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A Brownian Model for Recurrent Volcanic Eruptions: an Application to Miyakejima Volcano (Japan)

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Abstract The definition of probabilistic models as mathematical structures to describe the response of a volcanic system is a plausible approach to characterize the temporal behavior of volcanic eruptions, and constitutes a tool for long-term eruption forecasting. This kind of approach is motivated by the fact that volcanoes are complex systems in which a completely deterministic description of the processes preceding eruptions is practically impossible. To describe recurrent eruptive activity we apply a physically-motivated probabilistic model based on the characteristics of the Brownian passage-time (BPT) distribution; the physical process defining this model can be described by the steady rise of a state variable from a ground state to a failure threshold; adding Brownian perturbations to the steady loading produces a stochastic load-state process (a Brownian relaxation oscillator) in which an eruption relaxes the load state to begin a new eruptive cycle. The Brownian relaxation oscillator and Brownian passage-time distribution connect together physical notions of unobservable loading and failure processes of a point process with observable response statistics. The Brownian passage-time model is parameterized by the mean rate of event occurrence, μ, and the aperiodicity about the mean, α. We apply this model to analyze the eruptive history of Miyakejima volcano, Japan, finding a value of $44.2(\pm 6.5$ years) for the μ parameter and $0.51(\pm 0.01)$ for the (dimensionless) α parameter. The comparison with other models often used in volcanological literature shows that this pysically-motivated model may be a good descriptor of volcanic systems that produce eruptions with a characteristic size. BPT is clearly superior to the exponential distribution and the fit to the data is comparable to other two-parameters models. Nonetheless, being a physically-motivated model, it provides an insight into the macro-mechanical processes driving the system.

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Introduction

Volcanoes can be viewed as complex physical systems in which a completely deterministic description of the processes occurring before or during an eruption is practically impossible. This fact motivates the definition and development of probabilistic models as mathematical structures to describe physical phenomena: this is a response to the problem in which we do not have direct access to the physical processes, but we can have a record of the response of the system. In particular, if some characteristic properties of the response of the system (e.g. event times) can be associated with a random variable, and if it is possible to express a probability function for the random variable, then it is possible to define a probabilistic model for the response of the considered system.

A time series of eruptions from a single volcano can be treated as a stochastic point process with individual eruptions as (random) independent events in time. Statistical analysis of both repose time and erupted volume catalogs have been performed for a large number of volcanoes (e.g., Wickman 1976; Klein 1982; Mulargia et al 1985, 1987; De la Cruz-Reyna 1991; Burt et al 1994; Marzocchi and Zaccarelli 2006), mainly for those with frequent eruptive activity and where detailed catalogs exist. The main objective of this kind of analysis is to develop probabilistic models to understand the past eruptive activity of the volcano and to forecast its future behavior.

When treating eruptions as events in time, several simplifying assumptions must be made (Klein 1982; Ho 1991): although the onset date of an eruption is generally well constrained, the duration some times is harder to determine and/or is rarely reported. In our analysis, we ignore eruption duration since we take the onset date as the most physically meaningful, and measure repose times from one onset date (of an eruptive episode) to the next. In this way, our modeling intends to describe the waiting times of the long-term physical processes governing *renewed volcanic activity*. Once the volcanic system has been perturbed and a new eruptive episode has started, the short-term behavior of the eruptive activity may follow different patterns during the gradual decline of activity; in this context, sporadic eruptive activity in a short time window (with respect to the repose time) after the onset of a new eruptive episode cannot be described using the former long-term model. Thus, the definition of *repose time* from this point of view is not exactly equivalent to the classic concept of *non-eruptive period*; this assumption seems justified because most eruption durations are much shorter than typical effective repose intervals (e.g., Klein 1982).

Some distinct conceptual models have been proposed to describe the eruptive behavior of different volcanoes around the world. The most frequent solutions describe the eruptive activity in terms of (1) a homogeneous Poisson processes in the time domain (e.g., Klein 1982; De la Cruz-Reyna 1991; Marzocchi and Zaccarelli 2006), (2) Time-Predictable processes (e.g., Burt et al 1994; Sandri et al 2005; Marzocchi and Zaccarelli 2006) or (3) Size-Predictable processes (e.g., Burt et al 1994; Marzocchi and Zaccarelli 2006). In our analysis, none of these existing models successfully explains the eruptive activity of Miyakejima volcano, which seems to show a more regular behavior and a characteristic size for the erupted volumes. Other probabilistic models often used in literature were also tested, as for example the Loglogistic process (e.g., Connor et al 2003; Watt et al 2007), and a non-homogeneous Poisson process modeled using a Weibull process (e.g., Ho 1991; Bebbington and Lai 1996a,b). In this paper we present an interesting, physically-motivated, probabilistic model based on the Brownian passage-time distribution which has not been used before for volcanological problems; we apply it to analyze the time series of repose times (as defined), τ, of Miyakejima volcano, and compare its performance respect to other models often used in volcanology.

The Models

Most of the models considered here are based on generic distributions that characterize renewal processes that in general may have some physical meaning and applicability. A renewal process has the property that the inter-occurrence times are independent and identically-distributed, positive, random variables having a common distribution $F(\tau)$. We have also considered for test two non-renewal processes: the size-predictable (SPM) and the time-predictable (TPM) models.

Brownian Passage-Time Model (BPT)

A particularly interesting renewal model is the Brownian passage-time model (BPT). It was originally introduced by Matthews et al (2002) and Ellsworth et al (1999) to provide a physically-motivated renewal model for earthquake recurrence. It is based on the properties of the Brownian relaxation oscillator (BRO). A BPT model considers an event (earthquakes in Matthews' model or renewed eruptive activity in our case) as a realization of a point process in which new eruptive activity will occur when a state variable (or a set of them) reaches a threshold (X_f) and at which time the state variable returns to a base ground level (X_0). Adding Brownian perturbations to steady loading of the state variable X produces a stochastic load-state process. An eruption relaxes the load state to the characteristic ground level and begins a new cycle. The load-state process is a BRO, while intervals between events have a distribution known as *Brownian passage-time distribution*. Note that this is the name used in physics literature; in the statistics literature it is often known as *Inverse Gaussian* or *Wald* distribution (Matthews et al 2002).

In the conceptual model of Matthews et al (2002), the loading of the system has two components: (1) a constant-rate loading component, λt, and (2) a random component, $\varepsilon(t) = \sigma W(t)$, that is defined as a Brownian motion (where W is a standard Brownian motion and σ is a non-negative scale parameter). Standard Brownian motion is simply integrated stationary increments where the distribution of the increments is Gaussian (which might be motivated by central-limit arguments if we consider perturbations as the sum of many small, independent contributions), with zero mean and constant variance. The Brownian perturbation process for the state variable $X(t)$ (see figure 2 in Matthews et al (2002)) is defined as:

$$X(t) = \lambda t + \sigma W(t) \tag{1}$$

An event will occur when $X(t) \geq X_f$; event times are seen as “first passage” times of Brownian motion with drift (Matthews et al 2002), which means that eruptive episodes take place when the threshold X_f is first reached. The BRO are a family of stochastic renewal processes defined by four parameters: the drift or mean loading (λ), the perturbation rate (σ^2), the ground state (X_0), and the failure state (X_f). On the other hand, the recurrence properties of the BRO (repose times) are described by a Brownian passage-time distribution which is characterized by two parameters: (1) the *mean time* or period between events, (μ), and (2) the *aperiodicity* of the mean time, α, which is equivalent to the familiar coefficient of variation (defined in equation 3). The probability density for the BPT model is given by:

$$f(t;\mu;\alpha) = \left(\frac{\mu}{2\pi\alpha^2 t^3}\right)^{\frac{1}{2}} \quad \mathbf{e}^{\left\{-\frac{(t-\mu)^2}{2\alpha^2 \mu t}\right\}} \tag{2}$$

