Probabilistic volcanic hazard assessment and eruption forecasting: the Bayesian Event Tree approach

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Purpose

The purpose of this report is to discuss in detail the importance and prerogatives of quantitative volcanic hazard assessment and to describe the main features of a Bayesian model designed to achieve this goal. Ideas, models and results come out from the work made in the framework of the INGV-DPC V4 project, and partially from the application of the technique to Campi Flegrei (V3_2) and Vesuvius (V3_4). Here, we examine in depth the practical and philosophical implications of the approach, and report only a brief summary of the technical details that can be found on the cited literature.

General features of Probabilistic Volcanic Hazard Assessment (PVHA)

One of the major goals of modern volcanology is to set up a sound risk-based decision making in land use planning and emergency management.
Despite different scientific disciplines attribute disparate definition to the term "risk", in volcanology the most used definition reads (e.g., UNESCO, 1972; Fournier d'Albe, 1979)

risk = hazard x value x vulnerability

where *hazard* is the probability of any particular area being affected by a destructive volcanic event within a given period of time; the *value* is the number of human lives at stake, or the capital value (land, buildings, etc.), or the productive capacity (factories, power plants, highways, etc.) exposed to the destructive events; the *vulnerability* is a measure of the proportion of the value which is likely to be lost as a result of a given event.
The above equation points out that risk assessment involves different scientific expertise. As a matter of fact, any risk-based decision/action taken from authorities in charge to manage volcanic emergencies and/or risk mitigation

strategies has to account also for complex inter-plays between social and economic needs, and infrastructure capability to sustain them. In particular, it is necessary to evaluate the vulnerability of exposed infrastructure, facilities and property, the impact of eruptions on human beings, costs vs. benefits of proposed mitigation measures, and the level of "acceptable risk" for society. In addition, we need educational programs to improve the "risk perception" of the people living around volcanoes, and improved ways to communicate risk and associated uncertainties to those people, mass media, and local authorities. In this compound framework, the role of volcanology is mostly focused on providing a reliable volcanic hazard assessment.
As for the term *risk*, also the term *hazard* can lead to some misunderstanding. In English, *hazard* has the generic meaning "potential source of danger", but, as mentioned before, for more than thirty years (e.g., Fournier d'Albe, 1979), *hazard* has been also used in a more quantitative way, that reads: "the probability of a certain hazardous event in a specific time-space window". However, many volcanologists still use "*hazard*" and "*volcanic hazard*" in purely descriptive and subjective ways. For this reason, in order to minimize ambiguities, many researchers have recently proposed that a more suitable term for the estimation of quantitative hazard is "Probabilistic Volcanic Hazard Assessment" (PVHA, hereinafter; see Marzocchi et al., 2007).
Despite the still large use of "qualitative" and "subjective" volcanic hazard assessment, PVHA has undoubtedly many pivotal advantages:

a. A quantitative hazard assessment moves this branch of volcanology from pure (and mere) "speculations" into a "scientific" domain, because only quantitative hypothesis can be tested and compared.
b. A reliable PVHA becomes the rational basis for critical quantitative and transparent decision-making for safety and mitigating volcanic risk to communities, in the long-term, prior to onset of volcanic unrest, and, in the short-term, during volcanic activity and during "volcano crises". For instance, Woo (2007), and Marzocchi and Woo (2007) proposed a quantitative strategy to link PVHA with a cost/benefit analysis for calling an evacuation during an emergency. This approach sharply contrasts with the current common practice, where mitigation actions are usually based on subjective decisions of one or few researchers.
c. The description in terms of probability is particularly suitable for eruptive processes, as well as for any generic complex systems, that are intrinsically unpredictable from a deterministic point of view (at least over time intervals longer than hours/few days). Beyond the extreme complexity, nonlinearities, and the large number of degrees of freedom of a volcanic system (the so-called *aleatory* uncertainty), also our still limited knowledge of the process involved (the so-called *epistemic* uncertainty) make deterministic prediction of the evolution of volcanic processes practically impossible.
d. The probabilistic definition has also the merit to be quite general, therefore it allows a large variety of possible destructive phenomena, such as pyro-

clastic and lava flows, tephra, lahars, gas emission, ground deformation, volcano-seismic events, and so on, to be encompassed by PVHA. For instance, PVHA also includes the definition of Eruption Forecasting (EF), if the destructive event is the occurrence of a volcanic eruption (without considering the ensuing effects on the territory). In other words, EF can be seen as a branch of the more general problem of PVHA.

