Static stress drop as determined from geodetic strain rates and statistical seismicity

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Abstract

Two critical items in the energetic budget of a seismic province are the strain rate, which is measured geodetically on the Earth’s surface, and the yearly number of earthquakes exceeding a given magnitude. Our study is based on one of the most complete and recent seismic catalogues of Italian earthquakes and on the strain rate map implied by a multi-year velocity solution for permanent GPS stations. For each of 36 homogeneous seismic zones we use the appropriate Gutenberg Richter relation, which is based on the seismicity catalogue, to estimate a seismic strain rate, which is the strain rate associated with the mechanical work due to a co-seismic displacement. We show that, for each seismic zone, the volume storing most of the elastic energy associated with the long-term deformation, and hence the seismic strain rate, is inversely proportional to the static stress drop. The GPS-derived strain rate for each seismic zone limits the corresponding seismic strain rate, and an upper bound for the average stress drop is estimated. We show that the implied
regional static stress drop varies from 0.1 MPa to 5.7 MPa for catalogue earthquakes in the moment magnitude range [4.5 - 7.3]. The stress drop results are independent of the regional a and b parameters, and heat flow, but are very sensitive to the assumed maximum magnitude of a seismic province. The data do not rule out the hypothesis that the stress drop positively correlates with the time elapsed after the largest earthquake recorded in each seismic zone.

**Introduction**

The static stress drop represents the overall reduction in the average shear stress in or around an earthquake fault zone. The static stress drop $\Delta \sigma$ was introduced as a measure of the final fault slip $u$ as a fraction of the fault dimension $R$ ($\Delta \sigma \propto u/R$) and was estimated from geological observations (see, e.g., Kanamori and Anderson, 1975). Here, we propose the use of long-term geodetic measurements to compute the upper limit of the regional stress drop that can occur in a seismic area.

Following Brune (1970) and Madariaga (1976), who represented the radiated spectrum of seismic waves in terms of the source parameters, the stress drop is often estimated by measuring the corner frequency or source duration from seismic signals (see, e.g., Abercrombie, 1995). However, a finite rupture cannot be uniquely represented with an equivalent point-source, and therefore, the relation between high-frequency ground motion and static source characteristics is ambiguous (Beresnev and Atkinson, 1997), especially for large ruptures. As a consequence, most published stress drop values exhibit considerable scatter, even for the same earthquakes.

More recently, Kim and Dreger (2008) obtained a slip distribution by inverting the local strong motion waveform combined with GPS and InSAR data. They deduced the static
stress change using the method of Ripperger and Mai (2004). The static stress change on
the fault plane is linked to the dynamics of the earthquake rupture and hence also to the
associated energy release and seismic radiation. Knowledge of the static stress change is
therefore required for dynamic rupture modeling of past (and future) earthquakes (Peyrat
et al., 2001).

In this paper, we use the long-term geodetic deformation and relate it to the statistical
seismicity to infer the upper limit of the regional stress drop expected in different seismic
zones of Italy.

Geodesy yields the present-day (in the sense of an average over the past few years) rate
at which elastic energy is stored at or near the surface. Statistical seismology has been
used to characterize how elastic energy is released in the form of earthquakes over the
last few centuries. We investigate how a cycle of energy loading compares with a cycle of
energy release and take into account several sources of uncertainty. The geodetic
information is limited by the reliability of the strain rate estimates, which are often based
on only a few stations and velocities that are known with various degrees of accuracy. The
seismological data are limited by the completeness of the catalogues and their temporal
extension relative to a seismic cycle. Given the complexity and spatial variability of the
stress regimes in Italy, such a comparative study must be regionalized.

**Geodesy and seismic hazard in Italy**

Over one hundred permanent GPS stations have been in operation for the past several
years in Italy and neighboring areas, and horizontal velocities are known with formal
uncertainties well below 1 mm yr\(^{-1}\) (Anzidei et al., 2001; Battaglia et al., 2004; Caporali et
al., 2003, 2009; D’Agostino et al., 2005, 2008; Grenerczy et al., 2005; Serpelloni et al.,
Velocity profiles across areas of active deformation show detailed horizontal gradients that positively correlate with the deformation style implied by both field data and seismology. In areas with a dense population of GPS stations of known velocity, the horizontal strain rate tensor has principal directions that, in most, if not all, cases, successfully correlate with the horizontal projection of the PTN (Pressure, Tensional and Null) axes inferred from fault plane solutions (Caporali et al., 2003, D’Agostino et al., 2008, Caporali et al., 2009).