The state variable $X(t)$ is a formal parameter of a point process model and represents a constant-rate mean path that embodies a macroscopic view of a uniform loading of the volcanic system. It may summarize the macro-mechanics of the volcanic system controlled by one or more physical variables. An explicit definition of the driving physical parameters may be unrealistic and impossible to demonstrate from our analysis. Independently of its physical nature, the state variable should be a parameter that accumulates with time during repose episodes, up to a critical value beyond which the system becomes perturbed enough and a new eruptive process may be triggered. Then, the eruptive process *relaxes* the system and the state variable returns to a ground level and a new cycle starts. The perturbation factor $\varepsilon(t)$ represents the total sum of all other factors which may play a role in the recurrent eruptive process considered and/or that may randomly disturb the state variable producing the aperiodicity of the mean time between eruptions (e.g. effects from tectonic environment, changes in the magma rate supply, compositional changes).

Other models often used in volcanological problems

Poisson Process in the time domain: random model of eruption occurrences

The Poisson process is an important model often used to describe the patterns of eruption occurrences in volcanoes (e.g., Klein 1982; Mulargia et al 1985). It is mainly applicable to major eruptive activity involving a significant release of mass and energy (De la Cruz-Reyna 1991). In a Poisson process the repose times follow an Exponential distribution. It characterizes a *random volcano*, which is one that is ready to erupt at any time. An alternative possibility is that the eruption sequence is periodic to some degree, and that a certain repose time is favored; if eruptions were periodic, the distribution of repose times would be peaked instead of containing an exponentially decreasing number of larger times, as predicted by the Exponential model. In order to explore the degree of departure from a homogeneous Poisson process, we calculate the coefficient of variation, η, given by

$$\eta = \frac{\sigma}{\mu_\tau} \tag{3}$$

where μ_τ and σ are, respectively, the average and standard deviation of the repose times τ (Marzocchi and Zaccarelli 2006). The coefficient η may help us to quantify if and how much the statistical distribution of τ differs from a Poisson process: for a Poisson process (and then an Exponential distribution of τ), $\eta = 1$; more clustered distributions have $\eta > 1$ and for more regular recurrent times $\eta < 1$ (e.g., Cox and Lewis 1966; Marzocchi and Zaccarelli 2006).

Renewal models described by Weibull, Gamma, Lognormal and Loglogistic distributions

In renewal models, the random variable τ represents the lifetime or time to failure of a system. To analyze the intrinsic characteristics of this kind of probabilistic models, it is often more informative to consider the hazard function (also known as hazard rate, or intensity function) of the model than to look at the shape of the Probability Density Function (PDF) or Cumulative Distribution Function (CDF) directly; for this reason we make extensive use of the hazard function to compare the different renewal models considered. The hazard function describes the instantaneous failure rate, or the conditional density of failure at a given time, considering the information that no event occurred until that time. A more detailed description of the hazard function concept can be found forward in this paper.

The Exponential distribution (and its implicit homogeneous Poisson process) has a constant hazard function, highlighting its characteristic no-memory property. However, when processes like wearing, improvement, learning and growth are implicit in the physical system, then it is necessary to consider models where the hazard function must be a function of time. The Weibull, Gamma, Lognormal and Inverse Gaussian (the last one is described in the next section) are the models with those characteristics that are widely used in the literature.

The Weibull is one of the models most used in volcanological applications (e.g., Ho 1991, 1996; Bebbington and Lai 1996a,b). The Weibull process (WEI(ν,θ)) is one of the possible generalizations of the Exponential case; θ and ν may be interpreted, respectively, as the degree of clustering/periodicity and the underlying activity (e.g., Bebbington and Lai 1996a). If the volcanism is waning or developing, the model is generalized to allow the rate of volcanic events (which is constant in the homogeneous case) to be a decreasing or increasing function of time (Ho 1996). This can be defined as a non-homogeneous Poisson process (Bain 1978). The Gamma distribution (GAM(ν,θ)) provides an alternative generalization of the Exponential distribution but with different characteristics with respect to the Weibull; in fact, if we consider the hazard function for the Gamma model, the event rate may increase initially, but after some time the system would reach a stable condition and then it tends to a constant hazard rate (Bain 1978). This is considerably different in the Weibull model where, for $\theta > 1$, the hazard function tends to infinity as the time tends to infinity. The Lognormal model has been considered for periodicity tests by some authors (e.g., Bebbington and Lai 1996a). In this case, the hazard function has a similar behavior as the Gamma model but with the difference that the asymptotic event-rate goes to zero as the time goes to infinity. Finally, we also consider the possibility of a Loglogistic model, characterized by two parameters: μ, a location parameter, and σ, a scale parameter, which has been used by some authors (e.g., Connor et al 2003; Watt et al 2007) to successfully describe data in some volcanological applications.

Time predictable (TPM) and Size predictable (SPM) models

TPM and SPM are widely used in both seismological and volcanological literature. Both of them imply a functional relationship between size (of eruptions) and repose times. In the case of TPM, the time to the next eruption depends on the time required for magma entering the storage system to reach the eruptive level (Burt et al 1994). It can be described using a general definition of the form $\tau_i \propto [V_i]^{\beta}$, where $\propto$ stands for 'proportional to' (Sandri et al

2005). A reliable application of a TPM requires that the size (e.g. erupted volume, explosivity index, etc.) of the eruptions has to be significantly correlated to the logarithm of the time to the next eruption (Marzocchi and Zaccarelli 2006). The applicability of this model relies on two main assumptions: (1) eruptions occur when a threshold of the magma volume in the storage system is reached, and (2) the magma input in the storage system is a well defined function of the reservoir to be filled to reach that threshold; the specific case of $\beta = 1$ means that the input rate is constant (Marzocchi and Zaccarelli 2006).

On the other hand, in a SPM the duration of the repose time of the volcano (i.e. the time since the last eruption) is the parameter which is useful to forecast the size of the next eruption. As for the TPM case, the most general functional relationship between volumes and repose times for a SPM is of the form $V_i \propto \tau_i^{\beta}$ (e.g., Marzocchi and Zaccarelli 2006). In this case, the model relies on two main assumptions: (1) the output of each eruption is determined only by the magma accumulated since the last eruption, and (2) as in the previous case, the magma enters in the plumbing system at a rate described by a 'well defined' function of the magma volume in the reservoir.

Miyakejima volcano and data set

Miyakejima volcano

Miyakejima island, located about 200 km south of Tokyo (Fig. 1), is one of the most active basaltic volcanoes in Japan. Its recurrent eruptive behavior has been observed but up to now a detailed quantitative analysis based on its past activity has not been performed. In most historical eruptions, basaltic lava and scoria erupted mainly from flank fissures (Tsukui and Suzuki 1998) and most eruptions lasted a short time (a day to a month). The latest eruptive episode started in June 2000 and a caldera formed at the summit; since then, the volcano has been showing some seismic swarms accompanied by important gas emissions for more than 9 years. On the basis of surface phenomena observed, many authors divided the 2000-2010 eruptive period into at least four stages (e.g., Nakada et al 2005; Ueda et al 2005): (1) magmatic intrusion (1 day), (2) summit subsidence (10 days), (3) Explosion (40 days), and (4) gas emissions accompanied by small seismic swarms, deformation and explosions (>9 years). The total volume of tephra erupted was about 0.009 km^3 (DRE), which is much smaller than the volume of the resulting caldera (0.6 km^3) (Nakada et al 2005).