We conclude this paragraph giving emphasis to a couple of important issues. First, PVHA does not reduce in any way the importance of deterministic studies and the analysis of specific scenarios. The simultaneous use of physical models and data contrasts with what is sometimes encountered in seismic risk analysis, where deterministic and probabilistic approaches are often considered irreconcilable (e.g., Castanos and Lomnitz, 2002). In seismic hazard assessment, the terms "probabilistic" and "deterministic", contained in acronyms PSHA and DSHA, reflect the kind of strategy adopted, mostly evidence-based for PSHA and mostly based on physical models for DSHA. In volcanology, we do not see this conflict; we attempt to use all the information we have (models, data, and expert beliefs), and the term "probabilistic" in PVHA only emphasizes that the quantification of volcanic hazard takes account of associated uncertainties.
Second, we remark that the great importance of this scientific issue is due to its practical implications for society; in this perspective, no matter what probabilistic approach is used, it is fundamental that PVHA is "accurate" (i.e., without significant biases), because a biased estimation would be useless in practice. On the other hand, PVHA may have a low "precision" (i.e., a large uncertainty) that would reflect our scarce knowledge of some physical processes involved, from the preparation of an eruption to the derived impact on the ground of a specific threatening event (e.g., pyroclastic flow, lahars, etc.). An accurate PVHA can be realistically achieved by using some sort of "best picture" of the shared state-of-the-art, and by including all the existing uncertainties. This approach allows the potential bias associated to personal convictions and to lacks of knowledge to be minimized. In particular, we caution against the use of even sophisticated models that are not yet properly tested, because they certainly increase the precision, but they can introduce a significant bias making the estimation highly inaccurate.

Eruption Forecasting

As mentioned before, EF can be seen as a specific branch of PVHA. EF deserves to be considered separately because it is the main drive for important risk mitigation actions like evacuation of the people living in the surrounding of a volcano.

Despite some recent researches on short-term forecasting (from hours to few days) are based on a deterministic approach (e.g., Voight and Cornelius, 1991; Kilburn, 2003; see also Hill et al., 2001), the presence of complex and different precursory patterns for distinct eruptions, as well as the exigency to consider the possibility that a precursory pattern not necessarily leads to an eruption, suggest that a probabilistic approach could be more efficient in EF (e.g., Sparks, 2003). At this purpose, it is worth remarking that the probabilistic approach is not incompatible with the deterministic approach, because the former can include deterministic rules as limit cases, i.e., when the probability tends to one. In other words, the probabilistic approach is certainly more general, and it has also the merit to be applicable at different time scales; for instance, during a quiet period of the volcano, EF is estimated by accounting for the past activity of the volcano (long-term EF; see, e.g., Marzocchi and Zaccarelli, 2006); conversely, during an unrest, the method allows mid- to short-term EF to be estimated by considering different patterns of pre-eruptive phenomena (e.g., Newhall and Hoblitt, 2002; Aspinall and Woo, 1994; Aspinall et al., 2003; and Marzocchi et al., 2004, 2008).
The concept of short/long-term EF/PVHA deserves further explanations. The terms "short" and "long" are referred to the expected characteristic time in which the process shows significant variations; in brief, during an unrest the time variations occur on time scales much shorter than the changes expected during a quiet phase of the volcano. On the other hand, these terms are not linked to the forecasting time window (for instance, we can use a forecasting time window of one day, both for short- and long-term EF). The distinction between these two time scales, besides to reflect a difference in the physical state of the volcano (quiescence and unrest), is also important in a practical perspective; in fact, for example, the long-term PVHA (years to decades) allows different kinds of hazards (volcanic, seismic, industrial, floods, etc.) in the same area to be compared, which is very useful for cost/benefit analysis of risk mitigation actions, and for appropriate land-use planning and location of settlements. In contrast, monitoring on mid- to short-time scales assists with actions for immediate vulnerability (and risk) reduction, for instance through evacuation of people from danger areas (Fournier d'Albe, 1979).
As a general thought, we can say that a realistic EF is usually entangled by the scarce number of data, and by the relatively poor knowledge of the physical pre-eruptive processes. This makes any EF hypothesis/model hardly testable also in a backward analysis, overall for explosive volcanoes. On the other hand, the extreme risk posed by many volcanoes pushes us to be pragmatic and attempt to solve the problem from an "engineering" point of view: by this, we mean that the devastating potential of volcanoes close to urbanized areas forces the scientific community to address the issue as precisely as possible. This strategy can be summarized quoting Toffler (1990) that said "it is better to have a general and incomplete (*we add: general and incomplete, but precise!*) map, subject to revision and correction, than to have no map at all".