Although the surface kinematics derived from GPS data have been exploited in considerable detail, much less knowledge exists on how the geodetic data can be combined with data from statistical seismology. Early concepts pioneered by Westaway (1992) and Ward (1999) were applied by Caporali et al. (2003) to five seismically and tectonically homogeneous Italian macrozones. They concluded that, on average, 70 to 100% of the geodetic strain rate was released seismically. The correlation between the tectonic moment rate (mainly based on space geodetic data) and the number of events above a cut-off magnitude has been firstly analyzed within the Global Stress Rate Model by Kreemer et al., 2003 and subsequently applied in Europe in the Aegean-anatolian area (Kreemer et al., 2004).

Jenny et al. (2004) investigated the eastern Mediterranean area and determined seismic hazard parameters from historical seismicity, tectonics and geodetic data. Slejko et al. (2009) obtained 30-year earthquake probabilities by combining geodetic data with the seismogenic areas of known depth, strike, dip, rake, geologic slip rate and expected maximum magnitude derived from the Database of Italian Seismogenic Sources (DISS) (Basili et al., 2008). D’Agostino et al. (2009) concentrated on a dense GPS network in the
extensional and seismically active area in central Italy and concluded that there is a deficit in seismic activity relative to the geodetically measured deformation rate.

The working group for the seismic hazard map of Italy (Gruppo di lavoro MPS, 2004) and the S1 project sponsored for the period of 2004-2006 by the Department of Civil Protection of Italy and the Istituto Nazionale di Geofisica e Vulcanologia (INGV) (Meletti, 2007) reported 36 seismic zones (ZS9) that are homogeneous in deformation regime and depth (Meletti et al., 2008). The CPTI04 catalogue (Gruppo di lavoro CPTI, 2004) explicitly references the ZS9 and associates each event with a specific seismic zone. Details on CPTI04 and the ZS9 are available at [http://emidius.mi.ingv.it/CPTI04/](http://emidius.mi.ingv.it/CPTI04/).

**Velocity data**

The largest GPS time span covers an interval of 11 years (1998–2008), but most of the data come from the recent RING network (http://ring.gm.ingv.it) that was developed in Italy over the last five years by INGV and several other stations operated by other institutions. The GPS data processing follows the procedure proposed in Devoti et al. (2008).

The RINEX data was processed with Bernese software (Hugentobler et al., 2004) based on double difference phase observables and using IGS/EUREF guidelines on absolute antenna phase center modeling, orbits, earth rotation parameters and tides. The network is divided into clusters that are processed in daily batches. A priori uncertainties of 10 m were imposed to obtain the so-called loosely constrained solution. The daily loosely constrained cluster solutions were then stacked into a combined daily loosely constrained solution of the entire network by applying the classical least squares approach (Devoti et al., 2008). The daily combined network solutions were then rigidly transformed into the ITRF2005.
frame (Altamimi et al., 2007) to estimate translations and scale parameters and to apply minimum constraints to a set of conventionally defined stations.

The velocity field was estimated from the ITRF2005 time series of the daily coordinates. Annual signals and offsets at epochs of instrumental changes were removed, as in Devoti et al. (2008) and Riguzzi et al. (2009). Because the time series of coordinates are affected by white noise at high frequencies (up to about 2 cycles/yr) and colored noise (typically flicker phase noise) at lower frequencies, we could not rely on the formal least squares standard deviation to estimate the uncertainty in the velocities. This information is crucial for inferring the uncertainties in the strain rates. The most convenient way to reliably define the uncertainty in the velocity from the time series is to use the Allan variance (Allan, 1966), which is the $1\sigma$ probability that two consecutive, equal length, non-overlapping batches of time series data have the same slope. The Allan variance is taken as a function of the length of the batch. This approach uses the full spectrum of the noise in the time series and, as shown by Caporali (2003), yields estimates of the velocity variance that coincide with the so-called maximum likelihood estimates (Mao et al., 1999) when they exist (white noise + flicker phase, typically) and provide consistent variance estimates for other combinations of white and colored noise. We re-scaled the formal errors of the GPS rates using the mean scale factors estimated for each velocity component, as in Devoti et al. (2008) and according to the approach developed in Williams (2003). The GPS site positions and velocities given with respect to the Eurasian fixed reference frame defined in Devoti et al. (2008) and their re-scaled uncertainties are listed in Table AUX (auxiliary material) and reported in Figure 1.