(FIGURE 1 (MAP OF MIYAKEJIMA...) SOMEWHERE AROUND HERE)

Here we analyze a data set containing the repose periods and volumes of lava and tephra emitted by Miyakejima volcano based on the historical data published by Tsukui and Suzuki (1998) and from the Global Volcanism Program catalog (Simkin and Siebert 2002 onwards). The data set was updated introducing information of the last eruption (June 2000) from Nakada et al (2005) (Table 1).

Data set of eruptions and completeness of the catalog

In order to extract unbiased information from a catalog it is necessary to check for its completeness. This issue is well known in seismology where the completeness of catalogs is

Table 1 Summary of the eruptive history of Miyakejima (this table updates the catalog provided by Tsukui and Suzuki (1998). The volume of the 2000 eruption is from Nakada et al (2005); some dates missing in Tsukui and Suzuki (1998) are from the Global Volcanism Program (Simkin and Siebert 2002 onwards). The VEI values are calculated based on the DRE volumes using the criteria defined by Newhall and Self (1982). Eruption numbers accompanied by (*) mark are those considered in the present study (after the completeness analysis).

Num.	Kind	Date	DRE volume (km^3)	VEI
29*	2000 Scoria	2000 Jun 27 (AD)	0.009	3
28*	1983 Scoria + Lava	1983 Oct 03 (AD)	0.007	2
27*	1962 Scoria + Lava	1962 Aug 24 (AD)	0.006	2
26*	1940 Scoria + Lava	1940 Jul 12 (AD)	0.015	3
25*	1874 Scoria + Lava	1874 Jul 03 (AD)	0.010	3
24*	1835 Lava	1835 Nov 11 (AD)	<0.001	<2
23*	1811 Scoria	1811 Jan 27 (AD)	<<0.010	2
	1769 (?) Lava		0.001	
22*	1763 (?) Lava	1763 Aug 17 (AD)	0.001	3
	Shinmio Explosion Breccia (SMB)		0.031	
	1763 Scoria		0.033	
21*	1712 Lava	1712 Feb 04 (AD)	0.001	2
20*	1643 Scoria	1643 Mar 31 (AD)	0.009	3
	1643 Lava		0.003	
19*	Kamakata Lava (KKL)	1595 Nov 22 (AD)	<0.001	<2
18*	Benkenezaki Lava (BKL)	1535 Mar (AD)	0.003	2
17*	Enokizawa Lava (EZL)	1469 Dec 24 (AD)	0.002	2
16	Son-ei Bokujo Ash (SBA)	1154 AD (?)	0.040	3
	1154 Scoria		<<0.010	
15	Nanto Lava (NTL)	1085 AD (?)	0.012	2
	Kamane Scoria (KMS)	10-11C AD (?)	0.010	
14	Miike Explosion Breccia (MKB)	838-886 AD	0.040	3
	Oyama Scoria		0.030	
	Oyama Lava		0.012	
13	Kazahaya Scoria (KHS)	832 AD	0.007	2
12	Mitoribata Scoria (MBS)	1290 yBP	0.007	2
11	Daihannya-yama Scoria (DHS)	500 AD	0.010	3
	Anegakata Lava		0.001	
10	Sabigahama Explosion Breccia (SHB)	320 AD	(?)<0.010	2
	Togahama-south Lava (TSL)		<0.001	
9	Togataira Ash (TGA)	260 AD	0.040	3
	Togataira Scoria (TGS)		0.005	
	Usuki-west Scoria (UWS)		n.d	
	Igayazawa Scoria		0.010	
8	Tairayama Lava (TYL)	2050 yBP	0.001	3
	Tairayama Scoria (TYS)		0.020	
7	Izu Scoria (IZS)	600 BC	0.050	3
6	Hatchodaira Accrtionary Lapilli (HCA)	2500-3000 yBP	0.200	4
	Furumio Explosion Breccia (FMB)			
	Hatchodaira Scoria (HCS)		0.170	
	Nagane Scoria (NGS)	1450 BC	n.d.	
5	Tsubota Scoria (TBS)	3000 yBP(?)	0.010	2
4	Mizutamari Explosion Breccia (MZB)	3500 yBP(?)	0.062	3
3	Igaya-east Scoria (IES)	3660 yBP	<<0.010	2
2	Igaya Accrtionary Lapilli (IGA)	4000 yBP	0.090	3
	Izushita Lava (ISL)		(?)0.001	
1	Ofunato Explosion Breccia (OFB)	7000-8000 yBP	0.150	4

often checked by analyzing the Gutenberg-Richter law and/or the time evolution of the rate of occurrence of events. In volcanology however, the incompleteness of a catalog of eruptions may be more difficult to evaluate because there are only weak indications that a general power law applies (Simkin and Siebert 2002 onwards), and also because we know that volcanoes may have different eruptive regimes in their history and then the eruptive rate may change with time (e.g., Ho 1991; Marzocchi and Zaccarelli 2006; Coles and Sparks 2006). Because of this difficulty, when we talk about the *completeness* of a given eruptive sequence, we understand it as a period of "uniformity" in the data set.

To analyze the completeness of the catalog we plot the cumulative number of eruptions in time, as seen in Figure 2a, and identify changes in the statistics of the repose times. Changes in the statistics of the sequence are identified applying a *change point* strategy (CHPT). The change point hunting methods aim to find one or more statistically significant change points in a sequence of data. Here we use a method based on the two-sample Kolmogorov-Smirnov statistics (a non-parametric test for equal distributions), which has been proposed and tested by Mulargia and Tinti (1985) and Mulargia et al (1987).

(FIGURE 2 (CUMULATIVE NUMBER OF ERUPTIONS...) SOMEWHERE AROUND HERE)

The cumulative plot in Fig. 2a shows a curve with a slope changing with time. Changes in the slope may be due to different factors as changes in the eruptive regime and/or under-reporting of eruptions in past time (incompleteness of the catalog). Applying the CHPT strategy, we identify one change point (with 0.01 significance level threshold) between the eruptions 16 and 17 of the catalog (1154 and 1469 AD respectively, see table 1). It means that we can consider the catalog to be "uniform" in the period from 1469 up to now (13 eruptive episodes), and then our analysis is oriented to describe the eruptive regime of the volcano within this period. It is important to remark that volcanoes may change eruptive regime through time: our analysis and the derived eruption forecasting assessment is based on the eruptive regime shown by the volcano in the last (around) 540 years, and is valid only under the assumption that in the future it will behave in the same way.

Data Analysis and Results

Our analysis consists of four groups of tests which can be summarized as: (1) test of a SPM and a TPM; (2) test of a homogeneous Poisson process in the time domain, (3) tests of the Brownian passage-time model, and (4) test of other possible renewal models describing different processes (e.g. recurrent, non-homogeneous Poisson); in this case we have considered as possible candidate distributions four models often used in volcanology: Lognormal, Gamma, Weibull, and Log-logistic.

We estimate the model parameters for each candidate model using a Maximum Likelihood Estimate (MLE) approach, and use the Akaike Information criteria (AIC, Akaike 1974) for the model selection; the AIC is a tool based on the concept of entropy, and offers a relative measure of the information lost when a given model is used to describe some data (a trade-off between accuracy and complexity of the model). On the other hand, SPM and TPM models are tested by regression analysis of repose times and sizes (erupted volumes and VEI).

