On Local Causality and the Quantum-Mechanical State Vector.

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Bell's inequality deals with pairs of correlated systems: such systems are described by quantum mechanics in a two-fold manner, sometime with factorizable state vectors (also called state vectors of the first type, whose set will be denoted by S_{I} in the following), some other time with nonfactorizable state vectors (called of the second type, set denoted by S_{II}). This distinction of S_{I} from S_{II} is important because the elements of S_{I} always satisfy (while those of S_{II} sometimes violate) Bell's inequality (1).

This is not the only unpleasant feature absolute S_{II} because the famous paradox by EINSTEIN, ROSEN and PODOLSKY (2) is due to the presence of S_{II}. Furthermore all the unresolved difficulties met in trying to understand the description of the measurement given by q.m. arise from the need to get rid of nonfactorizable states from the final state of object plus measuring apparatus (3). Such a situation has led many physicists to believe that, perhaps, q.m. should be reformulated *without* S_{II}. Such an opinion has been expressed, for instance, by JAUCH (4): « We may thus say that the essence of our new notion of state is contained in the statement: *Mixtures of the 2nd kind do not exist* ». Other authors have linked strictly nonlocality and S_{II}: BARACCA (5) writes: « ... the states of the 2nd kind ... give the most precise physical meaning which may be actually attached to the concepts of « locality » or « nonlocality ».

We shared for a long time the first of the two previous points of view, but we will show in the present paper that it is actually untenable, because the concrete formulation of such a naïve point of view (drop S_{II} and keep only S_{I}) would lead to a theory with new types of paradoxical aspects.

We will furthermore show that local causality and states of the second kind are two quite different things because there are locally causal physical systems which cannot be described by state vectors of the first type.

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(1) J. S. BELL: *Physics*, **1**, 195 (1964); J. F. CLAUSER, M. A. HORNE, A. SHIMONY and R. A. HOLT: *Phys. Rev. Lett.*, **23**, 880 (1969); E. P. WIGNER: *Amer. Sourn. Phys.*, **38**, 1005 (1970); A. BARACCA, S. BERGIA and M. RESTIGNOLI: *Reinterpretation and extension of Bell's inequality for multi-valued observables*, to be published in *Inter. Sourn. Theor. Phys.*

(2) A. EINSTEIN, B. PODOLSKY and N. ROSEN: *Phys. Rev.*, **47**, 777 (1935).

(3) B. D'ESPAGNAT: *Conceptions de la physique contemporaine* (Paris, 1965).

(4) J. M. JAUCH: *Rendiconti S.I.F.*, Course IL (New York, N. Y., and London, 1971), p. 31.

(5) A. BARACCA: « *Proper* » *vs.* « *improper* » *mixtures. The key problem in the foundations of quantum mechanics*, University of Florence, preprint.

A point which has been investigated in recent papers is the search for « sensitive » observables, defined as those observables whose average is different according to whether the state of the system belongs to $S_{\rm I}$ or to $S_{\rm II}$. It has been show that the left-hand side of Bell's inequality is such an observable (6) and theorems have been proven which allow one to identify whole classes of sensitive observables (7). Two inequalities following from such considerations are (8)

$$|B| = |P(\boldsymbol{j}, \boldsymbol{i}+\boldsymbol{j}) - P(\boldsymbol{j}, \boldsymbol{i}-\boldsymbol{j}) + P(\boldsymbol{i}, \boldsymbol{i}+\boldsymbol{j}) + P(\boldsymbol{i}, \boldsymbol{j}-\boldsymbol{i})| \leqslant 2 \,, \tag{1}$$

$$\Gamma = -P(\boldsymbol{i}, \boldsymbol{i}) - P(\boldsymbol{j}, \boldsymbol{j}) - P(\boldsymbol{k}, \boldsymbol{k}) \leqslant 1 \,, \tag{2}$$

where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are three unit vectors along the x-, y-, and z-axes and $P(\boldsymbol{a}, \boldsymbol{b})$ are correlation functions for the two spin-$\frac{1}{2}$ particles which are being considered for measurements of spin-components along the directions $\boldsymbol{a}$ (for the first particle) and $\boldsymbol{b}$ (for the second particle).