Individual velocities cannot be interpreted as such, due to the noise and different uncertainties. The minimum variance algorithm described below was used to obtain the
average strain rates for each area by using a weighted summation over the local velocity data.

**The strain rate field**

The algorithm adopted here to estimate the strain rate is described in Caporali et al. (2003). It is based on weighted least squares collocation and requires an isotropic covariance function that is dependent on the distance between stations. Scattered velocities (east and north components) and their variances are interpolated at a point \( P \) by a weighted sum over all of the available data. The weight depends both on the uncertainty of each contributing velocity through the weight matrix and on its distance from the point \( P \) of computation through the covariance function.

The covariance function depends on one adjustable parameter: the scale distance \( d_0 \), which describes the typical correlation length. We used one value of the scale length for the entire data set. This approach has the advantage of providing a picture of the deformation pattern at the same wavelength everywhere. Taking into account the average distance between the GPS stations on the one hand and, on the other hand, the typical size of the ZS9s, we found that \( d_0 = 50 \text{ km} \) is a sound compromise. Table 1 contains an account of the number of stations that fall within each seismic zone and the number of stations that are located in a 50-km-radius circle centered on each ZS9. A minimum of three unaligned stations is required to estimate the strain rate components. As shown in Table 1, several of the seismic zones have insufficient coverage. In these cases, the strain rate was interpolated from the velocities of stations that were farther than 50 km from the ZS9. As described below, this approach resulted in small values for the strain rates.
because the least squares collocation acts as a low pass filter and tends to smooth the lateral changes in the velocity.

For each measured velocity, a weight factor was introduced based on the variance of the sample relative to the variance of the population. The covariance function and the weight matrix allowed us to map a specified point $P$ (in our case, $P$ is the center of each ZS9 seismic zone) and the variances of the two horizontal components of the velocity. These variances are required to estimate the uncertainties of the strain rate. The least squares collocation algorithm is a minimum variance algorithm and also allows the contributing velocities to be properly weighted according to their estimated uncertainty.

The strain rate eigenvectors that were interpolated at the center of each ZS9 are shown in Figure 2a. Figure 2b identifies the numerical labels for each of the 36 seismic zones. The extensional deformation style of the Inner Apennines is described very well by the GPS data. These data also show the compression in Friuli (NE Italy) and the transpression in east Sicily. For comparison, we used the MEDNET catalogue of fault plane solutions since 1976 (Pondrelli et al., 2006). As Figure 2a suggests, good qualitative agreement exists between the seismic and geodetic principal directions.

Relating present-day strain rate and historical seismicity

The earthquake catalogue CPTI04 consists of 2550 events in the momentum magnitude range $[3.9 – 7.4]$ in the time interval $[217 \text{ BC} – 2002 \text{ AC}]$. The interval of completeness for magnitude 4.5 and higher starts however from 1600 AC. The homogeneous ZS9 seismic zones represent the updated seismic zonation of the Italian territory. These seismic zones embody the most recent data on seismogenic sources available from the DISS 2.0 (Valensise and Pantosti, 2001) and reference the CPTI04 catalogue (Figure 3). The ZS9s
are characterized by the following properties: the geographical location of each zone, the
typical depth of the events, the maximum magnitude and the b value (Table 1). By ‘typical
depth’, we mean the depth at which the largest number of events takes place. By
‘maximum magnitude’, we mean the maximum expected magnitude. Because the
catalogue was complete over a period of time that is certainly smaller than a seismic cycle,
the maximum observed magnitude may underestimate the maximum expected magnitude
(see, e.g., Pisarenko et al., 1996). Any countermeasures for this lack of information
require some arbitrariness. We increased the maximum observed magnitude given by
Meletti (2007) and Meletti et al. (2008) by 0.3, and these increased values are reported in
Table 1 as the ‘maximum expected magnitude’. Assuming a b value b_s from the definition
of ZS9 zonation (Table 1), we estimated the a value a_s by fitting a straight line
\[ a_s = \alpha + \log(-b_s \cdot \ln(10)) \] to the catalogue events in the magnitude interval [4.5-5.5]. The
magnitude dependence of the logarithm of the number of events that exceed a given
magnitude is linear in most, if not all, cases (Figure 4). The intercept \( \alpha \) was estimated as a
free parameter. For each seismic zone, the geodetic shear strain rate is introduced
according to the following equation from Savage and Simpson (1997):