Estimating the parameters of the considered models

Fig. 3 shows the plots of erupted volumes against the time since the last eruption (Fig. 3a) for a SPM, and against the time to next eruption (Fig. 3b) for a TPM. In these figures we cannot see any clear relationship between sizes (i.e. volumes) and the times (from last eruption or to the next one). This is confirmed by the R-square statistic (reported in each panel), which in both cases is very low ($R^2 < 0.06$). Furthermore, if we consider the slope of the best fitted line and the associated errors (see Fig. 3), in both cases the slope of the fitted line does not significantly differ from zero; using a F-test (Ho : slope = 0), the hypothesis Ho cannot be rejected (at a significance level of 0.05). Then there is no strong evidence suggesting either a SPM or TPM as a successful model for Miyakejima, so at least from these data we cannot extract any information, even if we group sizes using the VEI. However, we should be aware of the potential problems that can be faced when working with erupted volumes due to the quality of the reported values and/or the real possibility to have an accurate quantification of the volume of magma involved during the process. The 2000 Miyakejima eruption is a nice example, since there are strong evidences that the explosive volume is just a fraction of the total volume of magma involved, because a large volume of magma moved laterally into a large dyke (e.g., Nakada et al 2005; Ueda et al 2005). This may be an unavoidable source of ‘noise’ that could eventually hide a potential relation among repose times and erupted volumes. Likewise, we should highlight here that this lack of evidence may be also a direct consequence of the eruptive behavior of the volcano; later in this paper we make a discussion about the erupted volume distribution and also about how TPM or SPM could coexist with a renewal model.

(FIGURE 3 (PLOT OF TIMES SINCE PREVIOUS...) SOMEWHERE AROUND HERE)

Next we test the hypothesis of a homogeneous Poisson process, in which the repose times τ follow an Exponential distribution. Fig. 4 shows the empirical cumulative distribution of observed data and the best (maximum likelihood) Exponential distribution fitting the data. A one-sample Kolmogorov-Smirnov test rejects this hypothesis (at a significance level of 0.05). Furthermore, if we calculate the coefficient of variation η (equation 3) we get $\eta = 0.51$, which confirms the non-random distribution and the possibility of a recurrent behavior (i.e. $\eta < 1$).

(FIGURE 4 (CDF OF THE EXPONENTIAL DIST....) SOMEWHERE AROUND HERE)

Now we test a set of trial probabilistic models (which were briefly described in a previous section). The characteristic parameters of each model (BPT, Lognormal, Gamma, Weibull and Log-logistic) were estimated using a maximum likelihood approach. Table 2 summarizes the functional form of the PDF, estimated (MLE) parameters (and uncertainties), and the AIC (Akaike 1974) for all the probabilistic models considered. Using a Kolmogorov-Smirnov test, we cannot reject the Weibull, BPT, Lognormal, Gamma, and Loglogistic hypothesis (at significance level of 0.05), which means that, from a statistical point of view, all these probability models successfully explain the observed data. Fig. 5 shows the Cumulative Distribution Function (CDF) of the candidate distributions and the empirical CDF of the observed data (τ); as reference, the CDF of the Exponential distribution is also included.

(FIGURE 5 (PLOT OF CDF OF BPT, WEI, LOGN...) SOMEWHERE AROUND HERE)

Based on AIC values listed in table 2, five out of six models have AIC values in the range $107.3 \leq \text{AIC} \leq 110.2$ (Weibull, BPT, Gamma, Lognormal and Loglogistic), being the Weibull the model with the lowest AIC. All these two-parameters models perform better than the exponential distribution, whose AIC value is significantly higher. To test it we simulate 1000 exponential catalogs and fit these catalogs using the exponential and each 2-parameters distributions; we find that more than 99% of the cases provide an AIC difference smaller than 2.5, which is much smaller than the 7-9 units of AIC difference that we have in the real catalog. On the other hand, the AIC differences found for the five two-parameters models are not significantly different. This fact demonstrates that the BPT model is able to describe the data with similar performance as the other models normally used in volcanological applications, however, the BPT model can be an interesting alternative to describe this kind of data since it may be directly linked to a physical system which may provide significant insights for the interpretation of the observed eruptive behavior. For instance, the mean repose time μ of the BPT (44.2 ± 6.5 years), or its reciprocal, the mean rate of occurrence, is the natural scale parameter of first-order interest, as it measures the typical frequency at which eruptions occur. Changing the mean re-scales time but does not otherwise alter the shape of the probability distribution. The aperiodicity ($\alpha = 0.51 \ \pm 0.01$) is chosen as a second parameter because it is a natural shape parameter of the BPT family, and it is a dimensionless measure of irregularity in the event sequence (Matthews et al 2002); in other words, it is a measure of the aperiodicity of the mean. As α tends to 0, the sequence tends to be perfectly periodic, while as α grows, the sequence tends to a (homogeneous or non-homogeneous) “Poisson-like” process. In particular, the higher α parameter (for $\alpha > 1$), the more possibility that a given “Poisson-like” sequence has a clustered character (i.e. non-homogeneous Poisson process).

Distribution analysis of erupted volumes

Using the eruption size data (volumes and VEI), it was not possible to find any evidence to support either a TPM or a SPM (highlighting that the unavoidable uncertainties in the volume estimation or the possible recurrent behavior of the volcanic system may play an important role to avoid the definition of a clear relationship between repose times and erupted volume). The question that arises is then, within this framework, how should the erupted volumes be distributed? We perform an analysis of the erupted volumes (within the period of completeness of the catalog) and the results are summarized in Fig. 6. A Lognormal distribution provides a good explanation of the erupted volume data (hypothesis not rejected using a one sample Kolmogorov-Smirnov test at a significance level of 0.05); it means that there exists a *preferred* or more common erupted volume (the mean erupted volume is 0.012 ± 0.004 km^3, assuming that not considerable bias exist in the volume database). In other words, we can consider that the logarithm of the erupted volumes are normally distributed. This result can support the hypothesis of a recurrent source model, as suggested by the Brownian passage-time distribution for the repose times, and may also explain the poor resolution of the TPM, since if there is a *preferred* size and a *preferred* repose time, then the data in a time-size space should tend to group in a cluster; from this point of view the BPT model would not be incompatible with a TPM model.

(FIGURE 6 (CDF OF ERUPTED VOLUMES...) SOMEWHERE AROUND HERE)

Table 2 Candidate distributions, PDF, estimated (MLE) model parameters and uncertainties, and Akaike Information Criteria (AIC)

Model	**Probability Density**	**Parameters (MLE)**	**AIC**
Weibull (ν, θ)	$\theta\nu^{\theta}t^{\theta-1}\,\mathrm{e}^{\left\{-\nu^{\theta}t^{\theta}\right\}}$	$\nu^{-1} = 49.9\ (\pm 5.6)$ y. $\theta = 2.7\ (\pm 0.7)$	107.3
Brownian Passage-time (μ, α)	$\left(\frac{\mu}{2\pi\alpha^2 t^3}\right)^{\frac{1}{2}}\mathrm{e}^{\left\{-\frac{(t-\mu)^2}{2\alpha^2\mu t}\right\}}$	$\mu = 44.2\ (\pm 6.5)$ y. $\alpha = 0.51\ (\pm 0.01)$	**108.3**
Gamma (ν, θ)	$\frac{\nu^{\theta}t^{\theta-1}}{\Gamma(\theta)}\mathrm{e}^{-\nu t}$	$\nu^{-1} = 9.1\ (\pm 3.8)$ $\theta = 4.9\ (\pm 1.9)$	108.3
Lognormal (μ, σ)	$\frac{1}{\sigma t\,\sqrt{2\pi}}\,\mathrm{e}^{\left\{-\frac{(ln(t)-\mu)^2}{2\sigma^2}\right\}}$	$\mu = 3.68\ (\pm 0.15)$ $\sigma = 0.51\ (\pm 0.11)$	109.1
Loglogistic (μ, σ)	$\frac{\mathrm{e}^{\frac{ln(t)-\mu}{\sigma}}}{\sigma\left(1+\mathrm{e}^{\frac{ln(t)-\mu}{\sigma}}\right)^2}$	$\mu = 3.72\ (\pm 0.15)$ $\sigma = 0.29\ (\pm 0.07)$	110.2
Exponential (ν)	$\nu\ \mathrm{e}^{-\nu t}$	$\nu^{-1} = 44.2\ (\pm 12.8)$ y.	116.9

