Are artificial satellites orbits influenced by an expanding Earth?

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Abstract
The possible role played by parameters linked to the expanding Earth on the effects we observe in the orbital motion of the artificial satellites is discussed here. The important result in this short note is the discrimination between the reality of the glacial rebound process and/or the relaxation of the 100 m excess of equatorial bulge testified by the high rate of $J_2$, and the improbable role that glacial rebound can have in driving PM and TPW. It is recommended that the new technology of drag-free satellites be used (Gravity-Probe B is the first step) to reveal possible residual orbital parameter variations ascribable to formerly unrecognized fictitious drag terms due to Earth radial increase.

Key words M and G time-variations – satellite geodesy – orbit perturbations – expanding Earth

1. Introduction

For many years unequivocal evidence has accumulated in favour of an expanding Earth (Scalera, 1993a,b, 1994, 1997, 1998, 2001, 2002, 2003c, 2006). I have tried to test this promising new global tectonics scenario, and in particular the branch that envisages the possibility that Earth expansion is directly linked to still unknown properties of the gravitational field – an old idea with roots in the works of Yarkovsky (1888), Beekman (2006) and Hilgenberg (1929, 1931, 1933), but with more ancient background in the mechanical explanation of gravity due to Georges-Louis Le Sage (1724-1803) (Edwards, 2002).

A first significant result has been the possibility to link the actual Polar Motion (PM) to the expected – and observed – extrusion of new crust with maximum rate in the Nazca triple point (Scalera, 2002, 2003a,c, 2006). A second result has been to become aware that many geophysical, geodetic and geological data and reasoning lead to the possibility that the Earth’s local gravity can increase through geological time (Scalera, 2003b).

In this short paper my purpose is to discuss the possibility that the anomalies detected in the orbits of the artificial satellites could be linked not only to the mechanical causes of the Earth’s shape variations – namely tidal deceleration and glacial isostatic rebound – but can be caused by other ongoing terrestrial physical processes, which are customarily neglected, but which can also play a role in explaining fine details in the detected secular trend of the involved observational quantities.

2. Neglected quantities in the $J_2$ variation estimations

Let us examine the further possibility that the observed anomalous higher value of $J_2$ variation could be a still unrecognized combination of gradients of different quantities. Initially, it can be recalled that the $J_2$ variation is usually detected...
from the precession of the artificial satellites’ orbits. If an equatorial bulge is present, the satellites experience a perturbing force that acts trying to transform the orbits to equatorial ones. But the reaction of the orbit is to precess and a retrogradation of the ascending node is observed. The retrogradation rate is not constant but because the Earth is moving toward a more spherical shape (due at least to the tidal deceleration and glacial rebound), the distance between two successive nodes is a series of values converging to zero – in the limit of a perfect spherical Earth and without considering the apparent drift of the node due to the tidal slowing down of the Earth and all the other Earth spin irregularities.

Measuring this deceleration of the node, \( \dot{\Omega} \), it is possible to evaluate \( J_2 \) using the following formula (Kaula, 1966; Caputo, 1967), neglecting higher order small terms and recalling that \( J_1 = -\sqrt{2I + 1} \cdot C_n \) (with \( C_n \) the harmonic expansion coefficients)

\[
\frac{\partial \Omega}{\partial t} = \dot{\Omega} = -\frac{3 \cos I \cdot N_0}{2(1 - e^2)^2} \left( \frac{R}{a} \right)^2 J_2 \quad (2.1)
\]

with \( N_0 = \frac{2\pi}{I} = \sqrt{GM/a^3} \) and \( a \) = satellite’s semimajor axis; \( e \) = orbit eccentricity; \( I \) = orbit inclination; \( N_0 \) = mean motion; \( R \) = Earth radius. With the value of \( I = 109.5^\circ \) for Lageso the node velocity is greater than zero and directed eastward. In this first order approximation the satellite’s semimajor axis is assumed constant and in the following considerations its variations will be assumed as caused only by the several types of drag affecting the satellites’ orbits.

