) and ground deformation (GD) data sets the 3D probability tomography approach, formerly used for the SP analysis (Di Maio et al., 2000).

2. Gravity tomography

2.1. The CF Bouguer anomaly map

A Bouguer anomaly \(B_s\) local map of the CF area was compiled by Cassano and La Torre (1987), who used a density of 1.4 g/cm³ for slab and terrain corrections and a N35°E gradient of 0.5 mgal/km for resinization. The top slice in fig. 2 shows a scaled version of the CF residual map, obtained by further subtracting the mean \(B_s\) value in order to better distinguish negative and positive anomalies.

The main features in the map of fig. 2 are: i) an intense positive anomaly just outside the CF calderas, at the SW corner corresponding to Monte di Procida; ii) a ring of low amplitude positive anomalies along the younger CF calderas rim; iii) an intense negative anomaly inside the CF calderas, mostly offshore in the Bay of Pozzuoli.

2.2. Gravity source centres tomography

The scaled residual \(B_s\) map was analysed by a new probability tomography method aimed at imaging the 3D configuration of GR sources (Mauriello and Patella, 2001a,b). An outline of the method follows.

Consider a coordinate system with the \((x,y)\)-plane at sea level and the \(z\)-axis positive downwards. The \(B_s(r)\) value measured at an observation point \(r\) on a surface \(S\) characterized by a height function \(z(x,y)\) a.s.l. is assumed to be due to the superposition of \(Q\) partial effects, say

\[
B_s(r) = \sum_{q=1}^{Q} \Gamma_q \cdot s(r_q - r)
\]  

where

\[
s(r_q - r) = \frac{z_q - z}{\|r_q - r\|^2}
\]

The \(q\)-th element has strength \(\Gamma_q\) equal to the gravitational constant \(G\) times the excess or deficit of mass concentrated at a point \(r_q = (x_q, y_q, z_q)\).

Using eq. (2.1), the power \(\Lambda\) associated with \(B_s(r)\) over \(S\) is written as

\[
\Lambda = \int_S \frac{B_s^2}{\epsilon(r)} dS = \sum_{q=1}^{Q} \Gamma_q \int_S B_s(r) s(r_q - r) dS.
\]

Applying Schwarz’s inequality to the \(q\)-th integral in eq. (2.3) leads to

\[
\left[ \int_S \int_{x=y} B_s(r) s(r_q - r) g(z) dx dy \right]^2 \leq \int_S \int_{x=y} B_s^2(r) g(z) dx dy \int_S \int_{x=y} s^2(r_q - r) g(z) dx dy
\]

where it was assumed that the projection of \(S\) onto the \((x,y)\)-plane can always be adapted to a rectangle with sides \(2X\) and \(2Y\) along the \(x\) and \(y\)-axis, respectively. The projection from the actual surface \(S\) to the rectangle \([2X, 2Y]\) requires a topographic surface regularization factor \(g(z)\)
expressed by

\[ g(z) = \sqrt{1 + \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2}. \]  

(2.5)

From eq. (2.4) a function \( \eta(r) \) can be defined as (Mauriello and Patella, 2001a,b)

\[ \eta(r) = C_s \int_{-1}^{+1} \int_{-1}^{+1} B_s(r) s(r, \theta) g(z) \, dx \, dy \]  

(2.6)

with \(-1 \leq \eta(r) \leq +1\)

where

\[ C_s = \left( \int_{-1}^{+1} \int_{-1}^{+1} B_s(r) g(z) \, dx \, dy \right)^{-1/2}. \]  

(2.7)

The function \( \eta(r) \) is interpreted as the probability a positive (\( \eta > 0 \)) or negative (\( \eta < 0 \)) \( \Delta \)-mass source located at \( r \) obtains as responsible of the observed GR field. A 3D tomographic image of this probability function can be thus constructed by scanning the subsurface e.g., along a sequence of vertical slices at different depths. This 3D image can then be used to retrieve a likely distribution of buried \( \Delta \)-mass sources fitting the observed \( B_s(r) \) map in a probabilistic sense.

