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Vertical electron density profiles from digisonde ionograms. The average representative profile

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Abstract
Profile calculations from ionograms using the Huang-Reinisch technique arrive at a set of boundary values and coefficients that describe the profile. From an ensemble of such sets an Average Representative Profile (ARP) is derived which is again expressed in terms of boundary values and coefficients.

Key words *electron density profile – true height – ionospheric modeling*

1. Introduction

Vertical electron density profiles for the bottomside ionosphere are the one output of ionosondes that is equally important for geophysics and for radio wave propagation applications. Advanced digital ionosondes like the Digisonde 256 (Reinisch *et al.*, 1989) and the Digisonde Portable Sounder (Haines, 1994) generate electron density profiles in real time at some 50 stations worldwide (fig. 1). Section 2 briefly reviews the Digisonde profile inversion technique and section 3 introduces a robust procedure that generates «Average Representative Profiles» (ARPs). Averaging may be done for a week or a month of profiles at a given hour, or it may be done for several profiles obtained within a short time interval, or for profiles simultaneously obtained at several moderately spaced ionosondes.

2. Review of the digisonde profile inversion technique

The profile for each ionospheric layer is expressed in the form (Huang and Reinisch, 1982; Reinisch and Huang, 1983)

$$h = h_m + \sqrt{g} \sum_{i=0}^{I} A_i T_i^* (g) \qquad (2.1)$$

where h_m is the layer peak height, I equals 4 for the F layers and 2 for the E layer, T_i^* are the shifted Chebyshev polynomials, and

$$g = \frac{\ln (f_N / f_m)}{\ln (f_s / f_m)} \qquad (2.2)$$

where $f_N/\text{Hz} = 9\sqrt{(N/\text{m}^{-3})}$ is the plasma frequency, f_s and f_m are the starting and critical

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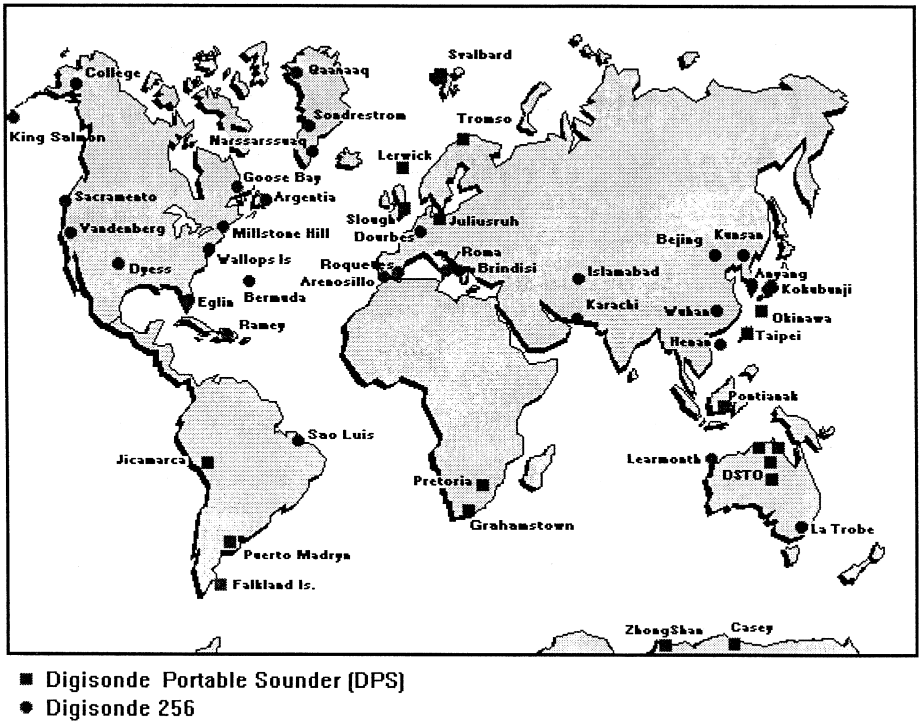