If we define

\[
P = -\frac{2(1 - e^2)^2}{3 \cos I}, \quad \text{and} \quad K = G \cdot \frac{1}{2} \cdot M \cdot \frac{R}{2} \cdot a^2 \cdot \alpha^2 \]

then we can write

\[
J_2 = -\frac{2(1 - e^2)^2}{3 \cos I \cdot N_0} \left( \frac{a}{R} \right)^2 \cdot \dot{\Omega} =
\]

\[
= P \cdot G \cdot \frac{1}{2} \cdot M \cdot R \cdot a^2 \cdot \alpha^2 \cdot \dot{\Omega} = P \cdot K \cdot \dot{\Omega} \quad (2.2)
\]

and then – in the classical way of evaluating – the \( J_2 \) variation is \( \delta J_2 / \delta t = J_1 = P \cdot K \cdot \dot{\Omega} \) and also \( C_{20} / C_{20} = J_2 / J_2 = \dot{\Omega} / \Omega \) finally holds.

While in (2.2) the parameter \( P \) can be considered constant, the value of \( K \) is affected at least by satellite altitude decay due to different types of drag – atmospheric drag, solar radiation drag, albedo, etc. – which are properly taken into account in the computations of \( J_2 \) variation that are commonly found in literature (Milani et al., 1987). It is evident in the above classic elaboration (2.1) and (2.2) that no possibility is envisaged for secular variation of \( M, G, \) and \( R \) – all contained in \( K \). But this is not the case if an expanding Earth with increasing mass and/or \( G \) is assumed.

If the time variations of \( G, M, \) and \( R \) are not neglected, it is easy to see that differentiating eq. (2.2), the expression for \( J_2 \) can be written

\[
J_2 = P \cdot K \cdot \frac{\dot{\Omega}}{\Omega} \left( \frac{\dot{\Omega}}{\Omega} - \frac{1}{2} \frac{\dot{G}}{G} - \frac{1}{2} \frac{\dot{M}}{M} - \frac{2}{R} \frac{\dot{R}}{R} + \frac{7}{2} \frac{\dot{a}}{a} \right)
\]

or

\[
\frac{\dot{J}_2}{J_2} = \frac{\dot{\Omega}}{\Omega} - \frac{1}{2} \frac{\dot{G}}{G} - \frac{1}{2} \frac{\dot{M}}{M} - \frac{2}{R} \frac{\dot{R}}{R} + \frac{7}{2} \frac{\dot{a}}{a} \quad (2.3a)
\]

which can also be rearranged

\[
\frac{\dot{\Omega}}{\Omega} = \frac{\dot{J}_2}{J_2} + \frac{1}{2} \frac{\dot{G}}{G} - \frac{1}{2} \frac{\dot{M}}{M} + \frac{2}{R} \frac{\dot{R}}{R} - \frac{7}{2} \frac{\dot{a}}{a} \quad (2.3b)
\]

From this last expression (2.3b) it is more clear that the astronomically observable quantity \( \dot{\Omega} \) is not determined only by \( J_2 \) – apart from the satellite decay \( \dot{a}/a \) – but it is also under the influence of variations of \( G, M \) and \( R \). Other factors can produce the total amount of \( \dot{\Omega} \) and then there exists the possibility that the value of \( \dot{J}_2/J_2 \) is in some amount incorrectly estimated because quotas of the observed \( \dot{\Omega} \) are not distributed to the other addends of the right side of (2.3b).