The results of this approach in the CF area are depicted in fig. 2. Left- and right-hand sequences of slices below the reference \( B_s \) map show the GR sources images at depths b.s.l. from 0.4 km to 2 km and from 2 km to 6 km, respectively.

In addition to the circular sequence of positive \( \Delta \)-mass sources appearing along the younger CF caldera rim in the first 0.8 km b.s.l. and distributed almost continuously in the western side of the area, the 3D probability tomography discloses the following two main features. The first is a positive \( \Delta \)-mass nucleus with \( \eta \)-values close to +0.8, appearing at the southwestern corner of the slices in the depth range 0.4-4 km b.s.l., in correspondence of Monte di Procida (see fig. 1 for geographic reference). The second feature is a negative nucleus with \( \eta \)-values close to -1, appearing in the central area in the depth range 1.6-2 km b.s.l., beneath the town of Pozzuoli.

2.3. Gravity source boundaries tomography

To further investigate the CF GR residual map, we used a \( \Delta \)-mass dipole tomography imaging approach aimed at highlighting the 3D probability distribution of the density contrast interfaces (Iuliano et al., 2001). We assume, in fact, that any density discontinuity can be locally assimilated to a \( \Delta \)-mass double layer, i.e. to a distribution of \( \Delta \)-mass dipoles located across the interface. Generalising, we can assume that among the sources of the GR field there are \( P \) dipoles with moment \( d_v \) (\( p = 1, 2, \ldots, P \)), whose total vertical GR field is written as

\[ B^{(d)}_v(r) = \sum_{p=1}^{P} (d_v \cdot \nabla) s(r, \theta, \varphi), \]  

(2.8)

We extract from the total power \( \Lambda \), given in eq. (2.3), the contribution \( \Lambda^{(d)} \) due to \( B^{(d)}_v(r) \) and write it as (Iuliano et al., 2001)

\[ B^{(d)}_v(r) = \sum_{p=1}^{P} \sum_{v=x,y,z} d_v^{(p)} \int B_v(r) \frac{\partial s(r, \theta, \varphi)}{\partial v} \, dS. \]  

(2.9)

Application of Schwarz’s inequality to the \( p \)-th term of eq. (2.9) allows a 3D \( \Delta \)-mass dipole probability of occurrence function to be now defined as

\[ \eta_v(r) = \]  

(2.10)

\[ C_{pv} \int_{-1}^{+1} \int_{-1}^{+1} B_v(r) \frac{\partial s(r, \theta, \varphi)}{\partial v} g(z) \, dx \, dy, \quad v = x, y, z, \]  

with

\[ C_{pv} = \]  

\[ \left( \int_{-1}^{+1} \int_{-1}^{+1} B_v^2(r) g(z) \, dx \, dy \right)^{-1/2}. \]  

(2.11)
Thus, at each \( r_n \), three occurrence probability figures are shown: (a) the mass source occurrence probability, (b) the Bouguer anomaly occurrence probability, and (c) the Bouguer anomaly occurrence probability (\( \Delta \)).

Figures 3(a-c) show the results of the new approach to the analysis of the PPRP data. In particular, Figs. 3a, 3b, and 3c show the Bouguer anomaly occurrence probability maps for 3D GR field, below the reference map.
source boundary images at depths b.s.l. from 0.4 km to 2 km and from 2 km to 6 km, respectively. The $\eta_1$ and $\eta_2$ $\Delta$-mass dipole tomographies (figs. 3a and 3b, respectively) give a clear picture of the lateral boundaries of the CF depression along the W-E ($x$-axis) and S-N ($y$-axis) directions, respectively. Generally, the highest probabilities occur at a depth of about 2 km, except for the boundary between the Monte di Procida dense structure and the light volcanics inside the CF caldera, which extends down to much greater depths. The $\eta_1$ $\Delta$-mass dipole
Fig. 3b. The gravity source boundaries probability tomography in the depth ranges 0.4-2 km b.s.l. (a) and 2-6 km b.s.l. (b) for the y-component of the source dipole occurrence probability function. The top slice is the residual Bouguer anomaly survey map (after Cassano and La Torre, 1987).