Implications of Brownian model for Eruption Forecasting Assessment

As emerged from the model comparison analysis performed, five of the considered models (Weibull, BPT, Gamma, Lognormal, and Loglogistic) successfully described the Miyake-jima data set. Weibull, Gamma, Lognormal and Loglogistic models have been widely used in volcanological literature, and their implications for the interpretation of the data have been discussed in many research papers (e.g., Bain 1978; Ho 1991, 1996; Bebbington and Lai 1996a,b; Connor et al 2003; Watt et al 2007). Here we concentrate in analyze the intrinsic characteristics of BPT model, and explore the implications that this tool may have for eruption forecasting.

Repose times for recurrent eruptive activity that may be described using a Brownian passage-time distribution may be used to define a model for time-dependent, long-term eruption forecasting. This distribution has some noteworthy properties as (1) the probability of having renewed eruptive activity at time $t = 0$ is 0 (i.e. just after the last eruptive period); (2) as $t \to \infty$ the hazard function is finite. In other words, it increases steadily from zero at $t = 0$ to a finite maximum near the mean recurrence time. The first property should be analyzed carefully since it may lead to misunderstanding if used improperly. As described in the introductory part, in our analysis we ignore eruption duration since we take the onset date as the most physically meaningful; then we measure repose times from one onset date to the next. Following this approach, our modeling intends to describe the waiting times of the long-term physical processes governing the onset of renewed volcanic activity. When an eruption starts, the short-term behavior of eruptive activity might follow different patterns during the gradual decline of activity; this means that sporadic eruptive episodes in a short time window after the onset of a new eruptive episode cannot be described using the former long-term model.

We now consider the random series of events $t_1 < t_2 < \ldots < t_i \ldots$, and the repose times $\tau_i = t_{i+1} - t_i$, $(i = 1, 2, \ldots)$. If the sequence of random variables $\{\tau\}$ is independent and distributed according to a function $F(\tau)$, then the original series of events $\{t_i\}$ is called a renewal process. For a history-dependent point process, the conditional intensity function $\lambda(t|H_t)$ of the form

$$\lambda(t|H_t) = h(x) = \frac{f(x)}{1 - F(x)} = \frac{f(x)}{S(x)} \tag{4}$$

for $x = (t - t_L)$, defines the hazard function. Here, H_t is a history of occurrence times $\{t_i;\, t_i < t\}$ before time t, including the information that no event occurred either in the intervals $(t,\ t_{i+1})$ or in the interval $(t_L,\ t)$, and where t_L is the last occurrence before the considered time t (e.g., Bain 1978; Ogata 1999). Then $h(x)$ is the ratio of the probability density function $f(x)$ to the survival function S(x) and it may be defined as the event rate at time t conditional on survival until time t (or later).

The hazard function describes instantaneous failure rate, or the conditional density of failure at time x given that no event occurred until time x. An increasing hazard function at time x indicates that an event is more likely to occur in a given increment of time $(x, x + \Delta x)$ than it would be in the same increment of time in an earlier age. It is also useful in the specification of a point process since it may be directly linked with the probabilistic forecast of an event occurrence.

Fig. 7 shows the hazard function of the BPT model for Miyakejima volcano (see also Fig. 5 for the corresponding cumulative distribution functions). Hazard functions of the other candidate models are also included for comparison. For the BPT model, the failure rate is zero (0) immediately after an event, then it grows to a peak and then asymptotically tends to a finite value at long times compared to the mean time. Fig. 7 can help to understand the different behavior of the different candidate models and to compare them with the BPT model. The main characteristic of the Exponential model is the constant hazard function, implying a random occurrence of volcanic events in time. All the other models are more or less similar up to the mean recurrence time, at which point their behavior diverges. For example, the hazard function of the Weibull model starts at zero and increases to infinity, while for the Lognormal model, the asymptotic failure rate goes to zero. The Gamma model also has a finite asymptotic failure rate, but the function grows more smoothly.

(FIGURE 7 (HAZARD FUNCTION OF BPT...) SOMEWHERE AROUND HERE)

We can calculate the conditional probability $Pr_{(x,\ x+\Delta t)}$ that an eruption will happen in a time interval $(x,\ x + \Delta t]$, given an interval of $x = (t - t_L)$ years since the occurrence of the previous eruption. Let $\mathbf{T}$ be the time to the next eruption, then

$$Pr_{(x,\ x+\Delta t)} = P(x < \mathbf{T} \leq (x + \Delta t) \mid \mathbf{T} > x) \tag{5}$$

for x being the time since last eruption, as defined before. If $F(\tau)$ denotes the cumulative distribution function of the repose times τ, then $F(x) = Pr(\mathbf{T} \leq x)$, and $F(x + \Delta t) = Pr(\mathbf{T} \leq$

$(x+\Delta t))$ for $x \geq 0$, while the survival time function $S(x)$ is $S(x) = 1 - F(x) = Pr(\mathbf{T} > x)$, for $x \geq 0$. Then, the probability that an eruption occurs in the next Δt interval is (e.g., Bowers et al 1997)

$$Pr_{(x,\ x+\Delta t)} = \frac{\int_x^{x+\Delta t} f(s)ds}{1 - F(x)} \tag{6}$$

(note that for small Δt, Eq. 6 may be approximated to $\frac{F(x+\Delta t)-F(x)}{1-F(x)}$. Equation 6 can be interpreted as the conditional probability that an eruption will occur in the time interval (x, $x+\Delta t$), given an interval of x years since the occurrence of the last event. We can use this equation to calculate probabilities of eruption and forecast future events. For example, Fig. 8 is the evolution of $Pr_{(x,\ x+\Delta t)}$ as seen from the time immediately after the last eruption in 2000 for different values of Δt.

(FIGURE 8 (ERUPTION FORECASTING FOR MIYAKEJIMA...) SOMEWHERE AROUND HERE)

Discussion

Renewal processes characterized by six different probabilistic models, plus a TPM and a SPM, have been applied to analyze the repose times between eruptive episodes of Miyakejima volcano during the last 540 years (the period for which the catalog has been considered complete). From our analysis we conclude that the two-parameters models (Weibull, Gamma, Lognormal, Loglogistic, and BPT) are able to explain the observed data, and show a better fit compared to the exponential distribution. While the former four models have been widely used in volcanological literature, the BPT model seems to be a new interesting alternative to describe volcanological data. The BPT is based upon a simple physical model (the Brownian relaxation oscillator), and is parameterized by the mean rate of event occurrence, μ, and the aperiodicity about the mean, α. The Brownian passage-time family differs from other usual candidate distributions for long-term eruption forecasting in that it may be more readily interpreted in terms of the volcanic processes. The Brownian relaxation oscillator and Brownian passage-time distribution connect together physical notions of unobservable loading and failure processes of a point process with observable response statistics (i.e. event recurrence in time).