\[ \dot{\varepsilon}_g = \max(\dot{\varepsilon}_1, \dot{\varepsilon}_2, |\dot{\varepsilon}_1 + \dot{\varepsilon}_2|) \]  

The numerical estimates of the geodetic shear strain rate and the related uncertainty are
given in Table 1. Figure 5 shows that the computed strain rate (Eq. 1) can assume
relatively large or small values when three or more stations fall in a circle of radius \( d_0 = 50 \)
km, i.e., the scale distance adopted for collocation. When no station is present within the
specified distance, the resulting strain rate goes to zero. Figure 5 suggests that the
number of stations, their geometry and the uncertainty in their velocities are sufficient to
provide a reliable estimate of the geodetic strain rate at the selected wavelength \( d_0 \).
To relate the catalogue data to the strain rate released seismically, we used a modified form of the Kostrov equation (Kostrov and Das, 1988):

\[ \dot{\varepsilon}_y = \frac{1}{2\mu}\sum M_0 \Delta t \]  \hspace{1cm} (2)

where \(\mu\) is the shear modulus, \(\Delta t\) is the time span of the seismic event catalogue, \(M_0\) is the seismic moment of each event, and \(V\) is the corresponding seismogenic volume. The summation is extended to the catalogue events that span the time \(\Delta t\) within each seismic zone. This formulation of the Kostrov equation has the disadvantage of being strongly dependent on the completeness of the catalogue over the time \(\Delta t\). Moreover, no formula exists to compute the seismogenic volume \(V\). The practice of defining \(V\) as the product of the area including the catalogue seismic events and the mean hypocentral depth implies that each seismic event, regardless of the seismic moment \(M_0\) that is released, is assigned the same seismogenic volume: (total area)*(mean hypocentral depth)/\(n\), where \(n\) is the number of seismic events in that area over a time interval \(\Delta t\). However, the total area is likely to be non-uniformly populated by events, and the events of larger magnitude (and hence of larger seismogenic volume) are given the same weight as smaller events, although the large magnitude events occur much less often in the selected time span.

A more convenient formulation of Eq. 2 requires using the Gutenberg Richter (GR) relation to approximate the seismic volume as the product of the slip area \(A\) times a thickness \(T\). The volume \(A*T\) then contains most of the deformation released by the seismic moment \(M_0\). Because \(M_0=\mu Au\), where \(u\) is the co-seismic displacement, we have:

\[ \dot{\varepsilon}_y = \frac{1}{\mu} \int_{m_{\text{min}}}^{m_{\text{max}}} d(m)T(m)da \]  \hspace{1cm} (3)
where $N$ is the number of events per year in the magnitude range $[m, m+dm]$. The total magnitude range is defined by $[m_{min}, m_{max}]$, where $m_{min}=4.5$ is the magnitude above which $\log[N(m)]$ is approximately linear in $m$ (Fig. 4), and $m_{max}$ is the maximum magnitude of the specific seismic zone (see Fig. 3 and Table 1). We assume the following:

\[
\log[N(m)] = a_s + b_s m; \quad \log[u(m)] = a_{AD} + b_{AD} m; \quad \log[A(m)] = a_{RA} + b_{RA} m
\]

(4)

where $a_s$ and $b_s$ are the $a$ and $b$ parameters of the GR relation (Table 1). The empirical relations between the average displacement (AD) $u$ (in meters) and the moment magnitude and between the rupture area (RA) $A$ (km$^2$) and the magnitude are as given in Wells and Coppersmith (1994). We use the coefficients for "all" slip-types as follows:

\[
a_{AD} = -4.60 \pm 0.57; \quad b_{AD} = 0.69 \pm 0.08; \quad a_{RA} = -3.49 \pm 0.16; \quad b_{RA} = 0.91 \pm 0.03
\]