Magnetic tomography

3.1. The CF magnetic anomaly map

For the MG analysis of the CF area we used a data set derived from the aeromagnetic survey
Fig. 3c. The gravity source boundaries probability tomography in the depth ranges 0.4-2 km b.s.l. (a) and 2-6 km b.s.l. (b) for the z-component of the source dipole occurrence probability function. The top slice is the residual Bouguer anomaly survey map (after Cassano and La Torre, 1987).

carried out in 1977 by the Italian Oil Company AGIP (Cassano and La Torre, 1987). A cesium optical pumping magnetometer was used to measure the total field intensity at a constant flight altitude of 1460 m.

The top slice in fig. 4 shows the residual MG total field map. The main features are two strong highs, one in the area of Monte di Procida and the other in the Astroni-Agnano area.
Fig. 4. The magnetic dipole source centres probability tomography in the depth ranges 0.4-2 km b.s.L. (a) and 2-6 km b.s.L. (b). The top slice is the residual total aeromagnetic field map (after Cassano and La Torre, 1987).

3.2. Magnetic dipole source tomography

Because the MG data are usually collected as total field intensities, as in the case of the CF study, a modification of the tomographic procedure was necessary. In fact, a linear superposition of source effects as in eq. (2.1) is now not possible. We followed the procedure of Manucci and Patella (2001c), which is briefly outlined.
We denote with $H$ the difference between the modulus of the measured MG field and the modulus of a reference primary field and expand $H$ in a Taylor series of the magnetic moment increments $\Delta m_{\nu}$. Truncating the series to the first partial derivative terms, we obtain

$$H(r) = \sum_{\nu=1}^{q} \left[ \frac{\partial H(r)}{\partial m_{\nu}} \Delta m_{\nu} + \frac{\partial H(r)}{\partial m_{\nu}} \Delta m_{\nu} \right].$$

(3.1)

The total power $\Lambda$ associated with $H$ over $S$ is written as

$$\Lambda = \sum_{\nu=1}^{q} \sum_{x=1}^{c} \Delta m_{\nu} H(r) s_{x}(r_{x} - r) dS$$

(3.2)

where the function $s_{x}(r_{x} - r)$ is the $\nu$-th component of the MG field at $r$ produced by a magnetic dipole of unit strength located at $r_{x}$.

Using the same procedure as for the GR tomography, we define the MG dipole source occurrence probability function as (Mauriello and Patella, 2001c)

$$\eta_{x}(r_{x}) = C_{x} \int_{\mathbb{R}^{2}} \frac{H(r)}{s_{x}(r_{x} - r)} g(z) dx dy$$

(3.3)

where $g(z)$ is the same surface regularization factor used in the GR tomography.

Thus, at each $r$, three $\eta$ values can be obtained, say $\eta_{x}, \eta_{y},$ and $\eta_{z}$. These values give the probabilities with which the three components of a MG dipole source at $r_{x}$ are responsible for the observed anomalous field.

The application of this imaging approach to the CF MG map is shown in fig. 4. In order to avoid useless dispersion of data, a unique 3D picture was elaborated by assigning at each $r_{x}$ the highest absolute value among the three probabilities $\eta_{x}, \eta_{y},$ and $\eta_{z}$, taken with its actual sign. The left-hand and right-hand sequences of slices below the reference aeromagnetic survey map show the 3D MG source dipole tomographies at depths b.s.l. from 0.4 km to 2 km and from 2 km to 6 km, respectively. The conclusions drawn from these plots are the high probabilities of occurrence of MG dipole sources south of Monte di Procida in the depth interval 0.8-2 km b.s.l. and in the Astroni-Agnano area at a depth not exceeding 0.4 km b.s.l. In both areas, the secondary MG dipole source would be aligned in such a way as to enhance the local MG primary field.

