Nonlinear heat conduction equations with memory: Physical meaning and analytical results

Journal of Mathematical Physics 58, 063501 (2017); https://doi.org/10.1063/1.4984583

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ABSTRACT

We study nonlinear heat conduction equations with memory effects within the framework of the fractional calculus approach to the generalized Maxwell-Cattaneo law. Our main aim is to derive the governing equations of heat propagation, considering both the empirical temperature-dependence of the thermal conductivity coefficient (which introduces nonlinearity) and memory effects, according to the general theory of Gurtin and Pipkin of finite velocity thermal propagation with memory. In this framework, we consider in detail two different approaches to the generalized Maxwell-Cattaneo law, based on the application of long-tail Mittag-Leffler memory function and power law relaxation functions, leading to nonlinear time-fractional telegraph and wave-type equations. We also discuss some explicit analytical results to the model equations based on the generalized separating variable method and discuss their meaning in relation to some well-known results of the ordinary case.