(5)

The work done to displace a slip area $A(m)$ by an amount $u(m)$ must equal the change in the potential energy within the seismogenic volume, which is the product of the stress drop $\Delta\sigma$ by the seismogenic volume $A(m)T(m)$. Therefore, we can estimate the thickness $T(m)$ (for reverse and normal faulting) or width (for strike slips) of the seismogenic volume as follows:

\[
\frac{\Delta\sigma A(m)T(m)}{\text{change in potential energy}} = \frac{\mu A(m)u(m)}{\text{mechanical work}} = \frac{\mu T(m)}{\Delta\sigma u(m)}
\]

(6)

Taking $\Delta\sigma=4$ MPa and $\mu=30$ GPa, it follows from Eq. 6 that, for example, $T=0.3$ km for $m=5$ and $T=8$ km for $m=7$.

With this model, the Kostrov strain rate (3) becomes:

\[
\dot{\varepsilon}_p = \frac{\Delta\sigma_a \log[A(m)T(m)] - \Delta\sigma_a \log[A_{max}T_{max}]}{2\mu (\Delta\sigma_a + \Delta\sigma_b)} b_{AD} \frac{a_{AD}}{b_{AD} - b_{RA}}
\]

(7)
where $a_{WC} = a_{RA} + a_{AD}$ and $b_{WC} = b_{RA} + b_{AD}$ ('WC' stands for 'Wells and Coppersmith').

In general, the geodetic strain rate (1) is expected to be greater than or equal to the seismic strain rate (3). Thus, the stress drop must satisfy the following inequality:

$$\Delta \sigma = \frac{2 \mu \varepsilon_{g}}{10^{\frac{a_{RA}}{10^{m_{max}}}} - 10^{\frac{a_{RA}}{10^{m_{max}}}} b_{RA}}$$

(8)

where $\Delta \sigma_{g}$ is the average stress drop that we expect in the case of the exact balance of the geodetic and seismic strain rates across the catalogue time span and the magnitude range, which, in our case, is [4.5 - 7.3] for each seismic zone.

In the right hand side of eq. 8 it should be noted that the ratio between the strain rate and the yearly number of events of zero magnitude $N(0) = 10^{-\infty}$, according to Table 1, varies over a wide range of magnitudes. This is balanced by the variability of $b_{s}$ and $m_{max}$ within each seismic zone, resulting in a slowly, laterally variable quantity.

The uncertainty in $\Delta \sigma_{g}$ can be obtained by linearly propagating the uncertainties in the parameters $m_{max}$, $\varepsilon_{g}$, $a_{s}$, $b_{s}$, $a_{RA}$, $b_{RA}$, $a_{AD}$, and $b_{AD}$. For $m_{max}$, we assumed a 3% uncertainty, which was derived propagating a 5% maximum error on the magnitude sample onto the maximum. For $\varepsilon_{g}$ and $a_{s}$, we used the values in Table 1, and for $b_{s}$ we assumed a 10% uncertainty, a conservative estimate. For the last four parameters, we used Eq. 5. We assumed that $m_{min}$=4.5. The resulting values are listed in Table 1.

**Discussion**

Figure 6 shows a plot of the estimates of the static stress drop from Eq. 8. Most of the seismic zones have a stress drop below 2 MPa. Higher values were reached in Zone 906-907, a left lateral shear zone in NE Italy that accommodates to the west the northwards...
indentation of the Adria plate into Eurasia. It is worth noting that in our computation of the maximum regional stress drop for zones 905 and 906, both with a maximum recorded magnitude of 6.6, the maximum regional stress drop in zone 905 appears smaller than in zone 906. This suggests that the assumed maximum magnitude does not necessarily bias the regional stress drop. Other zones of high stress drop are Zones 913, 923, 924, 925, 929 and 935. These zones are in central and southern Italy and are known to be seismically active. The epicenter of the Mw=6.3 Aquila earthquake of April 6, 2009 (hence, not included in the catalogue) is in Zone 923. Zone 924 includes the Mattinata fault, an E-W–trending feature that is ~50 km long. This fault cuts across the Gargano Promontory in the foreland of the southern Apennines (Argnani et al., 2009) and is thought to have high seismic potential (two Mw=5.7 events occurred in 2002 in rapid succession in that area). Zone 935 in Sicily includes the Iblean Malta escarpment. According to Table 1, the largest earthquakes in Italian history have occurred in Seismic Zones 929 and 935. Figure 7 shows that the static stress drop is uncorrelated with the $b_s$ parameter. A negative correlation between the stress drop and the $a_s$ parameter, being controlled by only one isolated point with a high stress drop, is unlikely.

