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# Implications of rate and state dependent friction for creep on shallow faults

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**Abstract**
The aseismic sliding on shallow strike-slip faults, under the assumption of a non linear constitutive equation (*velocity strengthening*), is here treated as a two-dimensional quasi-static crack problem whose equations are solved numerically (*boundary elements* method). Results are compared with the corresponding one-dimensional («depth averaged») model by a suitable choice of the effective stiffness of the fault. In the one-dimensional case also the inertial term was taken into account in the evolutive equation. The current results are in agreement with an earlier one-dimensional model for afterslip as long as the state variable evolution is neglected a priori and friction depends only on velocity. In general, if the state variable is allowed to evolve, the previous approximation is valid for velocity strengthening slipping section of faults extending down to several kilometers in depth. For smaller sections of fault the evolution of the state variable affects the coseismic and early postseismic phase and accordingly it cannot be neglected. Moreover, in the presence of rheological heterogeneities, for fault sections shallower than 1 km depth, the comparison between the two-dimensional and one-dimensional models suggests the need to employ the two-dimensional model, possibly taking into account inertial effects.

**Key words** *fault rheology – upper stability transition – crack models – afterslip – creep events*

## 1. Introduction

The aseismic slip which occurs on shallow faults after sudden changes in the stress state is a transient process often detectable at the Earth's surface. The study of this effect can provide a key to understand the general time dependent behaviour of a fault and its rheology (*e.g.* Nason and Weertman, 1973; Wesson, 1988; Marone *et al.*, 1991; Belardinelli and Bonafede, 1991, 1994). Evidence of aseismic sliding at the Earth's surface allows us to extrapolate the results of laboratory observations which necessarily involve small rock samples, during short periods of time. Several experiments of controlled sliding between rock samples (*e.g.* Dieterich, 1978, 1979) suggest that friction depends on normal stress, slip rate and, for time dependent slip-rate, on the slip history. This *fading memory effect* can be obtained through one or two phenomenological parameters, called *state* variables, describing the state of the sliding surface, which consequently evolve during the sliding process. Starting directly from the laboratory observations obtained through controlled variation in sliding velocity, it is possible to draw out (Rice and Tse, 1986) the one state variable friction law originally studied by Ruina (1983). This law entails only a small deviation from the classic concept of kinetic friction (*e.g.* Rice and Gu, 1983), nevertheless this slight difference is crucial in order to determine if the sliding is stable or not. In other words, the stability

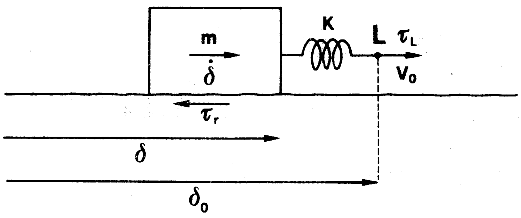


**Fig. 1.** *Spring-slider* scheme.

of sliding is linked to the possibility of a relative change of the resisting traction on varying the velocity and not to its absolute value. This effect can be analyzed by comparison between the evolution of a simple dynamical system called *spring-slider* and the laboratory observations. A *spring slider* is a sliding block pulled through a spring at a given velocity (fig. 1) and in the comparison with laboratory experiments the stiffness of the spring is thought to represent the elastic properties of the apparatus surrounding the sample.

Non linear frictional laws have often been employed in fault mechanics models such as nucleation (*e.g.* Dieterich, 1992), seismic cycle (*e.g.* Tse and Rice, 1986), and afterslip models (Marone *et al.*, 1991), whereby the instability of sliding specimens of rock at laboratory scale is identified with the earthquakes occurring on real faults. According to the *spring-slider* studies in quasi static conditions (Belardinelli, 1994) two characteristic times inversely depending on the applied velocity can be identified. This feature enables us to reproduce different time scales (provided that a time dependent applied velocity is used) keeping the same formal law: for instance in the mentioned application to fault mechanics both the seismic and aseismic behaviour in the same section of the fault might be explained. For instance in the case of the 1987 Superstition Hills (California) earthquake, several records of afterslip (till 5.5 years after the earthquake, Sharp, 1994, personal communication) and a non vanishing inferred slip at the Earth's surface (about 15% of the slip cumulated after 1 year) at 1 minute after the earthquake (William and Magistrale, 1989) suggest that both seismic and postseismic slip occurred in the same shallow section of fault.

In the following, a two-dimensional quasi-static algorithm for the solution of the constitutive equation, namely the equation for friction, and the equilibrium equation on the fault plane will be introduced. The solution is numerical and based on the discretization of the boundary integral equation (*boundary elements* technique). In problems of shallow aseismic faulting the proposed method allows us to simply vary the assumed constitutive equations and the rheological properties of the fault zone. The quasi static post-seismic evolution in a one-dimensional (depth averaged) model of fault, whose analog system is a *spring-slider* strongly perturbed from the stationary state, can be obtained by the same method as a particular case. Both the results from the two-dimensional and one-dimensional model have been obtained using the non linear frictional law of Ruina (1983). In the latter simpler model the inertia effect has been taken into account (dynamic evolution) and results are compared with the one-dimensional quasistatic model of Marone *et al.* (1991).

## 2. The state and velocity dependent constitutive laws

The friction laws depending non linearly on velocity and on the state variable in the Ruina (1983) formulation are given by

$$\tau_{\text{res}} = \tau_* + A \log\left(\frac{\dot{\delta}}{V_*}\right) + \theta \tag{2.1}$$

$$\dot{\theta} = \frac{-\dot{\delta}}{L}\left[\theta + B \ln\left(\frac{\dot{\delta}}{V_*}\right)\right] \tag{2.2}$$

where the dot means differentiation with respect to time, $\tau_*$ and $V_*$ are reference values for friction and slip rate respectively and $A$ and $B$ are parameters depending on depth through pressure, temperature and composition, $L$ is a characteristic length for slip. Dividing by the values of the normal traction $\sigma_n$ and using the classic second Admonton's law (*e.g.* Scholz, 1990) we can define a friction coefficient $\mu \equiv \tau_{res}/\sigma_n$ where two adimensional parameters appear: $a \equiv A/\sigma_n$, $b \equiv B/\sigma_n$, which are often regarded as independent from normal traction and velocity. On the other hand $a$ and $b$ vary sensibly with temperature: for instance $a - b$ at room temperature is generally negative and then the friction at steady state

$$\tau^{SS}(\dot{\delta}) = \tau_{res} \mid_{\theta = \text{cost}} = (a - b)\sigma_n \ \ln (\dot{\delta} / V_*) + \tau_* \qquad (2.3)$$

decreases with velocity (*velocity weakening*). If $T > 300$ °C, $a - b$ becomes positive and then $\tau^{SS}$ increases with velocity (*velocity strengthening*) (see *e.g.* Tse and Rice, 1986). The *velocity strengthening* materials are linearly stable, that is they are stable with respect to small perturbations, whereas the *velocity weakening* materials can be unstable if the surrounding media have a small enough elastic stiffness (*e.g.* Gu *et al.*, 1984). Then a sign change in $a - b$ and then in $A - B$ represent a transition in the stability properties. In general the scaling of the frictional parameters from the laboratory to crustal conditions is still a problem. Figure 2 shows the $a$ and $b$ profile as a function of depth used by Rice (1993): two important transitions in the rheological properties are evident: the thermally activated, so called (Scholz, 1990) lower stability transition, from velocity weakening to velocity strengthening at a depth around 10-15 km, and the upper stability transition, shown at about 2 km depth in fig. 2, due to the presence of shallow unconsolidated sediments (Marone *et al.*, 1990). The depths of these transitions correlate respectively with the cut-off of seismicity at medium crustal depth and at shallow depth in the case of faults with