This goal can be achieved at best by treating scientific uncertainty in a fully structured manner, and, in this respect, Bayesian statistics is a suitable framework for producing an EF/PVHA in a rational, probabilistic form (e.g., UNESCO, 1972; Gelman et al., 1995). Basically, the Bayesian approach described here starts from modeling the statistical distribution using our basic knowledge (or complete ignorance), and then it refines the distribution as long as new information come in as in a sort of "data assimilation" procedure.
As last remark, nevertheless the probabilistic tool used, we argue that only real data can reduce (epistemic) uncertainties in forecasting. The large uncertainty in EF is mostly due to the fact that the average level of knowledge of pre-eruptive phases is significantly lower than for syn- and post-eruptive phases. The most obvious reason is that all pre-eruptive processes occur deep inside the volcano, inaccessible to direct observation and, historically, early signs of impending eruption may have been marginal and not documented. Our eruption forecasting ability overall, and especially for long-quiescent explosive volcanoes, is still rather poor. Initiatives like the WOVOdat project (http://www.wovo.org/wovodat) will improve our capabilities. WOVOdat is a fledgling database that will serve as the primary resource for a new field of "volcano epidemiology" and will also aid associated research into how volcanoes prepare to erupt. During volcanic crises, it can be used to make queries such as "where else have X,Y,Z been observed and what happened?" or, more quantitatively, "how much do the given observations increase the probability of eruption today, tomorrow, and in the future?"

The Bayesian Event Tree (BET) applied to PVHA

General features

In this section, we describe a possible strategy for PVHA based on Bayesian Event Tree (BET hereinafter). Basically

BET translates volcanological input into probability
of any possible volcano-related event.

The "volcanological input" is every types of information relevant for the event under study. It ranges from models (i.e., ash fall model), to historical/volcanological information (i.e., eruptive catalogs), to monitoring measures (i.e., detecting magma movement), and so on…
A detailed description of the procedure can be found in Marzocchi et al. (2004; 2006; 2008), and Newhall and Hoblitt (2002). Other references on similar Bayesian strategy and, in general, on probabilistic approach are Gelman et al. (1995), Aspinall et al. (2003), Jacquet et al. (2006). Here, we report only the main features of BET that can be summarized in few general points:

a. Despite the probabilistic nature of the method, BET merges all kinds of relevant information, coming from theoretical/empirical models, geological and historical data, and monitoring observations. In brief, BET is a probabilistic model that transforms all of the input volcanological information into probabilities; such probabilities represent an homogeneous and quantitative synthesis of the present knowledge about the volcano.
b. BET has the most important characteristic for a model to be "scientific", that is, it gives the possibility to "falsify" the results provided; this important feature gives also an opportunity to make scientifically testable any scientific belief/hypothesis.
c. In general, BET does not rule out any possibility, but it shapes the probability distribution of the event considered around the most likely outcome accounting for all the information reported above. This is accomplished by dealing with *aleatory* and *epistemic* uncertainties (see above) in a proper way (see Woo, 1999; Marzocchi et al., 2004; 2008).
d. BET estimates short- and long-term PVHA/EF, depending on the present state of the volcano, providing a useful tool in several contexts: i) to compare different types of risks, ii) to carry out cost/benefit analysis of risk mitigation actions, iii) to indicate appropriate land-use planning and location of settlements, and iv) to suggest immediate vulnerability (and risk) reduction actions, such as the evacuation of people from danger areas (Fournier d'Albe, 1979).