The upper limits in (1) and (2) are the maximum values that the observable B and Γ can assume for state vectors to $S_{\rm I}$. The upper limit of (1) can also be obtained from local causality and (1) is essentially a particular case of Bell's inequality.

Quantum mechanics predicts in certain physical situations (those described by state vectors belonging to $S_{\rm II}$) values for B and Γ which are larger than the upper limits in (1) and (2). For the singlet state (which belongs to $S_{\rm II}$) one has

$$B = 2\sqrt{2} \,, \tag{3}$$

$$\Gamma = 3 \,. \tag{4}$$

It will be shown in the next paragraph that there is not a direct connection between (2) and local causality. The meaning of (1) and (2) is therefore very different since the former seems to be a consequence of local causality (9). The exact values of B and Γ can, of course, be found experimentally and this is what makes considerations about inequalities like (1) and (2) so interesting (10).

We will now proceed to show that it is possible to imagine physical situations which are fully compatible with local causality and which lead to a violation of the condition $\Gamma \leqslant 1$ deduced from q.m. with only state vectors belonging to $S_{\rm I}$. This suffices to show at least that q.m. with vectors belonging to $S_{\rm I}$ is not the most general formulation of local causality.

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(6) V. Capasso, D. Fortunato and F. Selleri: *Inter. Journ. Theor. Phys.*, **7**, 319 (1973).

(7) N. Cufaro Petroni: *On the observable differences between proper and improper mixtures*, University of Bari, preprint (1976).

(8) D. Fortunato, A. Garuccio and F. Selleri: *Observable consequences from second type state vectors of quantum mechanics*, to be published in *Inter. Sourn. Theor. Phys.*

(9) Some reservations about the fact that local causality be the most essential ingredient of Bell's inequality have been expressed in different ways by: A. Garuccio and F. Selleri: *Nonlocal interactions and Bell's inequality*, University of Bari, preprint (1976); O. Costa de Beauregard: *Compt. Rend.*, **282**, 1251 (1976).

(10) Contradictory results about Bell's inequality have been found for instance by S. J. Freedman and J. F. Clauser: *Phys. Rev. Lett.*, **28**, 938 (1972); E. S. Fry: Communication at the 1967 Erice Thinkshop on Physics; R. A. Holt and F. M. Pipkin: *Quantum mechanics vs. hidden variables: polarization correlation measurement on an atomic mercury cascade*, Harvard University preprint (1974). As far as the inequality (2) is concerned the experimental evidence is definitely against its validity (and in favour of quantum mechanics): J. F. Clauser: *Measurement of the circular polarization correlation in photons from and atomic cascade*, LBL preprint No. 4564 (1975).

(11) G. Scalera: *Fattorizzazione degli stati in meccanica quantica*, Tesi, Universitá di Bari (1976). The model discussed in this thesis is a generalization of the model introduced by Bell (ref. (1)).

It is not difficult to imagine visualizable models which always lead to correlation functions satisfying

$$P(\boldsymbol{a}, \boldsymbol{a}) = -1 \tag{5}$$

for an arbitrary direction specified by the unit vector $\boldsymbol{a}$. This results in $\Gamma = 3$ and therefore in a violation of inequality (2).

If such models are formulated consistently with local causality, as it is very easy to do, one instead has, necessarily, $|B| \leqslant 2$.

We will briefly sketch one such model, which has been developed in detail elsewhere ([11]). Two correlated particles, which quantum mechanics would describe with the singlet state vectors, are two classical spheres rotating in such a way that their spins lie on the same straight line but point in opposite directions. Their translation results in an increasing relative distance, until they interact with two different measuring apparata which accertain *the sign* of the spin projection along a common direction $\boldsymbol{a}$. Whatever apparatus is a dicotomic observable which can be assumed to be $+1$ for positive projection and -1 for negative projection. Obviously for any given $\boldsymbol{a}$ a total anticorrelation will be found and the measured correlation function will be given by (5) which leads to (4) which in turn violates (2).

Several mechanisms can be invented for the interaction between measuring apparata and rotating spheres, but we do not insist on them here because what we said that is probably enough to persuade the reader that a locally causal situation can well result in a ᴧiolation of (2).

The conclusion is thus that local causality and a theory with only state vectors of the first type are two rather different things.