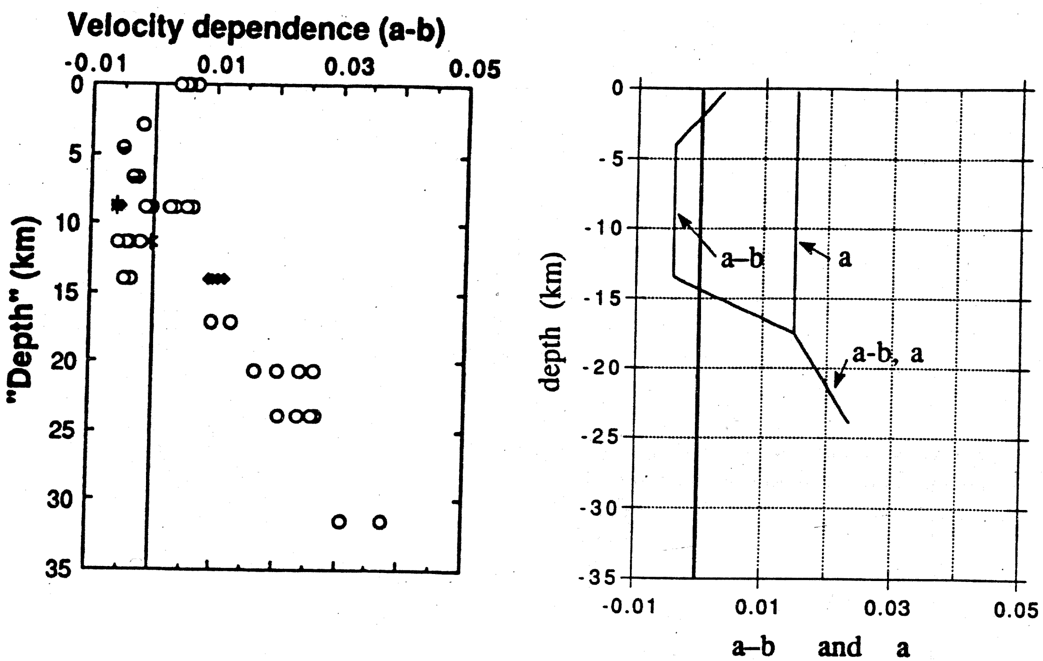


**Fig. 2.** Depth dependence of the difference $a - b$ inferred from laboratory observations at different temperatures and using the San Andreas geotherm (on the left) and rheological profiles adopted by Rice (1993) (on the right) where the $a$ distribution is arbitrary but compatible with data.

«well-developed» gouge (Marone and Scholz, 1988). The latter is an interesting topic since as we already noticed the sliding instability is associated with earthquake nucleation. In the following we will restrict our attention to the velocity strengthening region above the upper stability transition, which generally arrests the seismic rupture (*e.g.* Scholz, 1990) and where the traction concentration created by the earthquake is mainly released aseismically through *afterslip*, as noted since Smith and Wyss (1968). The velocity strengthening properties of a shallow section of fault after an earthquake, may also be ascribed to the experimentally observed frictional behaviour at high slip rate and low normal stress (Shimamoto, 1986; Kilgore *et al.*, 1993).

## 3. Crack models for aseismic forced slip

We consider the geometry shown in fig. 3: following an earthquake some aseismic slip $\delta$ occurs in the superficial part of a strike-slip, horizontally unbounded, vertical fault, extending down to a depth $x = h$, the Earth's surface being assumed free. In order to obtain the postseismic creep, the following set of equations for the time dependent distribution of slip $\delta(x, t)$ has to be solved (*e.g.* Wesson, 1988)

$$\tau_{ext}(x, t) + \tau_{dis}(x, t) = \tau_{res}(\delta, \dot{\delta}, x, t) \tag{3.1}$$

$$\tau_{dis}(x, t) = \frac{\mu}{2\pi} \wp \int_{-a}^{a} \frac{\partial \delta(x', t)}{\partial x'} \frac{dx'}{x' - x} \tag{3.2}$$

where $\wp$ indicates the principal value of the integral, $\tau_{res}$ is the friction traction expressed as a function of the slip, slip rate, etc. through the constitutive equation, $\tau_{ext}$ is the traction applied to the superficial part of the fault in the absence of postseismic slip, and $\tau_{dis}$ is the traction created by the postseismic slip or «self-traction» and is expressed by the boundary integral equation in a quasi-static crack formulation. In problems dealing with postseismic effects, the application of the external traction due to the earthquake occurrence, say at $t = 0$,

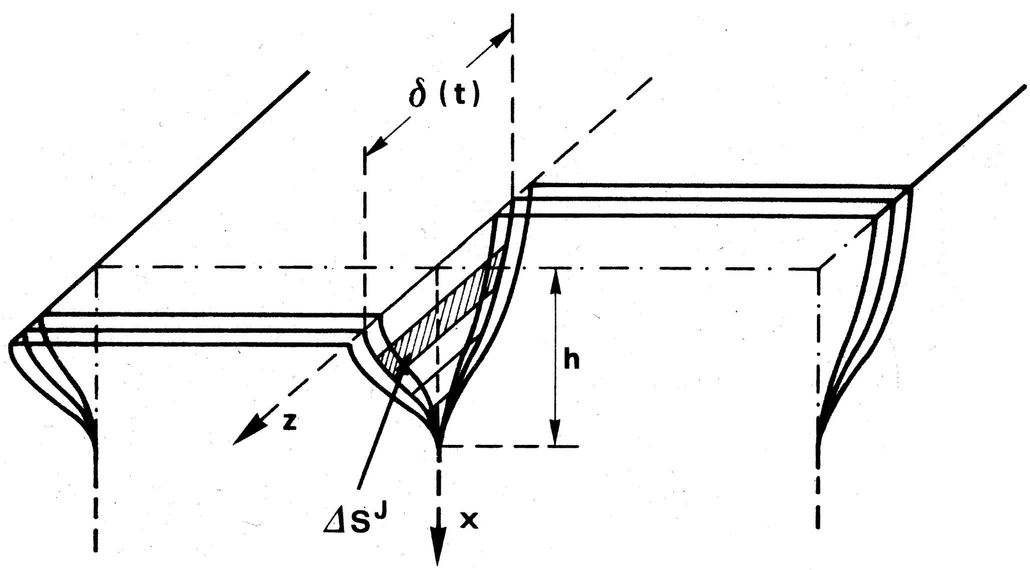


**Fig. 3.** Geometry of the problem.

is generally modeled as a time step function $\tau^i_{ext} = \Delta\tau^i U(t)$ (*e.g.* Rice and Gu, 1983).

The solution of (3.1) (3.2), together with the appropriate constitutive equation provides at each instant of time the slip, the self-traction and the slip-rate in each point of the fault. If the constitutive equations are given by (2.1) (2.2), the obtained set of equations is non linear and the numerical solution is the only feasible solution. The numerical solution here adopted consists of a spatial discretization of the unknown functions: the slipping section is divided into $n$ «nodes», namely $n$ horizontal strips $\Delta S^j$ (fig. 3), so narrow that on each of them the slip $\delta^j$ is assumed to be constant. Indicating also the discretized tractions with an apex referring to the fault node where they are defined, the set of eq. (3.1) (3.2) becomes

$$\tau^i_{ext}(t) + \tau^i_{dis}(t) = \tau^i_{res}(\delta^i(t), \dot{\delta}^i(t), t)$$

$$\tau^i_{dis}(t) = \sum_{j=1}^{n} S_{ij}\, \delta^j(t) \tag{3.3}$$

where the *influence* matrix $S_{ij}$ has been obtained by Dieterich (1992) in order to study the earthquake nucleation process on the surface of a two-dimensional, symmetric, plane *crack* with half-length $h$.

$$S_{ij} \equiv \frac{\mu n}{2\pi h}\left\{\frac{1}{(i-j)^2 - 1/4} + \right.$$

$$\left. -(1-\delta_{j1})\left[\frac{1}{i+j-3/2} + \frac{1}{i+j-5/2}\right]\right\},$$

$$i, j = 1, 2, \ldots, n. \tag{3.4}$$

For the antiplane case the use of this tensor allows us to keep the free surface condition in the first node. Further details of this numerical solution are discussed in Appendix A.

When $n = 1$ the set of eqs. (3.3) and (3.4) becomes the equation of motion in the quasi-static case of a *spring-slider* (fig. 1) subjected to a friction $\tau_{res}$ and pulled at a certain velocity $V_0 = \tau_{ext}/|S_{11}|$ through a spring with constant $k = |S_{11}|$, $\delta$ being the displacement. As far as quantities in the case of the $n = 1$ node algorithm are concerned, the apex for the node will be omitted. For $n = 1$, at high slip rates $\dot{\delta} > V_c$, where $V_c$ is a limiting velocity, with the current program it is possible to take into account also the inertial term in the *spring-slider* equation: in this case the following equation replaces (3.3)

$$m\,\ddot{\delta} = \tau_{ext}(t) + S_{11}\,\delta(t) - \tau^c_{res}(\delta(t), t), \quad \dot{\delta} > V_c \tag{3.5}$$

where for $\dot{\delta} > V_c$ one assumes $\tau_{res}(\delta, \dot{\delta}, t) = \tau^c_{res} = \tau_* - \theta - A \ln(V_c/V_*)/k$, with $\dot{\theta} = -(\dot{\delta}/L)[\theta + B \ln(V_c/V_*)]$ (Boatwright and Cocco, 1994), and $m = S_{11}(T_i/2\pi)^2$, where $T_i$ is a *vibration period of the analogous freely slipping system* (Rice and Tse, 1986).