The Event Tree

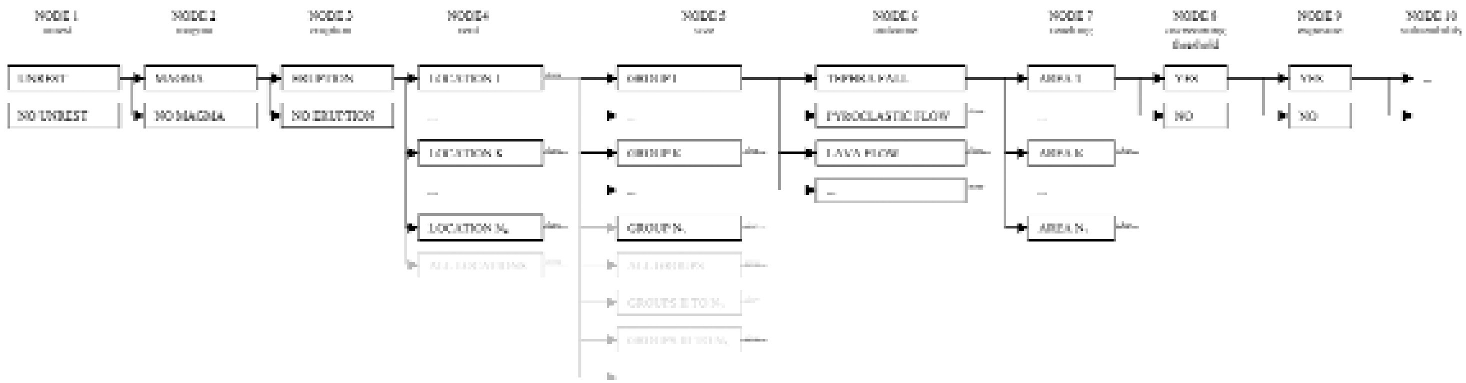
BET is based on the concept of event tree; the event tree is a branching graph representation of events in which individual branches are alternative steps from a general prior event, state, or condition, and which evolve into increasingly specific subsequent events. Eventually the branches terminate in final outcomes representing specific hazardous phenomena that may turn out in the future. In this way, an event tree attempts to graphically display all relevant possible outcomes of volcanic unrest in progressively higher levels of detail. The points on the graph where new branches are created are referred to as *nodes* (Newhall and Hoblitt, 2002; Marzocchi et al., 2004; 2006; 2008). In Figure 1, we report a general event tree for a volcano.

Note that since the definition reported above is mainly driven by the practical utility of the event tree, the branches at each node point to the whole set of different possible events, regardless of their probabilistic features. In other words, the events at each node need not be mutually exclusive.

At each node we have the following states:

- Node 1: there is unrest, or not, in the time interval $(t_0, t_0+\tau]$, where t_0 is the present time, and τ is the time window considered.
- Node 2: the unrest is due to magma, or to other causes (e.g., hydrothermal, tectonics, etc.), given unrest is detected.

Fig. 1. Event Tree for a generic volcano. See text for the explanation of each single node. In red, a pattern of a selected event is reported (The thickness of an ash fall produced by a VEI 5+ eruption overcomes a specific threshold in area 2).