4. Ground deformation tomography

4.1. The CF ground deformation anomaly maps

GD monitoring in the CF area started during the 1969-1972 unrest episode. Vertical GD data are now routinely provided by a network of 124 benchmarks covering the whole area subject to resurgence and subsidence cycles (Berrino et al., 1984).

The top slices of figs. 5 through 10 show the vertical GD measured during six different surveys carried out in the period January 1983-June 1984. At each station the reference ground level was that measured in January 1982. In all maps the main feature is a nearly circular GD anomaly centred over the town of Pozzuoli with maximum uplift of 1.79 m occurred on June 1984 (fig. 10).

4.2. Ground deformation source tomography

GD in volcanic areas is usually interpreted as the response of an elastic medium to a pressure increase in a magma chamber or to a magma intrusion (Mogi, 1958). However, large GD values in small areas without any apparent intrusion, as was the case in the CF area during the 1982-1984 unrest episode, would imply an unreasonably large pressure increase (Bianchi et al., 1986). Bonafede (1991) suggested a thermal advection mechanism to explain abnormal GD values in volcanic areas. He considered a pressure-temperature increase within a confined fluids reservoir producing a system of connected fractures during fluid migration towards the top boundary of the reservoir. Numerical simulations demonstrated that forced advection of hot fluids can be a very efficient GD source.
Fig. 5. The vertical ground deformation source centres probability tomography in the depth ranges 0.4-2 km b.s.l. (a) and 2-6 km b.s.l. (b). The top slice is the vertical ground deformation field map referred to the survey of January 1983 (after Berrino et al., 1984).

In order to image the distribution of GD sources in the CF area, we have adapted our probability tomography approach starting from the hot fluids model of Bonafede (1991) as follows. We put with $mod_i$, a reference model supposed to be a homogeneous, elastic and porous medium. If a small volume around a point $r_i$ of the subsoil is affected by a pressure and/or temperature variation, a point $r$ at the ground surface is displaced by an amount $\delta h(r)$ equal
Fig. 6. The vertical ground deformation source centre occurrence probability tomography in the depth ranges 0.4-2 km b.s.l. (a) and 2-6 km b.s.l. (b). The top slice is the vertical ground deformation field map referred to the survey of June 1983 (after Berrino et al., 1984).

to (Bonafede, 1991)

$$\delta h(r) = \frac{\delta M_y (z_x - z)}{4\mu |r_y - r|}$$  \hspace{1cm} (4.1)

where $\mu$ is the rigidity modulus of $mod$, and $\delta M_y$ is a coefficient depending on pressure and temperature in the source element and on the properties of $mod$ (Poisson's ratio, diffusivity and pore pressure). The $z$-axis is positive downwards.
Expanding the observed \( h(r) \) function in a Taylor series we obtain

\[
h(r) = \sum_{q=1}^{Q} \frac{\delta h(r)}{\delta M_q} M_q + \text{higher order terms.}
\]

Higher order terms contain information about the actual nature of the subsurface (e.g., presence of shallow aquifers) and can be calculated by introducing appropriate coefficients, which are generally unknown in advance. However, since we are interested in imaging only the oc-
Fig. 8. The vertical ground deformation source centres probability tomography in the depth ranges 0.4-2 km b.s.l. (a) and 2-6 km b.s.l. (b). The top slice is the vertical ground deformation field map referred to the survey of December 1983 (after Berrino et al., 1984).

currence probability distribution of hot fluids source elements, we can limit the analysis only to the first order terms.