The definition of a general model to describe eruptive activity is a difficult task due to different factors such as the intrinsic complexity of eruptive processes and the difficulty of getting complete catalogs with sufficient observations. The BPT model may be considered as a first-order approximation to describe different kinds of volcanic systems, which can span from random volcanoes (Poisson-like processes), up to perfectly periodic systems. The non-homogeneous Poisson process model of Ho (1991), characterized by a Weibull distribution, was a first attempt of generalization to describe with a single model different kinds of processes. However, the Weibull is a model that possesses some intrinsically undesirable features that are difficult to support from a physical point of view in volcanological applications. For example, hazard rate functions of Weibull variates (Fig. 7) either start at zero and increase to infinity or start at a finite value and decrease to zero. This asymptotic behavior may be unrealistic in many physical systems and specifically in a volcanological application may lead to unnecessary assumptions.

Conversely, the BPT model possesses many interesting features which make it a plausible model to describe the activity of different volcanoes. If we consider its hazard function, the failure rate is zero immediately after an event. Then it grows to a peak and then declines to a finite asymptotic rate at times long compared to the mean rate. These are unique properties among the set of candidate models considered. These properties provide a more realistic asymptotic behavior of the failure rate. The BPT model may be regarded as a delayed Poisson process (Ellsworth et al 1999), for which the failure rate is zero for a finite time following an event and then steps up to an approximately constant failure rate at all succeeding times.

To measure how much the BPT model approaches a Poisson-like or a periodic process, we can consider the α parameter. The more periodic the process, the more α approaches zero. The value $\alpha = 0.51 \ \pm 0.01$ found in this work for the aperiodicity in Miyakejima volcano indicates a clear recurrent behavior in this volcanic system. To compare eruptive activity of different volcanoes with the results obtained in Miyakejima, we analyzed some catalogs from published works in other volcanic areas: for instance, we considered the data from (1) Mt Ruapehu and (2) Mt. Ngauruhoe -New Zealand- (tables 2 and 3 in Bebbington and Lai (1996b)), (3) Kilauea and (4) Mauna Loa -Hawaii- (tables 1 and 2, respectively, in Klein (1982)), and (5) Mt. Etna (Marzocchi and Zaccarelli 2006).

Fig. 9 is a plot of the estimated parameters α and μ assuming a BPT model for the volcanoes cited before. The μ (y axis) is just a scale parameter measuring the mean recurrence time, whereas the (dimensionless) α parameter (x axis) may provide a good framework to compare different volcanic systems; for instance, if we consider the results in Fig. 9 it is evident that all considered volcanoes but Miyakejima have $\alpha > 1$. This is consistent with the different results provided by the authors; for example, if we consider Mauna Loa ($\alpha = 1.28 \pm 0.4$) and Kilauea ($\alpha = 3.02 \pm 1.49$) volcanoes, α parameter indicates that those volcanoes have more Poisson-like or a clustered behavior, which is in agreement with the results of Klein (1982) who concluded that Hawaiian eruptions are largely random phenomena displaying no periodicity. The high α value for Kilauea, could indicate clustering of eruptions (i.e. non-homogeneous Poisson processes). For Ruapehu ($\alpha = 1.26 \pm 0.36$) and Ngauruhoe ($\alpha = 1.4 \pm 0.33$) volcanoes, the α parameter indicates also a Poisson-like behavior, which is also in agreement with the results of Bebbington and Lai (1996b); in those cases, the authors examined both the homogeneous Poisson and Weibull as possible models to describe the eruptive patterns of both volcanoes, concluding that both of them are more Poisson-like processes even if they are satisfactorily modeled by a Weibull renewal process.

(FIGURE 9 (ESTIMATED PARAMETERS (ALPHA AND MU)...) SOMEWHERE AROUND HERE)

Another important consideration should be done with respect to the TPM/SPM models. As discussed in previous sections, it is not possible to define a clear relationship between repose times and eruption sizes from Miyakejima volcano; additionally, we found that there is a *preferred* erupted volume. Given the recurrent behavior of Miyakejima volcano (inferred from the α value of the BPT model), we argue that it is coherent that for *preferred* repose times it is possible to have *preferred* erupted volumes. It means that BPT model may be used in volcanoes that tend to produce eruptions of similar sizes. It implies also that it is possible that a TPM or SPM model could coexist within our BPT model for recurrent volcanic activity. In fact, The BRO may be extended to models that are not renewal processes; in particular stochastic versions of TPM and SPM may be derived from randomized boundary conditions in the Brownian oscillator (Matthews et al 2002).

Concluding remarks

The intrinsic complexity of volcanic systems motivates the definition of probability models as mathematical structures to describe the response of the considered systems. Here, we put forward a new model borrowed by seismology named Brownian Passage Time (BPT). The application to the past eruptive activity of Miyakejima volcano shows that BPT fits significantly better the eruptive time occurrence than the exponential distribution. Despite the fit is comparable to the one obtained by the other two-parameters models considered, BPT has some interesting features that allow us to interpret straightforwardly and more realistically the long-term behavior of the volcano.

The Brownian passage-time family of distributions describes the response of a conceptual physical system defined as a Brownian relaxation oscillator (BRO). BRO and BPT together connect physical notions of unobservable loading and failure processes of a point process with observable event-time statistics. BPT model is characterized by two parameters: the mean repose time (μ) and the aperiodicity of the mean (α). While μ is just an scale parameter that provides information about the typical frequency at which eruptive events occur, α is a dimensionless parameter that measures the aperiodicity of the mean response of the system, and for this reason this parameter may be useful to compare different volcanoes spanning from periodic-like to Poisson-like systems.

For the Miyakejima volcano, the mean repose time is $\mu = 44.2 \ \pm 6.5$ years, while the dimensionless aperiodicity parameter is $\alpha = 0.51 \ \pm 0.01$. This value for α parameter is an evidence of recurrent eruptive activity of Miyakejima volcano, and the first such recognized example.

The BPT model provides some insights for time-dependent, long-term eruption forecasting. For instance, if we consider the hazard function, some noteworthy properties can be defined: the probability of having renewed eruptive activity just after an eruptive cycle is very low, then it increases steadily from zero to a finite maximum near the mean recurrence time. Finally, for times greater than the mean recurrence time the hazard function tends to a finite constant value, indicating that for long repose times the system tends to behave as a Poisson process.