- Node 3: the magma will reach the surface (i.e., it will erupt), or not, in the time interval (t_0, t_0+t], provided that the unrest has a magmatic origin.
- Node 4: the eruption will occur in a specific location, provided that there is an eruption.
- Node 5: the eruption will be of a certain size (i.e., VEI), provided that there is an eruption in a certain location.
- Node 6: the occurrence of a specific threatening event (i.e., pyroclastic flow, lahars, etc.), given an eruption of a certain size in a certain location.
- Node 7: the area reached by the threatening event, given that the threatening event is occurred.
- Node 8: the overcoming of a threshold related to a certain threatening event in a certain area, given the threatening event has reached this area.

At each one of these states we attribute a probability function. As described in the following, the use of these probability functions (characteristic of the Bayesian approach) allows BET to estimate aleatory and epistemic uncertainties. Let us define θ^{E} as the probability of the conditional event *E* (note that each event reported above is conditioned to the occurrence of other events at previous nodes); therefore, for each one of the nodes we define [θ_1(unrest)], [θ_2(magma)], [θ_3(eruption)], [θ_4(location)], [θ_5(size)], [θ_6(threat)], [θ_7(area)], [θ_8(threshold)], where the square brackets stand for a generic "probability density function (pdf)". Since the first three nodes have only two possible states that are mutually exclusive and exhaustive (for instance, unrest or not), we set, for the sake of simplicity, [θ_1(unrest)]=[θ_1], [θ_2(magma)]=[θ_2], and [θ_3(eruption)]=[θ_3]. In other words, BET considers the conditional probability at each node as a random variable, therefore it estimates each probability through a pdf, not as a single value.

PVHA and EF

Given all the pdfs at each node, BET combines them in order to obtain the absolute probability of each event at which we are interested in. For instance, the pdf of the probability to have an eruption of VEI 5+ in a the time interval (t_0, t_0+t], at the *j*-th vent location, i.e, [Θ_1], is

$$[\Theta_1] = [\theta_1]\ [\theta_2]\ [\theta_3]\ [\theta_4^{(\mathrm{loc\ j})}]\ [\theta_5^{(\mathrm{VEI\ 5+})}] \tag{1}$$

In other words, [Θ_1] is a quantitative measure of EF. The probability to have the same eruption at any location, i.e, [Θ_2], is

$$[\Theta_2] = \Sigma_j\ [\theta_1]\ [\theta_2]\ [\theta_3]\ [\theta_4^{(\mathrm{loc\ j})}]\ [\theta_5^{(\mathrm{VEI\ 5+})}] \tag{2}$$

where the sum is on the number of all possible vent location considered. In this case we have assumed that the distribution of possible vents is a set of com-

pletely mutually exclusive events. The functional form of [Θ] is not determined analytically, but through a Monte Carlo simulation. In practice, we sample 1000 times each pdf, and we perform the calculation by using each sample. Therefore, we obtain 1000 values of [Θ] that are used to determine the functional form numerically. In this way, we propagate in a proper way both aleatory and epistemic uncertainties at all nodes, and we estimate best guess and errors of the absolute probability of any possible event in which we are interested.
In summary, BET provides quantitative estimations of PVHA through the evaluation of the pdfs of the nodes, by accounting for any kind of available information (i.e., *a priori* and theoretical beliefs, historical and geological data, and monitoring data). We remark that we do not estimate directly the probability of eruption, because in most of the cases it is much easier to estimate conditional probabilities. For the sake of clarity, in Figure 2 we report a scheme that describes the main logical steps of BET applied to EF. The generalization to the PVHA problem is straightforward.

Estimating the probability at each node: the volcanological input becomes probability