As hinted in the introduction, the *spring-slider*, and then the present $n = 1$ model, is equivalent to a one-dimensional quite schematic model of fault since *much interesting physics is lost in this reduction* (Rice and Gu, 1983). A fault model with distributed displacement such as the present one with $n > 1$ nodes, is called *two-dimensional* since it is able to produce displacement fields depending on two space variables (in the present antiplane case the distance from the fault and the depth). A *one-dimensional* model of fault, such as the $n = 1$ model, considers only a mean value or a «physically representative» value of the slip distribution on the fault plane so that at most it can provide the evolution of the corresponding value of the displacement field, which turns out to depend only on the distance from the fault; moreover we will see in the following that this is not true in general. Basically in the one-dimensional seismological applications (*e.g.* Dieterich, 1986), each parameter distribution is substituted with its mean value on the fault plane, and eq. (3.2), which represents the elastic interaction between a slipping zone and its surroundings, is substituted with an approximation of it. The latter is a relation of mere proportionality between the mentioned physically representative quantities (indicated between angular brackets «<>»), such as mean values or maximum values of the actual distribution on the fault plane

$$< \tau_{\text{dis}} > \; = k < \delta > , \qquad (3.6)$$

where the constant of proportionality $k$ will be called effective stiffness of the fault model. In the $n = 1$ model the effective stiffness of the fault is $S_{11}$, which represents the constant of proportionality between the modulus of the traction drop and the averaged slip of a crack where the same traction drop is uniform; accordingly this model as it stands could in principle provide the evolution of the mean value of the slip on the fault plane (*depth averaged model*). We will see in the following that if the traction drop distribution is non uniform, then a one-dimensional model, where the stiffness computed assuming a uniform stress drop is used, cannot reproduce the results of a two-dimensional model.

The effective elastic stiffness of the fault model simplifies the analogy between a one-dimensional model of fault and the *spring-slider*, providing a link between the forcing factor of spring-slider dynamics (the load point velocity $V_0$, see fig. 1), to that typical of fault mechanics (the distribution of external traction), as follows

$$\dot{\tau}_{\text{ext}} = kV_0 \, . \qquad (3.7)$$

The model by Marone *et al.* (1991) to which the results of the $n = 1$ procedure will be compared, is an example of a one-dimensional model for afterslip. It is a *depth-averaged* model, so that the quantities involved (between angular brackets) are supposed to be representative of depth averaged values. The superficial section of a fault, extending down to a depth $h$ (above the upper stability transition) is assumed to be *velocity strengthening*. The part of the fault plane at greater depth undergoes a strong variation in velocity from a high value $V_T$ during the earthquake, to a low value ($V_0 = V_S$) after it, and accordingly at the base of the velocity strengthening section, a coseismic slip $\delta_{0c}$ is assumed. Due to the elastic coupling between different parts of the fault, the upper part also undergoes a slip-rate variation with a coseismic slip $\delta_c$. Subdividing the slip $\delta$ in postseismic $\delta_p$ (whereby the secular part $V_S t$ was subtracted from it) and coseismic $\delta_c$, it is easy to conclude that

$$\lim_{t \to \infty} \delta = \delta_c + \delta_p \, U(t - T_d) = \delta_{0c} \qquad (3.8)$$

from which the trivial result that in a place with a greater amount of coseismic slip at the surface, then less postseismic slip occurs (see *e.g.* the Superstition Hills earthquake in William and Magistrale, 1989).

## 4. The one-dimensional model

### 4.1. *Comparison with a previous model at steady state*

In the following the $n = 1$ algorithm will be compared with the one-dimensional afterslip model by Marone *et al.* (1991) that will be indicated from now on as «MSB91». I recall that a one-dimensional model of fault, like MSB91 or the present model for $n = 1$, is described by the same equation as a *spring-slider* in quasistatic conditions. By use of (3.7) a step in external traction corresponds to a squared impulse in the loading point velocity of the *spring-slider*

$$V_0(t) = \begin{cases} V_S & , t < 0 \\ V_T & , 0 \le t < T_d \\ V_S & , t \ge T_d \end{cases} \qquad (4.1)$$

where $V_T \gg V_S$ and $T_d$ is the earthquake duration. Then the coseismic slip at the base of the fault and in the sliding section are $\delta_{0c} = V_T T_d$ and $\delta_c \equiv \delta(T_d)$ respectively.

The effective stiffness of the fault in MSB91 is assumed in the form $k = \mu/h$ and the system is equivalent to a *spring-slider* subject to a friction law given by (2.1), (2.2) with $A > B$, and with a load point velocity $V_0(t)$ given by (4.1) applied to a spring with elastic constant $k$. In the comparison I kept the effective stiffness $k = \mu/h$ adopted in MSB91 in place of $k = |S_{11}| = 2\mu/\pi h$ of the $n = 1$ algorithm. The mean value of the parameter $A - B$ over the depth $h$ is $< A - B > = (a - b)\sigma'_n h/2$, assuming a uniform gradient in normal stress $\sigma'_n$. In the following, dealing with the one-dimensional model, the angular brackets will be omitted and the specified value of $A$ and $B$ will be implicitly understood as a depth-averaged value. Accordingly by varying the values of $h$ we may obtain a variation of $A$, $B$, $k$, such that an increasing $h$ implies a linear increase of $A$ and $B$ and a decrease of $k$. In the following in most cases the values of $A$ and $B$ will vary with varying $h$ and keeping fixed the values of $\sigma'_n$, $a$ and $b$. The *spring-slider* equation is solved as in MSB91 in quasi static form, neglecting the inertial term.

We will refer mostly to the following parameters values

$$\begin{aligned} &L = 10\,\text{mm},\ a - b = 0.001 - 0.01,\ \mu = 10^5\,\text{bar} \\ &V_* = V_S = 2\,\text{mm/yr},\ V_T = 0.2\,\text{m/s},\ T_d = [0.5 - 1]\,\text{s} \\ &\sigma'_n = 150\,\text{bar/km},\ \tau_* = 0 \end{aligned} \tag{4.2}$$

which are also the values most frequently used in MSB91. As an initial state we assume $\dot{\delta}(0) = V_* = V_S$ and $\dot{\theta}(0) = 0$, accordingly the system is assumed in an initial stationary state. Since in MSB91 the value of $b$ or $a$ is not specified (they only specified the difference $a - b$), a value $b = 0$ was assumed and then $B = 0$. Allowing a non vanishing (arbitrary) value of $B$ the agreement with MSB91, based on estimates of coseismic slip, noticeably gets worse. As we will see in the following, for $B \neq 0$ the evolution tends to differ from the evolution for $B = 0$ even if $A - B$ and the other not specified parameters are the same, and the main difference occurs indeed near the coseismic stage. It is then necessary in general to specify the value of both the rheological parameters, and not only their difference, in order to univocally determine the evolution. Actually the case $B$=0 here considered is a very particular one: the friction reduces to a «viscous» term depending only on velocity, and, starting from stationary conditions as in the present case, the state $\theta$ is constant with time. Accordingly in the case $B = 0$, for our initial conditions, the state variable evolution is excluded *a priori*, namely the *steady state* condition is fulfilled.

Belardinelli (1994) found the exact solution for the *spring-slider* corresponding to the forcing term (4.1), in quasi static conditions, if the state can be assumed constant or, at least if its variation is such that it does not affect the system evolution, as when $A \gg kL$. This solution, denoted «ESS», will be used for testing the present numerical solution for $B = 0$ and $t > T_d$. The logarithmic law proposed by Scholz (1990) and used in MSB91 in steady state conditions is an approximation of this solution, holding for time small with respect to $T_{SS} = (A - B)/kV_S$, the transient duration in this condition. This solution will be denoted as «LSS». For the three different values of $h$ shown in fig. 4, the discrepancy between the numerical model and ESS is much smaller than between the numerical model and LSS: in the first case $r$, the ratio between the root mean square and the maximum plotted displacement, is about $10^{-5}$ (the difference between ESS and the numerical postseismic slip accordingly would not be detectable in fig. 4) whereas in the second case $r \simeq 10^{-2}$, and the discrepancy increases with time.