Fig. 1. Global digisonde network.

frequencies of the layer. The layer «half-thickness» is therefore,

$$h_m - h_s = -\sum_i A_i. \tag{2.3}$$

The inversion program NHPC outputs the boundary values f_s, f_m, h_m, and the coefficients A_0, A_1, ..., A_4 for each layer. A program THTABLE is available to calculate $N(h)$ from the boundary values and the coefficients. Program NHPC has been extensively tested by comparing thousands of ionogram-derived profiles with profiles from collocated incoherent scatter radar measurements (Chen *et al.*, 1991). Figure 2 shows an example from Millstone Hill. The next section describes how the boundary values and coefficients are applied to define the average representative profile.

3. Average Representative Profile (ARP)

There is no unique way of defining «the» average profile, and the new ARP technique is merely a robust, unbiased procedure that arrives at a representative profile, given the boundary values and coefficients for a set of ionograms. One month of Millstone Hill profiles for January 1990 are used to illustrate the new technique.

Each of the K F-layer profiles $h_k(f_N)$ displayed in fig. 3 is described by equations of the form:

$$h_k = h_{mk} + \sqrt{g} \sum_{i=0}^{4} A_{ik} T_i^* (g)$$

$$k = 1, 2, ..., K \tag{3.1}$$

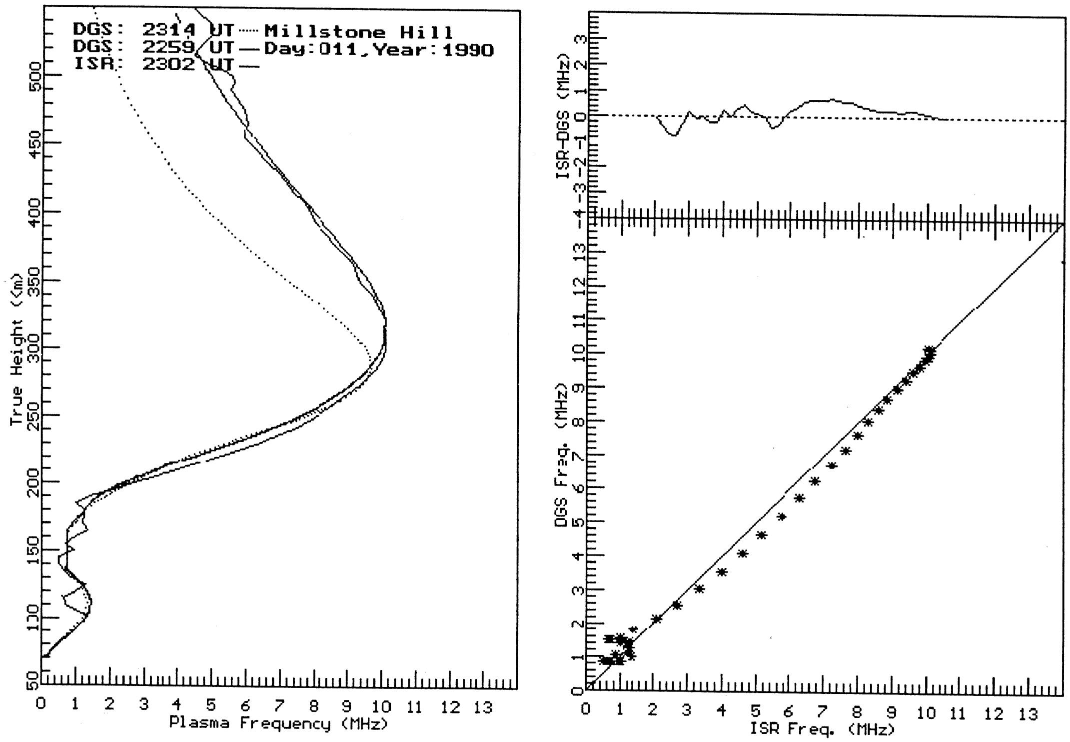

Fig. 2. Profile comparison at Millstone hill, digisonde *vs.* incoherent scatter radar profiles.