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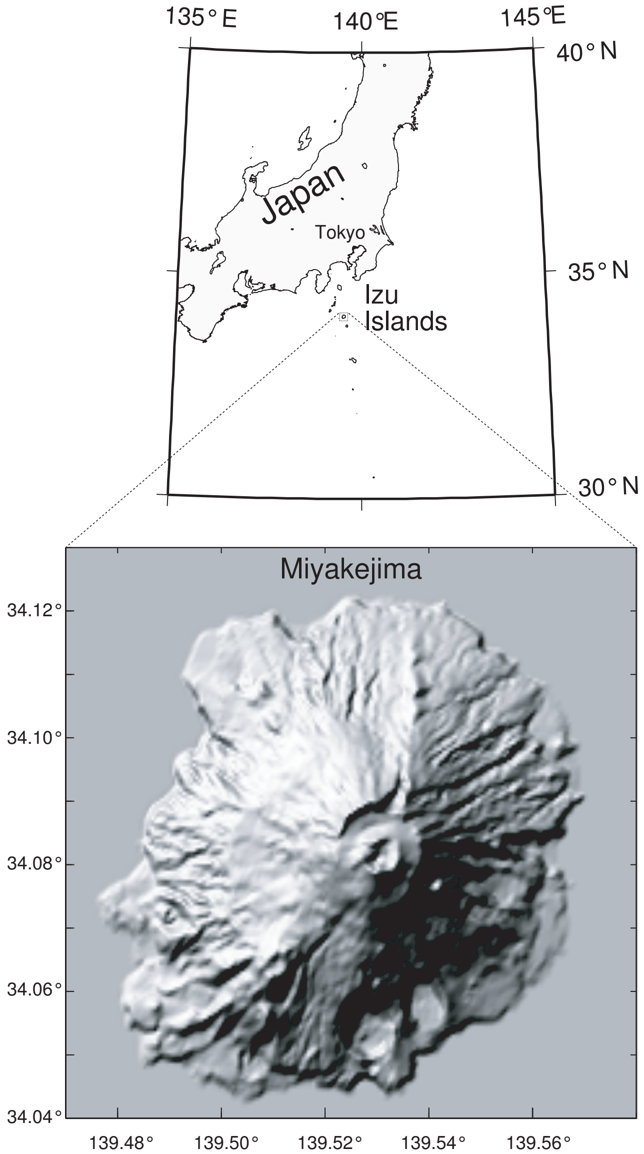

Fig. 1 Map of the Miyakejima volcano and location in the Izu island group, Japan.

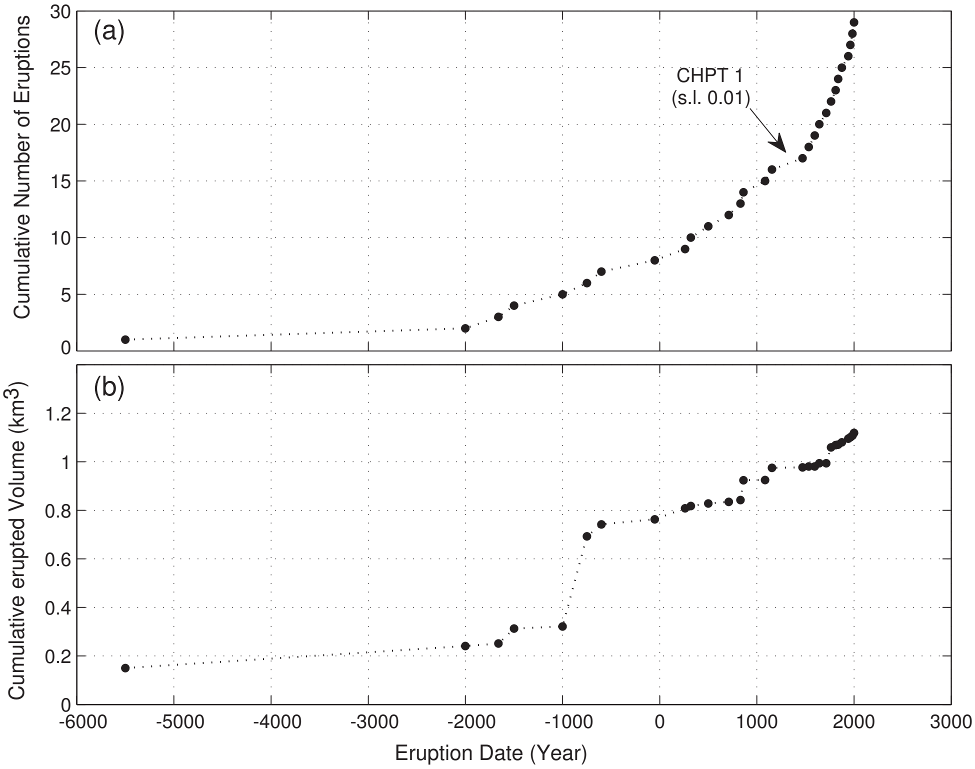

Fig. 2 (a) Cumulative number of eruptions with time, and Change Point analysis, Miyakejima volcano. (b) The cumulative volume of erupted material (DRE) is also shown for reference.

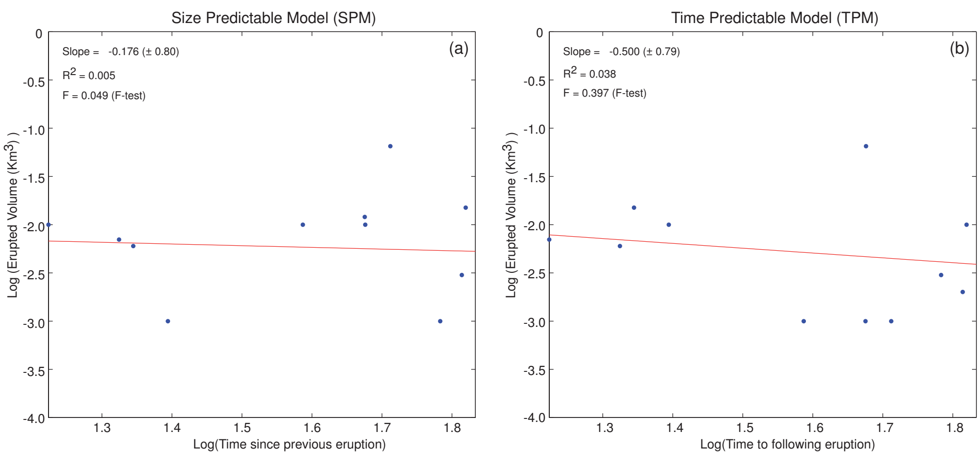

Fig. 3 Plot of times since previous eruption (a) and times to the following one (b) against the erupted volume (in km^3) of Miyakejima volcano, to test a Size Predictable Model (a) and a Time Predictable Model (b), respectively.

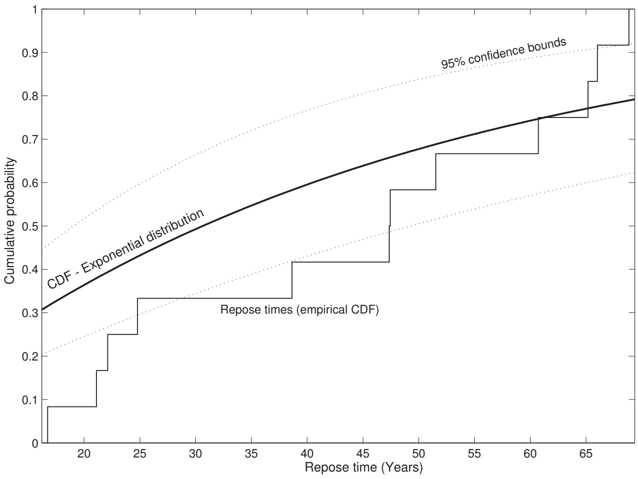

Fig. 4 Cumulative distribution function (CDF) of the best fitting Exponential distribution, and the empirical CDF of the observed repose times.