If $T_d$ is fixed as well as the other not explicitly varied parameters, some results by MSB91 can be summarized as follows: 1) if $k$ decreases, the «slider» starts to move late with respect to $T_d$ (when the «spring» starts to relax) and the coseismic slip tends to disappear, at the same time the postseismic slip, say the slip at one year after the earthquake, increases due to the complementarity relation (3.8); after the disappearance of the coseismic slip, for a further decrease of $k$ the postseismic slip. *i.e.* the total slip at 1 year, also decreases; 2) similar effects are produced with increasing $A - B$. If both $A - B$ and $k$ are varied through the variation of the parameter $h$ (I recall that both the parameters depends on $h$ in the MSB91 model) the transient duration $T_{SS}$ of the process at steady state varies with the square of $h$ and accordingly in steady state conditions the effects (1) and (2) strengthen very fast so that beyond a certain value of depth extension $h = h_1$ the total disappearance of the coseismic slip oc-

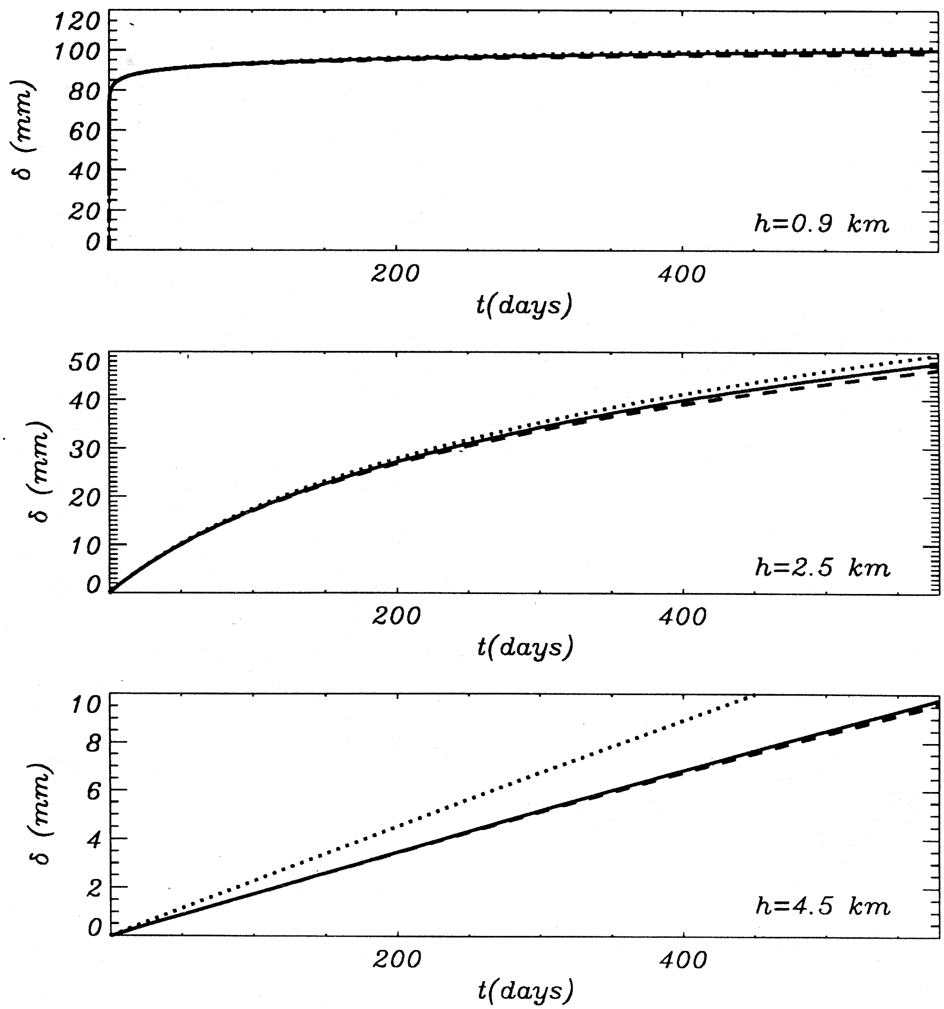


**Fig. 4.** Slip histories (solid lines) at steady state ($B = 0$) for different values of $h$, compared to the closed-form law LSS (dashed and dotted lines) for afterslip suggested by Marone *et al.* (1991). The following relations were used: $k = \mu/h$, $A = a\sigma'_n h/2$. $T_d = 0.5$ s $b = 0$ (steady state), and $a = 0.005$ (the other parameters have the values specified in eq. (4.2). In all the represented curves the coseismic displacement ($t = 0$) was computed numerically with the present model.

curs, and similarly beyond a depth extension $h_3 > h_1$ the postseismic slip after one year has decreased. In fig. 5a-f we can see in particular the effect of varying the $h$ value on the coseismic slip of the system. In each case the coseismic peak in velocity is evident, and an attendant coseismic slip occurs only in the case shown in fig. 5a where there is a small value of $h$ and then also $T_{SS}$ is short *i.e.* the system evolution is rather fast. The dependence of coseismic and postseismic slip on the parameter $h$ is summarized in fig. 6a where we may note the disappearance of the coseismic slip at a value $h_1$ comparable with that obtained by MSB91

(~ 1.2 km). The decrease in postseismic slip occurs beyond $h_3 \sim 2$ km and seems to be more drastic than in MSB91.

### 4.2. *Inertia and state evolution effects*

Figure 6a was obtained without taking into account the inertial effect, since the limiting velocity used is $V_c = 1$ m/s, a value too high to be reached by the system. If the inertial effect is enabled, keeping the same conditions used in the previous section and switching the limiting velocity $V_c$ to a lower value (fig. 6b), a total disappearance of postseismic slip occurs if the sediments thickness is less than $h_2 \sim 0.9$ km. The latter value is higher than the value declared in MSB91 (0.5 km) where inertia is not

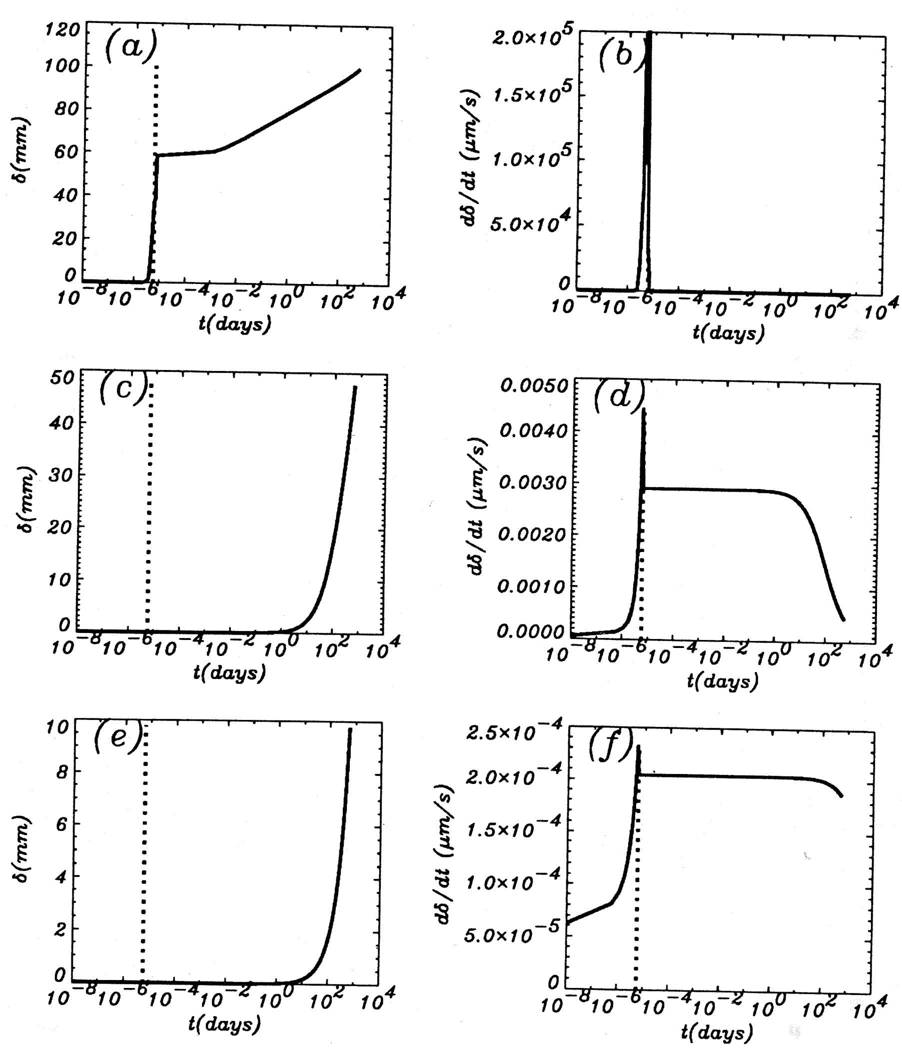


**Fig. 5a-f.** Slip and slip rate as functions of time numerically determined in the cases of fig. 4: (a,b) $h = 0.9$ km, (c,d) $h = 2.5$ km, (e,f) $h = 4.5$ km. The dotted line indicates $t = T_d = 0.5$ s.