From what reported until now, it is easy to understand that the main technical problem in BET is to estimate the pdf for each node. This estimation is the core of the procedure because it translates volcanological input for each node into probability. Since each node deserves a careful discussion about how such a probability is estimated, here we report only the general features common for each node. A full and detailed description of the technical details can be found in Marzocchi et al. (2004, 2006, 2008).
Generally speaking, we have two broad classes of information that can be considered in EF/PVHA: measurements from monitoring (dataset M) and all the other kinds of data/information (dataset $\bar{M}$). This subdivision is mainly due to the fact that, usually, these two types of information have different weights in different states of a volcano. During an unrest, monitoring data may be the most relevant for EF/PVHA purposes, while the same data do not carry relevant information about EF/PVHA during a quiet period, apart from telling that the volcano is at rest. At the same time, it is obvious that monitoring data contain fundamental information that must be used to quantify mid- to short-term PVHA. For these reasons, we introduce these kinds of information through two different functions. At the generic *k*-th node, the pdf of the *j*-th event ([$\theta_\kappa^{(j)}$], see notation used before) is

$$\left[\theta_k^{(j)}\right] - \gamma_k\left[\theta_k^{(j)|M|}\right] + (1-\gamma_k)\left[\theta_k^{(j)\{\bar{M}\}}\right] \tag{3}$$

where γ_k is a variable in the interval [0,1], $\left[\theta_k^{(j)\{M\}}\right]$ and $\left[\theta_k^{(j)\{M\}}\right]$ have the

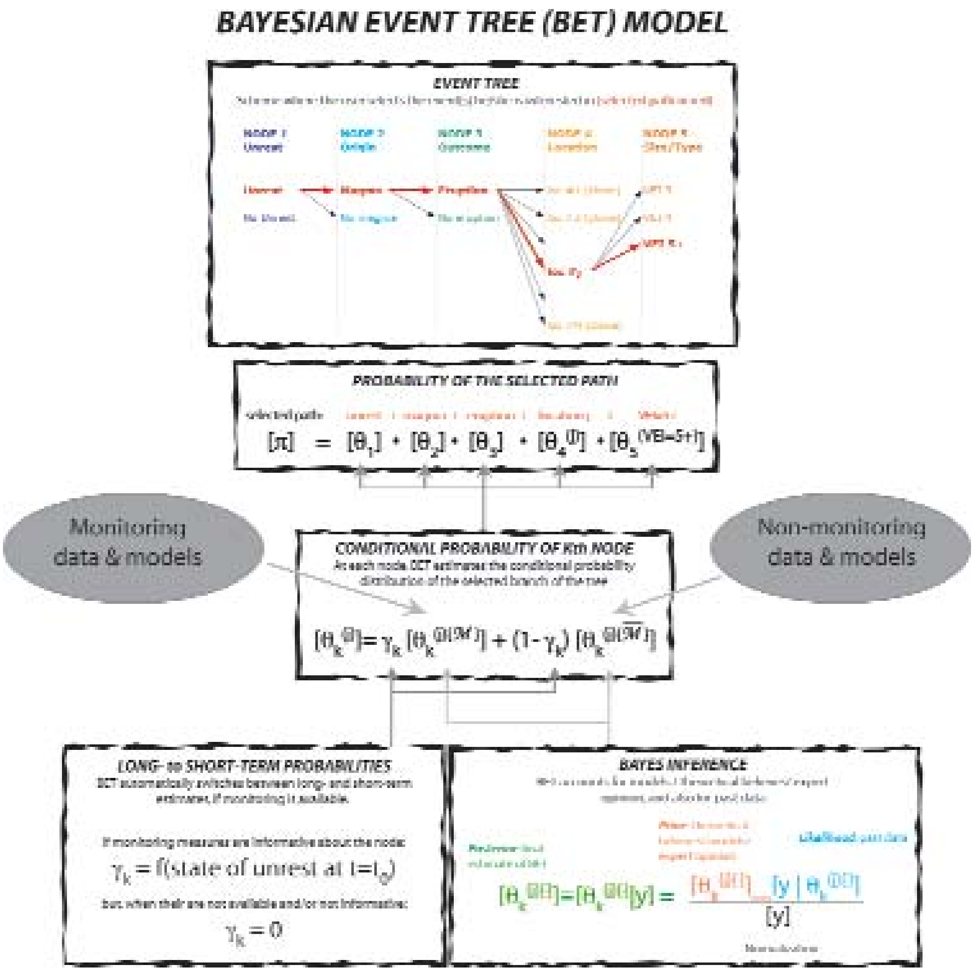


Fig. 2. General scheme of BET. Beginning at the top, there is (1) the selection of a path within the event tree; (2) the computation of the probability of the path; (3) the computation of each conditional probability from all of the monitoring and nonmonitoring information, (4) the computation of the weight of the monitoring part, and (5) the Bayesian inference core of BET.