3. Discussion

Let us examine the typical values that can be assigned to \( \dot{\Omega}/\Omega \) and to the five addends which appear in the righthand side of the expression (2.3b), listed in table I. Now it can be considered:

i) Among the mechanical causes of the decreasing polar flatness of the Earth the isostatic glacial rebound is today considered a main...
cause, but recently some doubts have been raised – on geologic and geomorphologic basis – about the real cause of the uplift of the Fennoscandian Mountains (not due to deglaciation) (Lidmar-Bergström et al., 2000).

ii) Different causes – linked to expanding Earth – can be envisaged on the origin of the Polar Motion (Scalera, 2002, 2003a,c, 2006). Polar Motion was formerly explained as being due to the deglaciation of an irregularly distributed ice cap (Peltier, 1976, 1981; Sabadini et al., 1982, 1983; Peltier and Jiang, 1996) (also this irregularity has been criticized: the main difficulty in this view is the estimation of the extensions and thickness of the paleo-icecap on Canadian and Siberian shields respectively. See Clark et al., 2001), but this phenomenon is incapable of explaining PM during the deep geological time (True Polar Table I. Typical values that can be assigned to $\dot{\Omega}/\Omega$ and to the five addends which appear in the right hand side of the expression (2.3b).

<table>
<thead>
<tr>
<th>Kind</th>
<th>Quantity</th>
<th>Value</th>
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| Observed        | $\dot{\Omega}/\Omega$ | $3.3 \cdot 10^{-8}$ | If $\dot{\Omega} = -15 \text{ marcs/yr}^2$ (based on LLR modelling of UT1) (other elaborations of observational data span from $-10 \text{ marcs/yr}^2$ by Rubincam to $-18 \text{ marcs/yr}^2$ based on BIH modelling of UT1; see Yoder et al., 1983). Considering orbital parameters for Lageos:  
- Period $= 3.758 \text{ h}$;  
- Semimajor axis $= a = 12270 \text{ km}$;  
- Eccentricity $= e = 0.004$;  
- Inclination $= I = 109.94^\circ$;  
- Equatorial Node Period $= 1046 \text{ days}$.
| Actual value    | $J_2$    | $0.00108263$ | Two order of magnitude higher than the expected value.                    |
| Estimated value | $\hat{J}_2$ | $-2.8 \cdot 10^{-11}$ | Expected value taking into account only the decreasing flatness due to the increasing LOD (Varga, 2002). |
| Expected value  | $\hat{\hat{J}}_2/\hat{J}_2$ | $-5.5 \cdot 10^{-13}$ | Taking only the value coming from the slowing down of the Earth spin. Neglecting the isostatic glacial rebound contribution. |
| Estimated value | $\dot{J}_2/J_2$ | $-2.6 \cdot 10^{-8}$ |                                      |
| Expected value  | $\dot{\hat{J}}_2/\hat{J}_2$ | $-5.1 \cdot 10^{-10}$ |                                      |
| Upper limit     | $\dot{G}/G$ | $-10^{-9} \div -10^{-10}$ | While precise orbital estimates provide a value of $-10^{-11}$ (Wesson, 1978; Müller, 1991), these estimates do not consider the possibility of a variation of the involved masses and consequent compensations in the searched effects. |
| Upper limit     | $\dot{M}/M$ | $10^{-9}$ | From geological records of heap of uncremented materials (Mann and Kanagy, 1990; Scalera, 2003b), and from palaeogeography linked to secular polar motion (Scalera, 2003a,c, 2006). |
| Reasonable value| $\dot{R}/R$ | $0.8 \cdot 10^{-10}$ | If $\dot{R} = 0.5 \text{ mm/yr}$ from considerations on an actual low tectonic activity in the Recent, following the Map of the spreading rates in the geologic time (Müller et al., 1997; McElhinny and McFadden, 2000). |
| Mean value      | $\dot{R}/R$ | $2.4 \cdot 10^{-9}$ | If $\dot{R} = 1.5 \text{ cm/yr}$ from palaeogeography on a geologic time window of 220 Myr from Triassic to Recent (Scalera, 2001, 2003c). |
| Typical value   | $\dot{a}_a/a_a$ | $-3.8 \cdot 10^{-8}$ | If $\dot{a}_a = -1.3 \text{ mm/day} \approx -47 \text{ cm/yr}$. |
Wander lasts hundreds of Myr and cannot be sustained by a glacial rebound that lasts a few Myr or a fraction of a Myr).

iii) The absurd situation is that the same phenomenon, PM and TPW (TPW should be considered the prolongation of PM in the geological past), is explained today in two different ways, in the Recent as due to glacial rebound and before Recent attributed to the convective motions in the mantle.