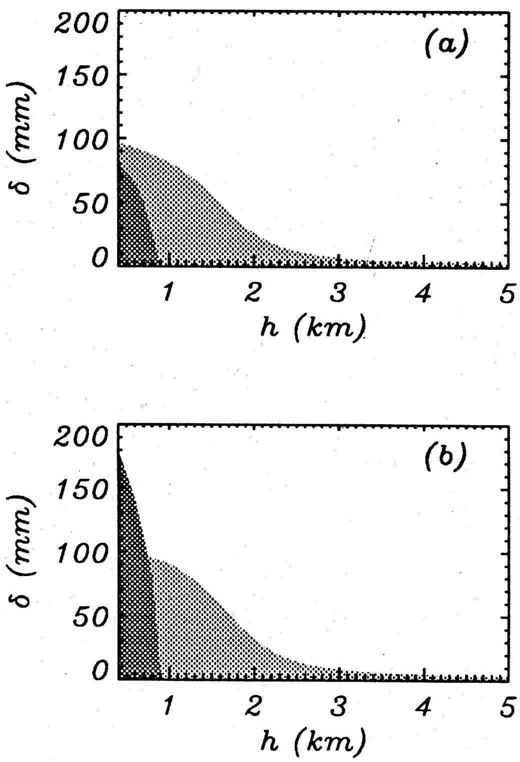


**Fig. 6a,b.** Slip $\delta$ as a function of the sediments thickness at one year after the earthquake (curve marking the upper boundary of the light gray area) and at $t = T_d$ (curve marking the upper boundary of the dark grey area). $B = 0$, $T_d = 0.5$ s, $a = 0.01$ and $T = 2\pi\sqrt{m/k} = 5$ s, (a) $V_c = 1$ m/s (non inertial evolution) and (b) $V_c = 0.1$ mm/s (inertial evolution) and $T_i = 5$ s (Rice and Tse, 1986). The other parameters have the values specified in (4.2). The load point displacement is then $\delta_{0c} = 10$ cm.

taken into account, on the contrary the thickness $h_1$ is not altered by inertia introduction. In fig. 6b the slip, and particularly the coseismic one, is greater than in fig. 6a and it is evident that for the least thicknesses represented the slip may overcome $\delta_{0c}$, due to the so-called phenomenon of *overshooting*, where the equilibrium point of the spring is overcome by the slider with compression of the spring. It was not possible to lead the computation to thicknesses less than those shown in fig. 6a,b due to the very short times that characterize the transient of the evolution in these cases, where it cannot be distinguished from a real instability. The insertion of terms of damping due to seismic radiation emission (Rice, 1993) in the tractions balance (eq. 3.1) is likely to allow a computation also at low values of $h$, by subtracting energy to the system and making its evolution slower.

In the comparison with MSB91 the state variable evolution was ignored by putting $B = 0$ (see eq. 2.2) even if in the same model this hypothesis is not made explicitly. As already hinted, if $B$ is not kept to a null value the estimate of coseismic displacement could vary with respect to what can be inferred for instance from fig. 5a-f. Due to the state variable evolution for $B \neq 0$, even if $A - B$ is the same, the system does not follow exactly the forcing term as in the case $B = 0$ when only the coseismic velocity peak at $T_d$ is evident and, as one can see from fig. 7e-f, a further peak in velocity during the postseismic phase may appear. On the base of the considerations developed in Appendix B, assuming that in the postseismic phase the slider velocity is much greater than the loading point velocity so that the latter is negligible, and that the coseismic phase occurs at steady state, one can see that in order to have a maximum in velocity during the postseismic phase, irrespective of $A - B$, it is necessary that

$$kL/B < y_c \simeq \ln (V_T/V_S), \text{ if } V_T \gg V_S \qquad (4.3)$$

which implies that $B$ must be greater than a finite value: by assuming $V_S = 2$ mm/y, $V_T = 0.2$ m/s and the effective stiffness used in MSB91, the previous condition entails $B > [1/h(\text{km})] \times 4.5 \times 10^{-2}$ bar. This «delayed» maximum is strictly linked to the state variable evolution and is closely followed by a marked minimum of $\theta$ (fig.7a,b). The instant of the maximum occurrence during the postseismic phase depends on the value of $B$, $A$ and $k$ and in particular it increases with increasing $B$ and/or increasing $A/k$. If the delayed peak occurs the coseismic peak reaches a lower, if not null value with respect to the case $B \sim 0$ with the same $A - B$. In fig. 7a-f we can observe an extreme case (for very low values of $a - b$ and $b$) where increasing $B$, with $A - B$ fixed, the coseismic peak in

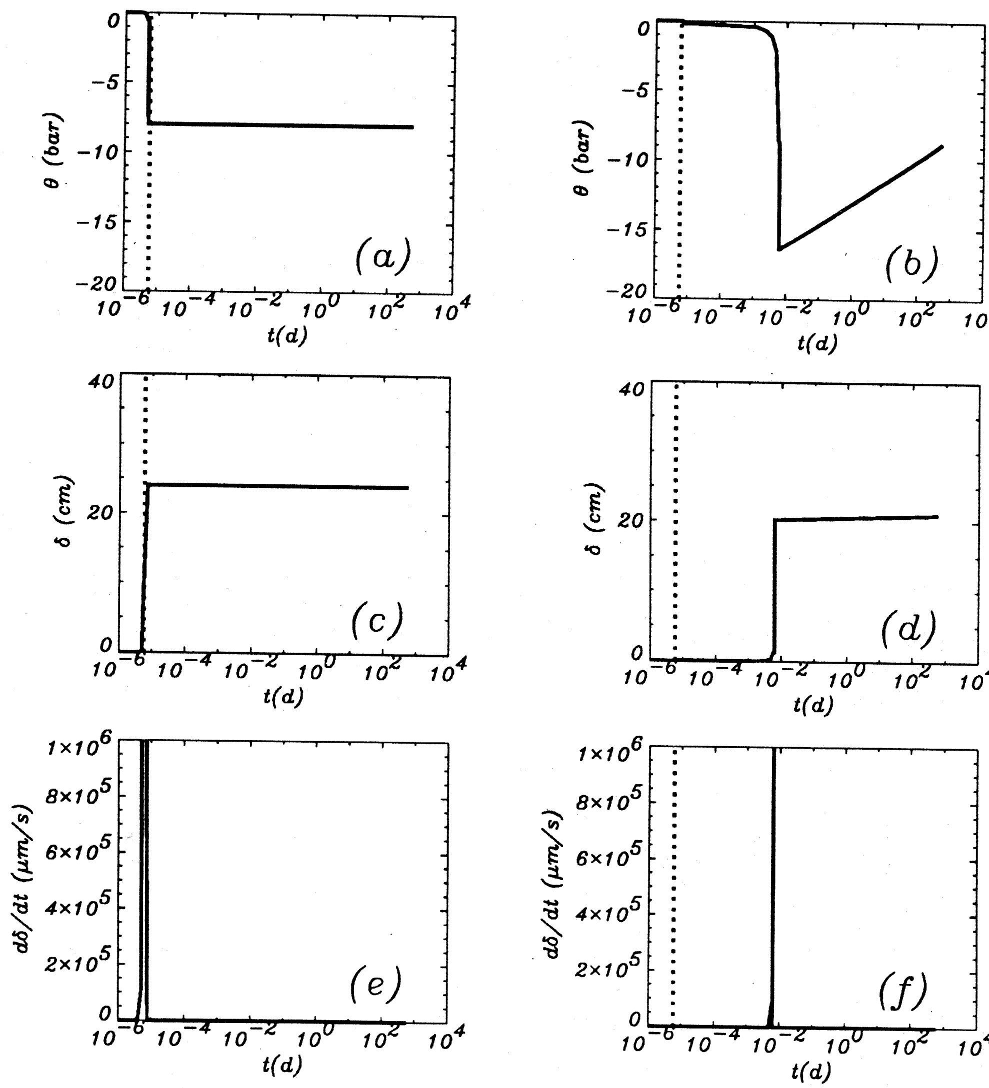


**Fig. 7a-f.** State, displacement and velocity evolution for $k = 8.3 \times 10^5$ bar/km, $T_d = 0.5$ s with $a - b = 1 \times 10^{-3}$ fixed, with varying $a$: $a = 2 \times 10^{-3}$ (a, c, e), $a = 4 \times 10^{-3}$ (b, d, f). The value of the other parameters involved is specified in (4.2).

velocity and then the coseismic displacement disappears (fig. 7d-f).