same meaning as $[\theta_k^{(j)}]$, but they are defined by using only monitoring information and all the other kinds of information (non-monitoring, hereinafter), respectively. In other words, $[\theta_k^{(j)}]$ is a linear combination of the probabilities based on these two types of knowledge, weighted with γ_k and $(1-\gamma_k)$ respectively.

It follows that the unknowns of BET are the pdfs of the conditional probability of the selected events at the nodes. The evaluation of each one of these probabilities passes through the estimation of the three unknowns in equation (3), i.e., γ_k, $\left[\theta_k^{(I)\{M\}}\right]$, and $\left[\theta_k^{(I)\{M\}}\right]$. In particular:

1. The parameter γ_k sets the degree at which monitoring data (useful for short-term EF) control the posterior probabilities with respect to the non-monitoring part (useful for long-term EF); for the nodes where monitoring parameters are informative, γ_k is a function of the "state of unrest" (see Marzocchi et al., 2008) which, in turn, is a *fuzzy* parameter (Zadeh, 1965) in the interval [0,1] that indicates the degree at which unrest is detected by the monitoring observations at $t=t_0$. In practice, through γ_k, BET switches dynamically from long-term (when the volcano is found to be at rest) to short-term (during unrest) probabilities.

2. $\left[\theta_k^{(I)\{M\}}\right]$ is the monitoring part in equation 3, i.e., the leading term in short-term probability evaluation. It is determined through Bayes' rule, which combines estimated probabilities from monitoring measures at time t_0 and monitoring measurements from past episodes of unrest (if any). Here, the present and past monitoring measurements are transformed into probabilities by means of a physical model that depends on the node considered.

3. $\left[\theta_k^{(I)\{M\}}\right]$ is the non-monitoring part in equation 3, i.e., the leading term in long-term probability evaluation. It is determined through Bayes' rule, which combines estimated probabilities from all our knowledge based on theoretical models and/or beliefs, and past data, i.e., past frequencies of occurrence.

The estimation of these three unknowns requires the use of two important technical concepts, namely the Bayesian inference and the fuzzy approach (Zadeh, 1965). The Bayesian inference is necessary to merge together theoretical models/beliefs with data, and to deal with aleatory and epistemic uncertainties. The *fuzzy* approach is used to manage monitoring measures into the probability calculations and to define the state of unrest. The state of unrest is used to detect unrest from monitoring measures, and then to define γ_k. The specific estimation of γ_k, $\left[\theta_k^{(I)\{M\}}\right]$, and $\left[\theta_k^{(I)\{M\}}\right]$ for each node of BET (equation 3), and all technical details are reported in Marzocchi et al. (2008).

Final Remarks on BET

To summarize, in this section we report some central features of BET

- BET is a tool to calculate and to visualize probabilities related to PVHA/EF. In particular, BET "dynamically" assesses long-term (useful for land use planning, and for comparing the volcanic hazard with other different kinds of hazard), and short-term (useful during emergency to help managing short-term actions aimed to reduce risk) eruption forecasting.
- BET estimates probabilities by using all of the available information such as models, state of the volcano, geologic/volcanologic/historic data, present and past monitoring observations, expert opinion, and theoretical beliefs.
- BET takes properly and explicitly into account the epistemic (data- or knowledge-limited) and aleatory (stochastic) uncertainties. This guarantees reliable outputs, given reliable input information.
- BET is a scientific tool because it provides probabilities that can be used to test any hypothesis/models contained in BET.
- BET is a quantitative and "transparent" tool that allows to move from subjective choices made, for example, during an emergencies, to more quantitative, objective, and clear rules that can assist decision-makers in handling at best emergencies and land-use planning.

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