The values obtained for the lateral variation of the static stress drop are in the range of the estimates of the dynamic stress drop determined from spectral analysis of the seismic source (Allmann and Shearer, 2009). Figure 8 shows the static stress drop and its correlation with the seismic zones. Specific examples of seismic sequences in NE Italy and western Slovenia (Bressan et al., 2006) exist; our value of $0.31 \pm 3.47$ MPa for ZS905 compares well with the 0.05-2.30 MPa range of the 1996 Claut swarm, the 0.5-7 MPa range of the 1998 Kobarid sequence and the 1.4-7.8 MPa range of the 2002 M.te Sernio sequence. In the Auxiliary material we provide additional plots similar to Figure 8, but with
the contours controlled only by the significantly non zero (i.e. non zero within one standard deviation) values of the stress drop, under the same assumptions on the maximum magnitude as in Figure 8 (i.e. $m_{\text{max}} + 0.0$, $m_{\text{max}} + 0.3$, $m_{\text{max}} + 0.5$ respectively).

Two Mw=5.7 events that occurred further south in 2002 in Molise (ZS924) were analyzed by Calderoni et al. (2009). Their estimated range of the dynamic stress drop (0.6 – 2.5 MPa) is in good agreement with our estimate of $3.20 \pm 1.18$ MPa given in Table 1.

Cocco and Rovelli (1989) reported stress drops on the order of several tens of MPa, and they argued that thrust faults (e.g., in Friuli) have stress drops about three times larger than normal faults. However, these large values may indicate the maximum stress drop. Therefore, the inconsistency with the average values given in Table 1 is only apparent.

In the classical treatment of the slider block problem (e.g., Turcotte and Schubert, 2002), the stress drop is considered to be proportional to the lithostatic load, and the proportionality constant is taken as the difference between the static ($f_s$) and the dynamic ($f_d$) friction coefficients as follows:

$$\Delta \sigma = 2(f_s - f_d)\rho gh$$  (9)

where $\rho$ is the crustal density, $g$ is the acceleration due to gravity, and $h$ is the thickness of the slider. The lowest values of the static stress drop are independent of the typical depth $h$ (Fig. 7), and higher values require larger depths (i.e., larger lithostatic loads). The fact that our upper limits for the stress drop are relatively small, at most, a few MPa, implies that the dynamic friction coefficient is very close to the static friction coefficient. This situation in turn suggests that very little or no shear stress is left on the faults, on average, after an event. In Fig. 9, the map of the static stress drop is superimposed on the contours of the heat flow (Della Vedova et al., 2001). In principle, the higher the heat
flow, the lower the friction. This expectation was verified in Seismic Zones 923 (with a slight offset) and 924, but not in several other areas. This result leads us to conclude that a correlation between the static stress drop and the heat flow is either non-existent or very small in this plot. We cannot however exclude that better data for the Italian area, or a similar analysis for other areas, make the correlation between heat flow and regional stress drop more convincing.