Keeping in mind that the equalities I will write below cannot be perfect – because this is a first order analysis and serves only to make general considerations – I attempt heuristically to neglect completely the glacial rebound contribute with the aim to make evident other possible contributions to the anomalous high value of node deceleration, writing in (3.1b) only the expected value of the addend \( \frac{J_2}{J_2} \), and assigning arbitrarily a rate of radius increase of 0.5 mm/yr.

\[
\frac{\dot{\Omega}}{\Omega} = \frac{J_2}{J_2} + 1 \frac{\dot{G}}{G} + 1 \frac{M}{M} + 2 \frac{\dot{R}}{R} - 7 \frac{7 \dot{a}}{2 a} \quad (3.1a)
\]

\[
3.3 \times 10^{-8} = \frac{J_2}{J_2} - \frac{1}{2} \cdot 10^{-9} + \frac{1}{2} \cdot 10^{-9} + 2 \cdot 0.8 \cdot 10^{-10} + \frac{7}{2} \cdot 3.8 \cdot 10^{-8} \quad (3.1b)
\]

or, in the case of a rate of radius increase of 1.5 cm/yr (the average rate determined by paleogeography on a Triassic-Recent time window; Scalera, 2001, 2003c)

\[
3.3 \times 10^{-8} = \frac{J_2}{J_2} - \frac{1}{2} \cdot 10^{-9} + \frac{1}{2} \cdot 10^{-9} + 2 \cdot 2.4 \cdot 10^{-9} + \frac{7}{2} \cdot 3.8 \cdot 10^{-8} \quad (3.1c)
\]

From (3.1c) it is possible to become aware that the dominant term is the last one, the drag term, which produces an acceleration of the node, and that the \( M \) and \( R \) – if both increasing – reinforce this accelerating effect. It is to be noted that what is really observed is acceleration, because of the dominant effect of the spiralling of the satellites down to lower altitudes where the effect of the equatorial bulge is stronger. The observed value of the nodal deceleration has to be corrected by adding the term of acceleration coming from drag effects.

### 4. Conclusions

From the above considerations and from an examination of the magnitude orders of the quantities in (3.1b) and (3.1c) it can be concluded:

i) Only several centimetres of annual radius increase can have a non-negligible effect on the total of the summation (3.1a). Indeed, from the global palaeogeography, the mean rate on the geological time window Triassic-Recent, nearly 220 Myr, is \( \approx 1.5 \) cm/yr (Scalera, 2001, 2003c), and this value does not seem sufficient to have important effects on the (3.1a) balance, remaining the relative addendum one order of magnitude below the required \( 10^{-8} \). Due to the low rate of the present global tectonic activity (map of the oceans expansion rate in: Müller et al. 1997; McElhinny and McFadden, 2000) we should expect a rate in the Recent in the order of a few millimetres. Moreover the effect of an increasing radius can only be an increase of the velocity of the node (acceleration), while a deceleration is the reality. Consequently, a role of increasing radius in forming the observed value of the node deceleration can be excluded.

ii) The glacial rebound seems to be the real phenomenon capable of causing the ongoing higher than expected decreasing of the Earth’s flatness, but additional contributions to \( J_2 \) variations, impossible at present to be separated, can come from the relaxation of the well known excess of equatorial bulge (Caputo, 1965, 1967; Alessandrini and Papi, 1987; Alessandrini, 1989), and from variations in the distribution of the Earth’s waters – also underground water – (Dickey et al., 1999; Rodell and Famiglietti, 1999; GRACE project).