## 5. The two-dimensional model

The model with distributed displacement, *i.e.* with $n > 1$ nodes, is finally employed in order to compute the mean slip and slip rate and compare them with the results of the one-dimensional («1-D») model, where the same mean values of $A$ and $B$ were kept. The parameters $A$ and $B$ in the eqs. (2.1) and (2.2) are assumed to depend on depth whereas $L$, $\tau_*$ and $V_*$ are assumed uniform on the fault for the sake of simplicity. In the present case, we assume a uniform external traction on the fault plane and the temporal step function is simulated through two ramps with different slope, as follows

$$\dot{\tau}^{i}_{\text{ext}}(t) = \begin{cases} \dot{\tau}_T & , t < T_d \\ \dot{\tau}_S & , t \geq T_d \end{cases} \quad , \quad i = 1,2 , ..., n \qquad (5.1)$$

where $\dot{\tau}_T$ is such that the negative traction drop on the postseismically slipping section $\Delta\tau^i \equiv \int_0^{T_d} \dot{\tau}^{i}_{\text{ext}} dt$ due to an earthquake which occurred nearby, is a fraction (~ 0.05 - 0.5) of the traction drop due to the earthquake in the coseismic region (~ 100 bar).

Only if the distributions of $A$ and $B$ are uniform it is possible to obtain comparable mean values of slip and slip rate from the 1-D and the two-dimensional («2-D») models, provided that the effective stiffness $| S_{11} |$ in place of $\mu/h$ is used (Belardinelli, 1994). The same occurs if the superficial displacement is compared with the 1-D model and an effective stiffness $\mu/2h$ is used; the latter is the appropriate one for this case according to crack theory. This value is half the spring stiffness used by MSB91 and represents exactly the constant of proportionality between the maximum displacement and the uniform stress drop in an antiplane crack. I suggest that this expression of the stiffness is more appropriate than that used by Marone *et al.* (1991) if one is interested in the comparison between the model output and the superficial afterslip, at least in a crack context. For instance, the estimates of the stress drop realized aseismically through superficial afterslip can be changed by a factor of two with respect to those given by MSB91, by changing the stiffness in the mentioned way.

On the contrary, if heterogeneous profiles of $A$ and $B$ are used, the model with $n > 1$ nodes is obviously more accurate than the 1-D model and moreover the averaged slip and slip rate of the former model differ from the slip and slip rate given by the latter one. In fig. 8a we show the slip distribution on the fault plane for the profile of $A$ and $B$ represented in fig. 8b, where

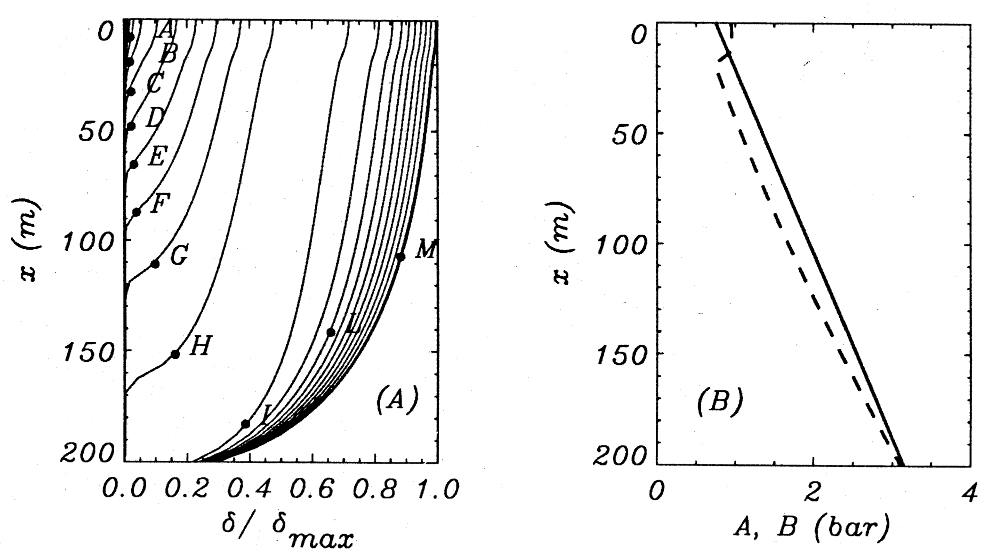


**Fig. 8a,b.** Slip snapshots (a) for the rheological vertical profile shown in (b) where $A(x)$ is the solid line and $B(x)$ is the dashed line. $n$ = 33 nodes were used, $\Delta\tau$ = 5 bar, $\dot{\tau}_S = 3.1 \times 10^{-9}$ bar/s, $L$ = 1 mm. The times of figure (a) are: 0.2 days (A), 0.3 days (B), 0.4 days (C), 0.6 days (D), 0.7 days (E), 1 day (F), 1.3 days (G), 1.7 days (H), 2.3 days (I), 3 days (L), 23-30 days (M). $Slip_{\max}$ is 22.4 mm.

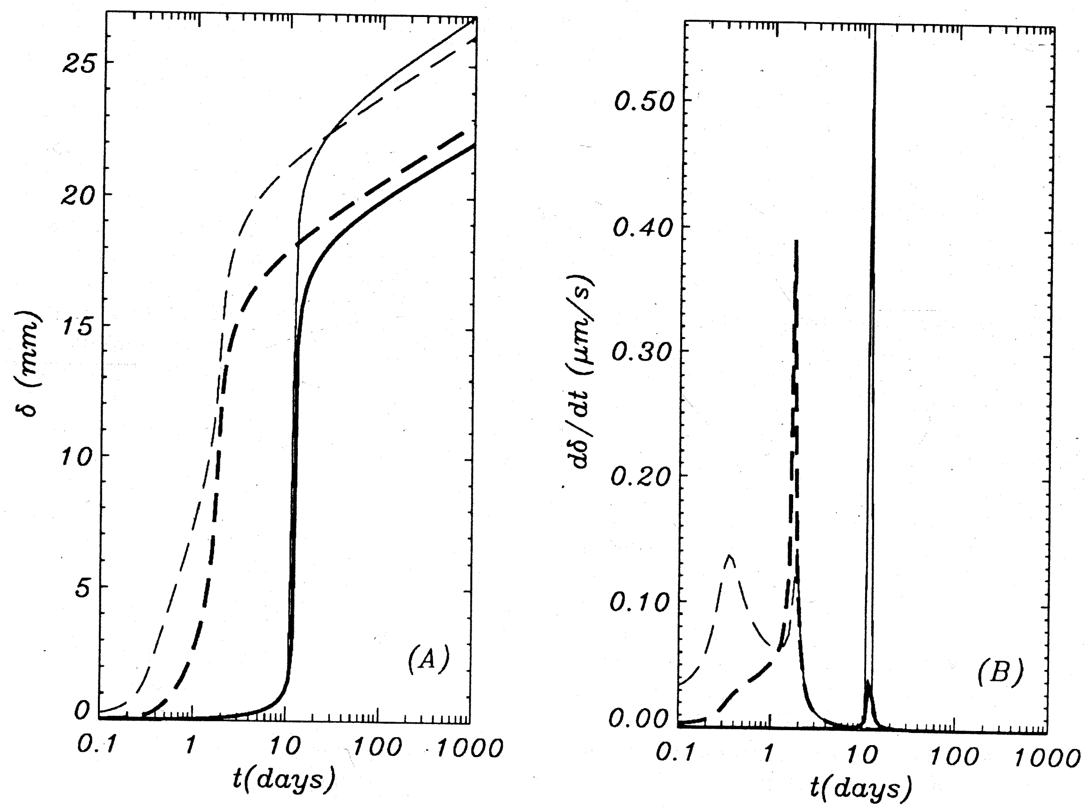


**Fig. 9a,b.** For the same case illustrated in fig. 8a,b: slip (a) and slip rate (b) at the Earth's surface (thin lines) and their averaged values on the fault plane (thick lines) with a two-dimensional model (dashed lines) and a one-dimensional model (solid lines). In the one-dimensional model the averaged slip (thick, solid line) and the superficial one (thin, solid line) are obtained with the corresponding suitable effective stiffness (see text).