Had we computed the seismic volume in the Kostrov formula (Eq. 6) as the product of the area enclosing the epicenters times some average hypocentral depth, the resulting volume would have been very large. This volume would also be dependent on the selection of contributing events. This subjective element was emphasized by Caporali et al. (2003). D’Agostino et al. (2009) showed that different assumptions for the seismic depth and/or the tension factor used in the spline interpolation of the velocities (the tension factor plays a role similar to that of our scaling distance \(d_0\) in the collocation algorithm) could lead to contradictory results for the energy balance of a seismic province. Our approach defines the seismic volume as an integral across the magnitude spectrum and recognizes that each magnitude involves a different deforming volume. The deformation scales with the rupture area \(A\) and a characteristic thickness (or width, for strike slip faults) \(T\) perpendicular to the rupture area. Therefore, the volume is:

\[
V = \int_{m_{min}}^{m_{max}} A(m) T(m) \, dm
\]

The value of \(V\) depends on the maximum magnitude assigned to the seismic province, but defining the maximum magnitude is less of a subjective choice than the boundary definition of the seismic province. However, our results indicate that two provinces with a
population of earthquakes of the same maximum magnitude can have different seismogenic volumes $V$, depending on the lateral variations of the stress drop.

The static stress drop is usually introduced in terms of the slip $u(m)$, a scale distance $T$ and a geometric factor $C$ that depends on the form of the fault (e.g., Scholz 2002, ch.4) as follows:

$$\Delta u = C \mu \frac{u}{T}$$  \hspace{1cm} (11)

The ratio $\mu/T$ may be considered as a stiffness relating the stress drop to the slip $u$. $T$ represents the scaling distance for the fall-off of the deformation moving away from the slip area. As a consequence, $T$ is proportional to $u$, and the scaling factor can be estimated from the data. The form factor $C$ can be included in the definition of the stress drop.

Fletcher and McGarr (2006) have mapped the static stress drop and stiffness over the slip surface of the Northridge ($m=6.7$), Landers ($m=7.3$) and Kobe ($m=6.9$) earthquakes, using a technique (McGarr and Fletcher, 2002) which requires the knowledge of the apparent stress and of the ratio between far field and near field energy. They report average values of the static stress drop of 17, 11, and 4 MPa respectively, with large variability across patches of the faults. These values are systematically larger than those obtained by simple crack models, essentially because slip –and hence the distribution of rupture of asperities- are laterally varying on the slip area. Our approach extends the concept of stress drop to a seismic zone, rather than an individual fault or slip area, and accounts for the variability of the amount of slip in each seismic zone by considering the lateral changes in the maximum expected magnitude.
Figure 10 (top) shows the stress drop increasing with the time elapsed since the occurrence of the earthquake of highest magnitude in each seismic zone according to the CPTI04 catalogue. Although the statistical significance of this correlation plot is made uncertain by the large scatter and error bars, the slope of 2 kPa yr$^{-1}$, when divided by a Young modulus of 70 GPa, yields a strain rate of $3 \times 10^{-9}$ yr$^{-1}$, which is the average strain rate in Italy as determined from GPS geodetic measurements. If this correlation could be made more precise, there could be interesting implications for a better understanding of recurrence times and seismic hazards. A more precise correlation will result both from better data and, possibly, from a more homogeneous definition of certain seismic zones (e.g., Abruzzo and Irpinia, see Table 1) which include a large number of active faults. Alternatively, the correlation which is implied in this figure could well be an artifact resulting from a misinterpretation of old, large events in the Catalogue.

Figure 10 (center) shows that the geodetically measured strain rate has a similar correlation with the time elapsed since the largest earthquake as the static stress drop. Figure 10 (bottom) finally shows that the mean time between earthquakes of zero magnitude (that is $1/N(0)$, where $N(m)$ is the Gutenberg Richter law for each seismic zone) has no appreciable correlation with the time since the last largest event.

The combination of geodetic and historical seismicity data provides constraints on the static stress drop. Deconvolution of the source parameters from the spectra of measured seismic waves provides estimates of the dynamic stress drop. Although the two stress drops do not necessarily coincide, they are expected to be of the same order of magnitude. A detailed analysis of the waveform spectra of earthquakes located in each of the 36 seismic zones, wherever feasible, can, to some extent, validate our predictions.
Conclusions