Therefore, following the same procedure as in the previous sections, we define the occurrence probability function \( \eta(r_e) \) for GD source analysis as

\[
\eta(r_e) = C_y \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(r) s(r_e - r) g(z) d\sigma dy
\]  
(4.3)

where \( s(r_e - r) \) is expressed by eq. (2.2) and
A probability tomography approach to the analysis of potential field data in the Campi Flegrei caldera (Italy)

![Vertical ground deformation (cm)](image)

**Fig. 9.** The vertical ground deformation source centres probability tomography in the depth ranges 0.4-2 km b.s.l. (a) and 2-6 km b.s.l. (b). The top slice is the vertical ground deformation field map referred to the survey of March 1984 (after Berrino et al., 1984).

\[
C_v = \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} h^2(r)g(z)dx\,dy \int_{-\infty}^{\infty} s^2(r, z)dx\,dy \right)^{1/2}.
\]

The results of the application of this GD tomographic approach to the CF data are depicted in figs. 5 through 10. The left- and right-hand sequences of slices beneath the reference GD survey maps show the source images at depths b.s.l. from 0.4 km to 2 km and from 2 km to 6 km, respec-
Fig. 10. The vertical ground deformation source centres probability tomography in the depth ranges 0.4–2 km b.s.l. (a) and 2.6 km b.s.l. (b). The top slice is the vertical ground deformation field map referred to the survey of June 1984 (after Berrino et al., 1984).

5. Discussion and conclusions

Compiling the results from the previous SP tomographic imaging (Di Maio et al., 2000) and the present GR and GD tomographic images, the following features are outlined in terms of
point sources:
  a) A pressure-temperature source centre not less than 5 km deep accounts for the wide negative nucleus shown by the SP tomography under the Bay of Pozzuoli.
  b) A pressure-temperature source centre at around 3-4 km of depth explains the notably less extended positive nucleus shown by the ground deformation tomography under the Bay of Pozzuoli (figs. 5 through 10).
  c) A mass deficit source centre at a depth around 2 km accounts for the negative nucleus shown by the GR tomography under the Bay of Pozzuoli (fig. 2).

Bianchi et al. (1986) maintain that exceedingly high ground uplifts, like those detected in the CF area (over 1.7 m in both crises of 1969-1972 and 1982-1984) without any apparent magma intrusion, cannot be explained by pressure increase in a shallow magma chamber. Bonafele (1991) suggested that forced advection of hot fluids in a reservoir overlying a magma chamber can be a very efficient GD source. Furthermore, Orsi et al. (1999c) maintain that hot fluids advection can account for the intense seismic activity during uplift and its absence during subsidence. Earthquakes during resurge would be associated with fracturing, whereas their absence during subsidence would be explained by lateral fluid diffusion without regression of source pressures.

Combining these last phenomenological aspects with the results of the potential fields tomographies, we can suggest the following conclusions:

a) The SP source with hypocentral location at a depth not less than 5 km may be related to a primary source structure, e.g. a magma chamber. Electric currents can in fact be generated by temperature gradients (Thomson effect) and/or pressure gradients (streaming potential) without necessarily invoking a massive fluid motion (Di Maio and Patella, 1991).

b) The GD tomographies clearly show a source for ground uplift centered at a depth around 3-4 km. We can thus corroborate the hypothesis of existence of a shallower geothermal reservoir, where hot fluid advection is activated due to heating from an underlying heat source (magma chamber).

c) The GR tomography allows the observed $B_j$ low under the Bay of Pozzuoli to be explained in terms of a less dense body with hypocentral location at a depth around 2 km. The same body was shown by MT and DG soundings as a highly resistive structure overlying a very conductive layer. Thus, we can conclude that the bottom of this lighter, resistive body is the top limit for hot fluids upward migration before lateral diffusion. Resurgence of La Stanna resistive block is of course favoured by its relative lightness with respect to the surrounding rocks.

Acknowledgements

The authors wish to thank Giovanni Orsi and Andreas Tzanis, who carefully reviewed the manuscript and gave useful suggestions to improve readability. The work was performed with financial grants from the European Community (TOMAVE project) and the Italian Group of Volcanology, Department of Civil Defence.

REFERENCES