a small velocity weakening section near the surface is evident and below it the rheology is velocity strengthening. This rheological profile is chosen arbitrarily but the adopted values of $A$ and $B$ are generally in agreement with those shown by Kilgore *et al.* (1993) taking into account a very low normal stress (about 5 MPa) such as that near the Earth's surface. The corresponding slip and slip rate at the Earth's surface and their mean values on the fault section as functions of time are represented in fig. 9a,b. Here they are compared with the results produced by the 1-D model with $n = 1$ and $k = |S_{11}|$, $k = \mu/(2h)$ respectively, assuming the mean values of $A(x)$ and $B(x)$. It is clear that the details of the evolution at the Earth's surface cannot be reproduced by the 1-D model even if a suitable stiffness according to crack theory is used, nor the «depth averaged evolution» on the fault is in agreement with the 1-D model, at least during the transient stage. For instance the peak in velocity due to the state variable evolution (in the 2-D model this is the second peak in the velocity at the Earth's surface, since the first one is due to the fast relaxation of the weak, shallow part of the fault) is predicted by the 1-D model always at times later by an order of magnitude with respect to the 2-D model.

## 6. Discussion and conclusions

Some proposed models of afterslip (Scholz, 1990, MSB91) assumed that the state variable can be regarded as a constant during sliding. Accordingly, in this case friction would depend only on the instantaneous velocity. The steady state *spring-slider* evolution depends on the difference $A - B$ and not on the separate value of $A$ and $B$. Even if the state varies with time but $A \gg kL$, then the system evolution depends only on the difference $A - B$ and can be regarded as equal to that for $B = 0$, $\dot{\theta}(0) = 0$, and/or $\theta(0) = 0$, provided that $A - B$ is substituted to $A$ (Belardinelli, 1994). Moreover the parameter $A - B$ is simpler to be determined experimentally than the separate values of $A$ and $B$ (*e.g.* Kilgore *et al.*, 1993), and indeed the linear stability analysis for the *spring-slider* can wholly be defined through the specification of the parameters $A - B$, $L$ and $k$. Perhaps for these reasons often in models such as MSB91 only the difference $A - B$ is specified. Nevertheless, as far as a strongly perturbed system is considered (*e.g.* Rice and Gu, 1983) the role of both rheological parameters become significant during its evolution.

The condition $A \gg kL$ for which the state evolution can be ignored since it does not affect the slider evolution, referring to values similar to those in (4.2), is equivalent to having a value of $h$ greater than about 1 km, where $h$ is the thickness of the section involved in aseismic slip or the shallow velocity strengthening layer of the sediments. In general, if the previous condition is not met, as during transient processes in thin fault sections (*creep events, e.g.* Wesson, 1988), the state evolution is meaningful and can lead to a delayed effect, such as the velocity peak appearance during the postseismic phase. In this case the whole set of eqs. (2.1) (2.2) in the Ruina friction law has to be taken into account and a model of such a kind of process could in principle give some constraints on the parameter values in this constitutive equation. If the state variable evolution is not properly taken into account, coseismic quantities such as slip and slip-rate may be overestimated, above the real values (fig. 7a-f). At the same time, the inertial term of the evolving equation can also play an important role in the estimates of coseismic quantities (see MSB91) and if this term is not taken into account, they may be underestimated (fig. 6a,b).

The steady state assumption, on the contrary, is valid for the afterslip processes suggested by the common use of the term, where the sliding section can be expected to extend within the first kilometers of sediments (see *e.g.* Crook, 1984). Then for the afterslip processes the evolution does not differ from what can be obtained with a «viscous» constitutive law, logarithmic in velocity, which some weeks after the earthquake can be well reproduced by a linear relation between slip rate and resistant traction, as that used in Belardinelli and Bonafede (1991, 1994).

Moreover, as seen at the end of the previous section, the study of transient creep on shallow faults where the state evolution may be significant, especially if a heterogeneous distribution of the rheological parameters is assumed, cannot be well approximated by a one-dimensional model even in a depth-averaged sense. Previously, a two-dimensional study of nucleation processes was compared with the results of a one-dimensional or «patch» model (Dieterich, 1992), and a good agreement was found, however, in that case only random heterogeneities in the initial state of traction were considered. Since rheological heterogeneities can be expected in real fault surfaces (*cfr. e.g.* Boatwright and Cocco, 1994; Rice, 1993), the analysis of a relaxation process on a shallow fault section should rely upon at least a two-dimensional model in order to correctly estimate the transient features of the process.

In conclusion: a two-dimensional numerical algorithm for the study of the aseismic slip on a shallow strike-slip fault was presented and in particular a non linear friction law was considered. The method consists of a spatial discretization of the fault plane in several «nodes». When used with only one node and with a homogeneous distribution of traction on the crack surface, it directly supplies the mean displacement on the fault plane as in a one-dimensional model of fault. The current results are in agreement with the one-dimensional model of Marone *et al.* (1991) as long as the state variable evolution is neglected a priori and friction depends only on velocity. In general if the state variable is allowed to evolve, the latter condition is not fulfilled unless the slipping section extends down to several kilometers in depth. If the state variable evolution is not negligible, then for instance the coseismic slip is lower with respect to steady state conditions, due to the fading memory effects induced by the same state variable. Moreover, the difference between a one-dimensional and a two-dimensional model are shown in the presence of rheological heterogeneities. In this case the former cannot reproduce the results of the latter, nor as far as depth averaged variables are concerned. Accordingly, in order to correctly simulate the evolution of fault sections shallower than 1 km depth, a two-dimensional model, possibly taking into account the inertial effects, is suggested.

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## Appendix A

In this appendix I illustrate the numerical solution to the set of eqs. (3.3) where $\tau_{\text{res}}$ is expressed by some constitutive equation in terms of $\delta$ and $\dot{\delta}$. If

$$\tau_a^i(t) \equiv \sum_{\substack{j=1 \\ j \neq i}}^{n} S_{ij}\, \delta^j(t) \tag{A.1}$$

were known in each node, eq. (3.3) would be reduced to a set of first order decoupled differential equations for the slip $\delta^i$ at each node and from the first of (3.3) (in the following referred as *constitutive equation*) we would obtain the slip and the friction traction at a certain instant of time in each point of the fault. Actually, in the current problem $\tau_a^i(t)$ is not known a priori and it must be determined together with $\tau_{\text{dis}}^i$ taking into account the second equation of (3.3), here called *boundary integral equation*. This can be done following the method adopted by Dieterich (1992) and based on a cycle of iterations, shown in fig. A1.

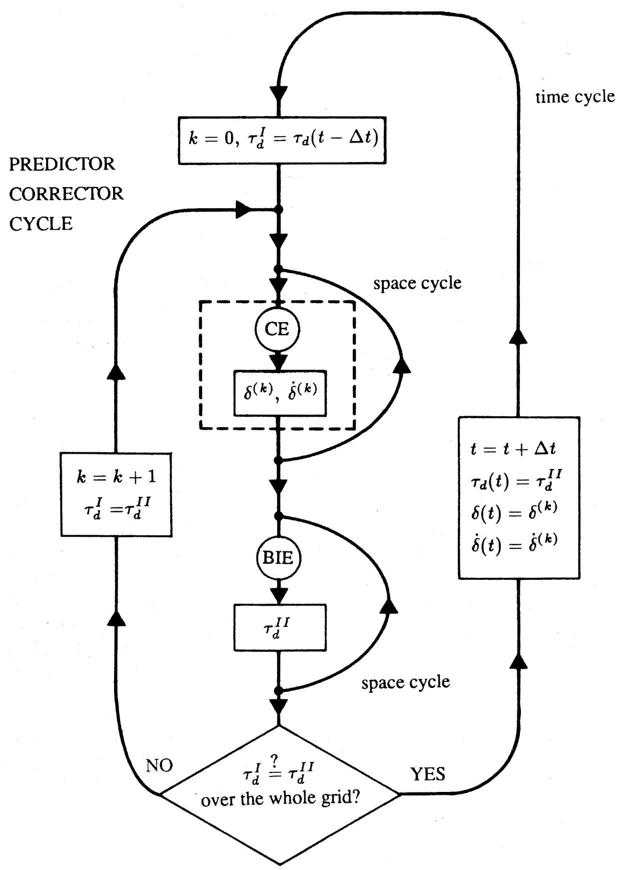


**Fig. A1.** The *predictor-corrector* cycle used to solve the set of equations (3.3). «CE» = constitutive equation; «BIE» = boundary integral equation. $k$ is the counter index of the «predictor-corrector» cycle.