iii) We can continue to consider Polar Motion (and True Polar Wander) and decreasing Earth’s flatness as independent phenomena. Indeed both the following statements can hold: 1) PM-TPW can be driven – with a common explanation – by causes linked to an expansion of the Earth (Scalera, 2002, 2003c, 2006); 2) ice cap melting can easily explain the decreasing flatness of the Earth, but only using complicated modelling can it provide an explanation of PM – that however cannot be prolonged into the deep geological time to explain TPW. Thus polar ice caps melting is a suspicious explanation.
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of PM because it produces a lack of unitary cause for clearly identical phenomena.

iv) Ice cap melting is also a suspicious cause of uplift of mountains because geological and geomorphological clues indicate different causes, eventually analogous to the causes of uplift of mountains at latitude far from the glacial circle parallel (Lidmar-Bergström et al., 2000; Ollier, 2006).

v) A decreasing $G$ gives an addend in ((3.1a) to (3.1c)) that can contribute only a minute term – in the order of $10^{-9}$ or less (Olive and Qian, 2004) – to the summation. The same holds for an increasing planetary mass, but with the same unfavourable trend (node acceleration instead of deceleration) as the increasing radius. Then $G$ and $M$ play an irrelevant role in this problem and an expansion of the Earth, whether due to inner changes of phases or to an increase in mass by an unknown cosmological process, cannot be distinguished by present artificial satellite technology.

vi) Finally, we may consider the improbable possibility that the increasing Earth radius could episodically reach a rate able to produce an acceleration of the node nearly equal to the mean deceleration – a situation of absence of deceleration really observed in the twenty-five years of data (1979-2003) in the four years time window 1997-2001 and still unexplained (Cazenave and Nerem, 2002; Cox and Chao, 2002). To produce a similar total compensation the radius time gradient should be of several cm/yr, which I think may be resolved without ambiguity from astrog- odelic global baseline measurement techniques (VLBI, GPS, ...). Moreover an episodic radius variation should not be compensated, as that concerns consequences on global angular moment, by deep Earth material ongoing differentiation, which is a different and independent process. Then we would have to expect a strong effect on LOD that has not been observed.

vii) The problem of the difficulty in separating the global expansion, with its effect of an increasing radius on the equator, and the glacial isostatic rebound and/or relaxation of the ≅100m excess of equatorial bulge (Caputo, 1965, 1967; Alessandrini and Papi, 1987; Alessandrini, 1989) with their effect of a decreasing equatorial radius, is still open (Scalera, 2003c).

viii) The problem of a precise separation of the drag driven orbital decay from the global uplift of the Earth surface (global expansion) is not resolvable at present. The annual rate of radius increase is only from 1/10 (a few centimetres, too optimistic) to 1/100 (a few millimetres, more realistic) of the annual orbit decay (several tens of centimetres). Thus variation of the radius, of $G$ and of $M$ can be easily contained in the decay as a small additional unrecognized term. The only way to detect the non-classic parameters variation rate in the satellites’ orbits is to eliminate all sources of drag. In this light, the new experimental development of GRAVITY PROBE B (Anonymous, 2005), which – besides the investigation of minute relativistic effects – may be the first example of an orbiting mass shielded from the causes of drag, promising to open a new epoch in global positioning satellite geodesy.

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REFERENCES

Dickey, J.O., C. Bentley, R. Bilham, J.A. Carton, R.J. Eanes, T.A. Herrings, W.A. Kaula, G.S.E. Lager-


HILGENBERG, O.C. (1933): Vom wachsenden Erdball (The Expanding Earth) (Giessmann & Bartsch, Berlin), pp. 56.


YARKOVSKI, J. (1888): Hypotèse cinétique de la gravitation universelle et connexion avec la formation des éléments chimiques (Kinetic hypothesis of the universal gravitation and its connection with the formation of chemical elements), (Chez l’Auteur, Moscou), pp.139 (in French).