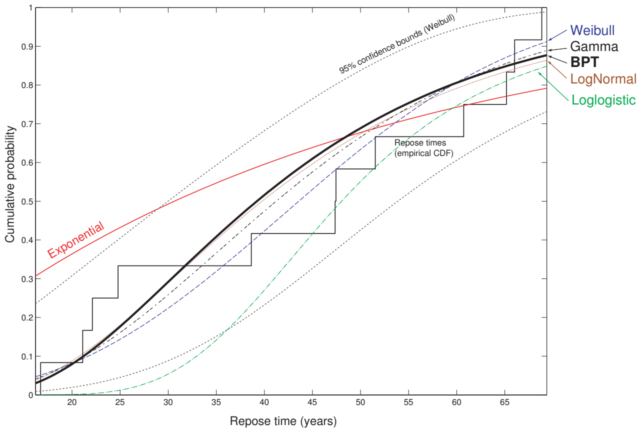

Fig. 5 Plot of the Cumulative Distribution Function (CDF) of Brownian passage-time, Weibull, Lognormal, Gamma and Loglogistic models, and the empirical CDF of the observed repose times. The CDF of the Exponential model is also included for reference.

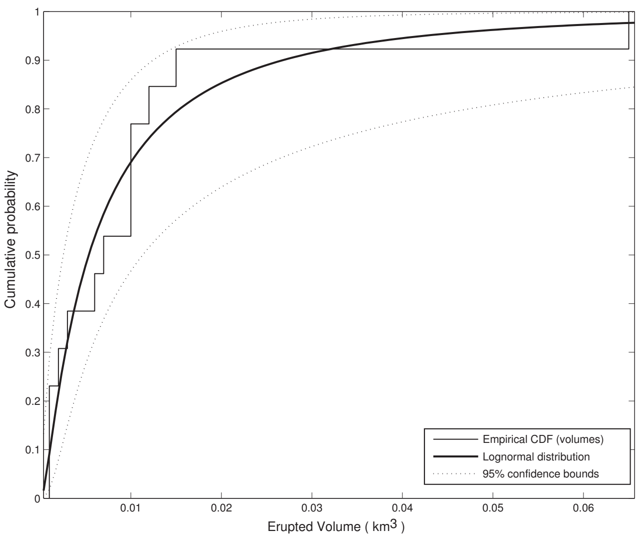

Fig. 6 CDF (empirical and Lognormal distribution) of the erupted volumes in Miyakejima volcano from 1469.

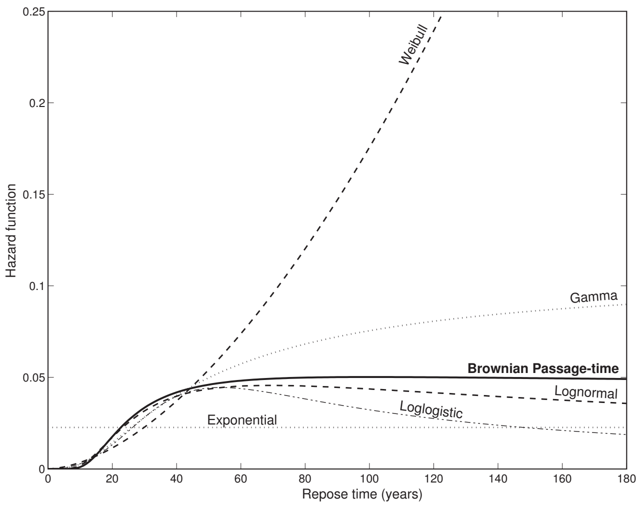

Fig. 7 Hazard function of the Brownian passage-time (BPT) model and all the other candidate distributions (Exponential, Weibull, Gamma, Lognormal and Loglogistic).

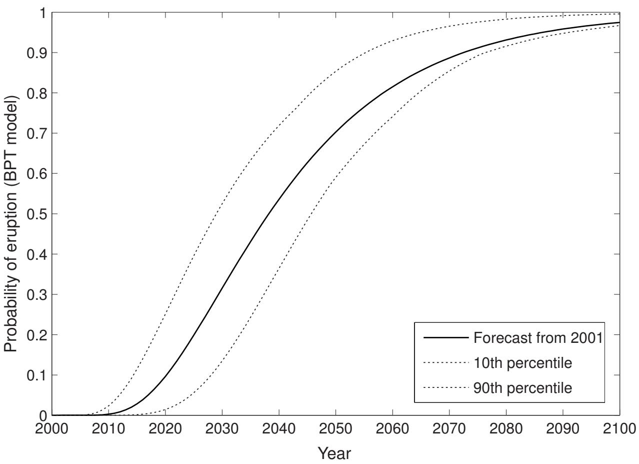

Fig. 8 Eruption Forecasting for Miyakejima volcano, using a Brownian passage-time model. Probability of eruption from 2001 (just after the last eruption) evaluated for Δt values in the interval [1, 100]

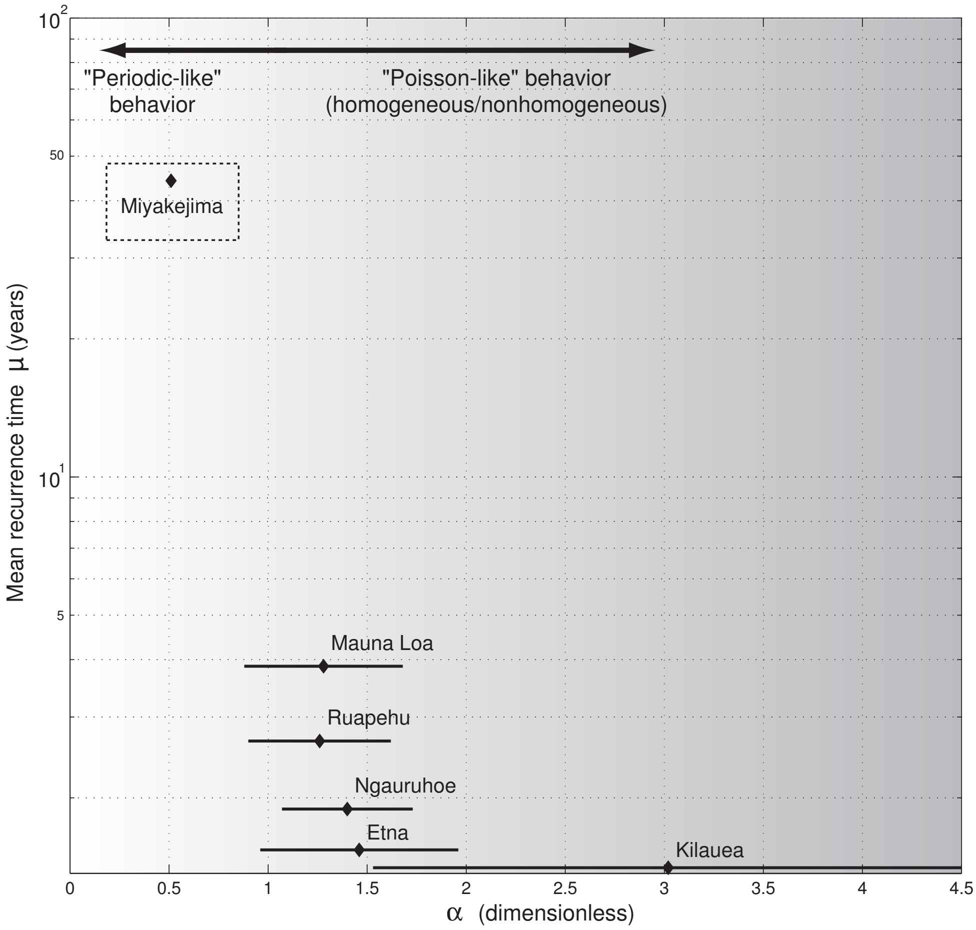

Fig. 9 Estimated parameters (α and μ) of the Brownian passage-time model for different volcanoes: Miyakejima (Japan), Etna (Italy), Ruapehu and Ngauruhoe (New Zealand), Mauna Loa and Kulauea (Hawaii). For the source of the data see the text.