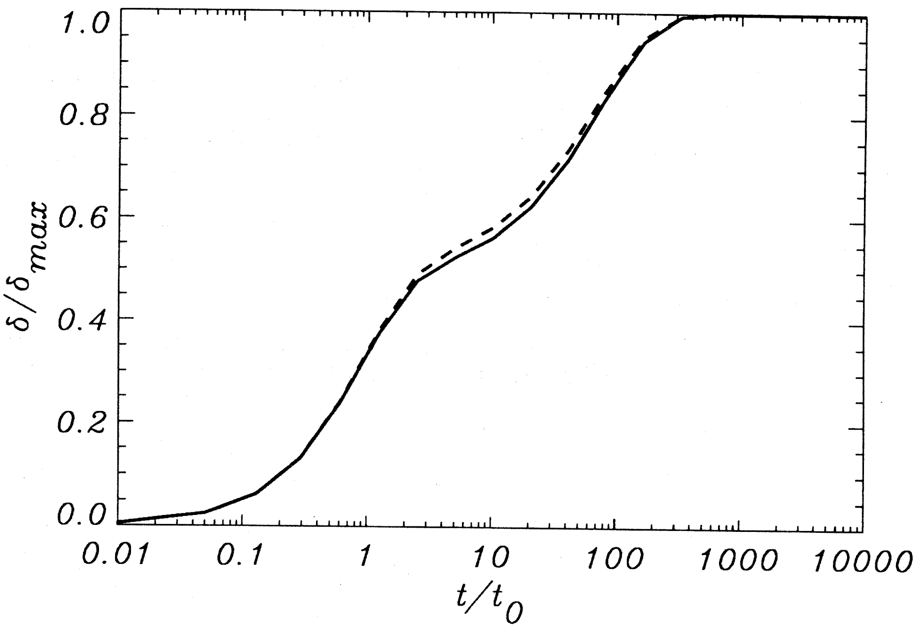


**Fig. A2.** Comparison between the numerical model with $n$ = 33 nodes (solid line) and the semi-analytical model (dashed line) by Belardinelli and Bonafede (1991) with a truncation order $M$ = 90, for the post-seismic superficial slip with $\delta_{max}$ as a final displacement. A contrast of viscosity of two orders of magnitude passing from the superficial half part of the fault (with low viscosity) to the deeper one, is assumed. Time is scaled by means of a characteristic time $T = 4\pi\alpha\mu/a$, where $\alpha$ is the constant of proportionality between friction and slip-rate and scales with the mean viscosity on the fault.

The method followed is called *predictor-corrector* because it starts with a trial prediction on the value of the self-traction at a certain instant of time $\tau^{iI}_{dis}$, $i$ = 1, ... $n$, from this the value $\tau^{i}_{a}$, is obtained and put in the constitutive equation (dashed block in fig. A1) in order to get the slip at the first ($k$ = 1) iteration, the latter is substituted in the boundary integral equation in order to obtain a second estimate of the self-traction $\tau^{iII}_{dis}$, $i$ = 1, ... $n$. If the second estimate does not coincide with the initial estimate, the latter becomes the initial estimate for the subsequent iteration ($k = k + 1$). The iteration cycle is left when $\tau^{iI}_{dis}(t) = \tau^{iII}_{dis}$, $i$ = 1, ... $n$ within a certain precision, then the variable values are up-to-date and the algorithm goes to the subsequent instant of time.

This kind of solution can obviously be used with constitutive equations different from (2.1) (2.2), by simply changing the dashed block in fig. A1. If, for instance, in the first of (3.3) a viscous friction is considered as in Belardinelli and Bonafede (1991)

$$\tau^{i}_{res}(t) \propto \eta^{i}\,\dot{\delta}^{i}(t), \quad \text{if} \quad \tau^{i}_{ext}(t) + \tau^{i}_{dis}(t) > Y^{i} \tag{A.2}$$

where $Y^i$ is a threshold traction for sliding, then the structure of the dashed block is quite evident: the slip rate is obtained from the first of (3.3) and then the slip is found, by trivial temporal integration. In order to text the algorithm I tried to reproduce the results by Belardinelli and Bonafede (1991) obtained by series expansion on Chebychev polynomials of the spatial part of the slip solution. The comparison between this semi-analytical method and the current one in terms of superficial slip is shown in fig. A2. The comparison seems to be satisfactory: the discrepancies near the curve «shoulder» being reducible by increasing the truncation order of the expansion in the former model.

The algorithm for $n = 1$ comes down to a twice iterated execution of the dashed block in the *predictor-corrector* loop. The block in this case represents the solution of the constitutive equation at each node or the quasi-static equation of the *spring-slider* (since $\tau_a^i$ is null). The predictor-corrector cycle is executed $k = 2$ times for each instant of time due to the fact that the value of $\tau_{\text{dis}}(t)$ is brought to up-to-date. The program solving the *spring-slider* evolution and the algorithm for $n = 1$ differ in that two iterations of the predictor-corrector loop are effected at each instant of time in the latter case. When the constitutive equations (2.1) (2.2) were considered, the program for the *spring-slider* evolution with non linear frictional law by Boatwright and Cocco (1994) was used as a subroutine of the current one. In the former program the numerical integration of the set of first order differential equations is made through a step adaptive Runge-Kutta algorithm (subroutine RKQC in Press *et al.*, 1989).

## Appendix B

In this appendix we will obtain a necessary condition for a maximum in the *spring-slider* velocity during the postseismic phase $t > T_d$, under the assumption that $\dot{\delta}$ is much greater than the loading velocity $V_0$ (assumed constant), and that the coseismic phase occurs at steady state. It is possible to show (Belardinelli, 1994) that in quasi-static conditions if $\dot{\delta} \gg V_0$ and $V_0$ is assumed constant, the locus of the maximum occurrence along each trajectory in the plane (ln $w$, $y$), where

$$y \equiv \frac{\tau_{\text{res}} - \tau_* - \tau^{SS}}{A} \qquad w \equiv \frac{\dot{\delta}}{V_0} \tag{B.1}$$

is given by

$$y = \frac{kL}{A} + \frac{A-B}{A} \ln w\,. \tag{B.2}$$

Indeed from (2.1) (2.2), substituting the definitions (B.1) and taking into account that in quasi static conditions (*cfr.* Belardinelli, 1994) $\theta = (y - w) \ln\left(\frac{V_0}{V_*}\right)$, one has

$$\dot{y} = \frac{kV_0}{A}(1-w) \qquad \dot{y} = \frac{\dot{w}}{w} - \frac{V_0}{L} w\left[y + \left(\frac{B}{A} - 1\right) \ln w\right] \tag{B.3}$$

and then equating if $\dot{\delta} \gg V_0$, and then $w \gg 1$

$$\frac{\dot{w}}{w} = -\frac{kV_0}{A}w + \frac{V_0}{L} w\left[y + \left(\frac{B}{A} - 1\right) \ln w\right] \tag{B.4}$$

from which the (B.2) for $\dot{w} = 0$.

Assuming that the coseismic transition occurs at constant state the point representative of the system moves along the locus of constant state of the plane ($\ln w$, $y$) (see Rice and Gu, 1983)

$$y = \ln w + \text{cost}\,, \tag{B.5}$$

in the direction of increasing $w$. After this transition, only if the point is above the maximum locus (see fig. 9a,b in Rice and Gu, 1983) then along the subsequent trajectory a maximum in slip rate will occur. If the point starts from the stationary state ($\ln w = y = 0$ at $t = 0$) then the constant in (B.5) is null and the condition to have a maximum in the postseismic phase is that at $t = T_d$

$$\ln w > \frac{A-B}{A} \ln w + \frac{kL}{A}\,, \tag{B.6}$$

or, putting $y_c = \ln w_c$, where $w = w_c$ meets the previous condition,

$$y_c > kL/B\,, \quad \text{where} \quad y_c \equiv (\tau_{\text{res}}(T_d) - \tau_{SS} - \tau_*)/A \sim \ln (V_T/V_S) \quad \text{if} \quad V_T \gg V_S \tag{B.7}$$

which is eq. (4.3) of the text. In the previous equation the approximate expression of the adimensional coseismic friction traction $y_c$ is taken from the analytical solution developed by Belardinelli (1994).