A static stress drop which is defined for a seismic province of known geodetic strain rate, which covers a time interval of some centuries and which covers events distributed over a range of magnitudes can prove a viable mean of integrating geodetic and historical seismic data. The Italian area, with a detailed map of geodetic strain rates and long term record of historical seismicity, is an ideal testing ground, but other areas such as Southern California or the Anatolian Fault in Turkey, for example, can serve as well. We provide an expression (eq. 8) where the geodetic strain rate, the a and b parameters of the regional Gutenberg Richter law and the value of $m_{\text{max}}$ (maximum magnitude) are area dependent parameters which constrain the maximum value of the stress drop released by the seismic zone. The assumption that the geodetically measured strain rate is greater than or equal to the strain rate released seismically (in the magnitude range of 4.5 to 7.3) leads to upper limits of a static stress drop for each seismic zone. Lateral variations of the stress drop results are in the range of 0.1 to 5.7 MPa in 36 seismic zones in Italy. Our data give some indication of a correlation between the stress drop and the time elapsed since the last largest earthquake in each seismic zone. This hypothesis should be investigated further in Italy and other seismic areas and, if confirmed, could have far reaching implications in several aspects of earthquake mechanics and seismic hazards.

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References


Table and Figure Captions

Table 1: The first nine columns list the properties of the ZS9 as defined by Meletti et al. (2008). The symbol $m_{\text{max}}$ indicates the maximum observed magnitude + 0.3. The geodetic shear strain rate and its uncertainty were computed from the eigenvalues of the geodetic strain rate tensor according to Savage and Simpson (1997). The static stress drop is defined in Eq. 12. We indicate the area of each seismic zone, the number of GPS stations falling within each seismic zone and the number of GPS stations within $d_0=50$ km of the geographic center of each seismic zone.

Table AUX (Auxiliary material): The velocities of the INGV multi-year solution relative to the rigidly rotating Eurasian plate. Error ellipses are 1σ.

Figure 1: The velocities of the INGV multi-year solution relative to the rigidly rotating Eurasian plate (see Table AUX for the numerical values). Error ellipses are 1σ.

Figure 2: (a) Strain rates interpolated at the center of each of the 36 ZS9 seismic zones (polygons in yellow are labeled 901 to 936 in the index map). For comparison, we plot the CMT’s for the fault plane solutions of events of $m>5.5$ since 1976. Yellow cones represent 1σ uncertainty in azimuth and absolute value for the largest eigenvector of the strain rate tensor. (b) Identification of the 36 seismic zones (yellow polygons).

Figure 3: The seismic events in the CPTI04 catalogue (open circles) and their geographical relation to the geometry of the ZS9 (yellow polygons).

Figure 4: Plots of the logarithm of the yearly number of events exceeding a given magnitude for each ZS9 according to the CPTI04 catalogue.

Figure 5: The geodetic strain rate at the center of each ZS9 seismic zone vs. the number of stations within a distance $d_0=50$ km from the geographical center of each zone. The low correlation is an indication of the stability of the algorithm. When no station is within the assigned distance, the algorithm tends to generate strain rates close to zero.

Figure 6: Estimates of the static stress drop defined by Eq. 8 as a function of the seismic zone. The color codes refer to the dominant deformation regime (Meletti et al., 2008). The assumed maximum magnitude is, for each seismic zone, the maximum recorded magnitude + 0.3.

Figure 7: Graphical representation of data in Table 1. The static stress drop shows a weak negative correlation with the a parameter (sub plot a) and is uncorrelated with the b parameters of each seismic zone (sub plot b). The stress drop is independent of depth at low values (sub plot c). Higher values appear at larger depths or lithostatic loads. The stress drop has a positive correlation with the maximum magnitude with a large scatter.
The resulting correlation law is $m_{\text{max}} = 0.2 \Delta \sigma + 6.4$, $\Delta \sigma$ being the maximum stress drop in MPa.

Figure 8: Geographical interpolation of the lateral variations of the static stress drop under three assumptions: (top) the quantity $m_{\text{max}}$ in eq. 8 is the maximum magnitude recorded in each zone in the CPTI04 catalogue; (center) the maximum value in the catalogue is increased by 0.3 in all zones; (bottom) the maximum value in the catalogue is increased by 0.5 in all zones.

Figure 9: Static stress drop and heat flow (contours in mW/m$^2$).

Figure 10: The stress drop (top), geodetic shear strain rate (center) and recurrence time of zero magnitude events (bottom) vs. time elapsed since the largest earthquake in each seismic zone. The dashed line in the top plot represents the regression line. The error bars of the strain rate in the center plot tend to be smaller than the size of the plot symbol.