where K is the total number of profiles over which the average is taken. The average representative profile is to be represented in the same form:

$$\mathrm{ARP} = h = h_m + \sqrt{g} \sum_{i=0}^{4} A_i T_i^* (g) \qquad (3.2)$$

with the restraint given by (2.3)

$$h_s = h_m + \sum_{i=0}^{4} A_i \qquad (3.3)$$

(I was set to 4, assuming an F layer). The ob-

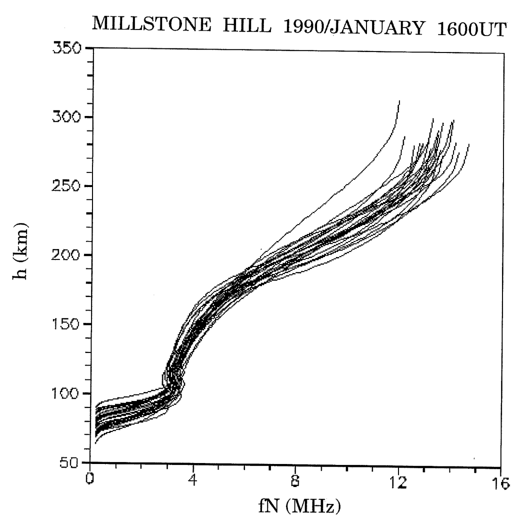

Fig. 3. Monthly set of profiles at Millstone Hill for January 1990, 1600UT (1100LT).

jective now is to find representative values f_s, h_s, f_m, h_m and the coefficients A_i. For the starting and maximum heights, h_s and h_m, we chose the medians of h_{sk} and h_{mk}; and for the starting and critical frequencies, f_s and f_m, we choose the medians of f_{sk} and f_{mk}. In fig. 4, the F-layer profiles are plotted *vs.* g rather than f_N and the medians $h(1) = h_s$ and $h(0) = h_m$ are shown as black dots. It now remains to determine the coefficients A_i so that eq. (3.2) «optimally» represents all profiles and the restraint (3.3) is satisfied. A least squares technique with a Lagrangian multiplier is used to find the A_i. Defining the function

$$F = \sum_{k=1}^{K} \int_0^1 dg \, .$$

$$\left[h_m + \sqrt{g}\sum_i A_i T_i^*(g) - h_{mk} - \sqrt{g}\sum_i A_{ik} T_i^*(g)\right]^2$$

$$+2K\lambda\left[h_m - h_s + \sum_i A_i\right] \qquad (3.4)$$

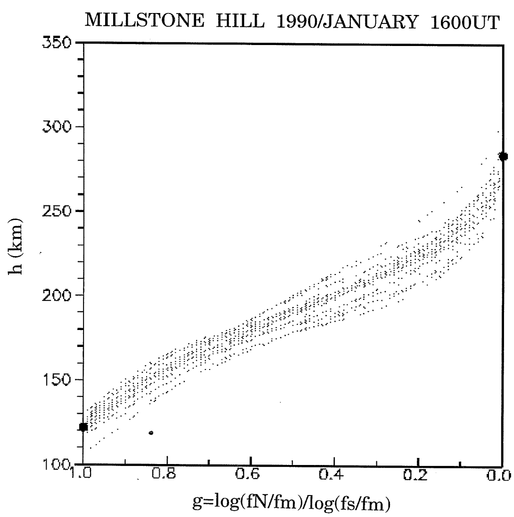

Fig. 4. F-region profiles as function of $g = \ln(f_N / f_m) / \ln(f_s / f_m)$.

the conditional minima are found from

$$\partial F / \partial A_j = 0, \qquad j = 0, 1, \ldots, 4 \qquad (3.5)$$

and

$$\partial F / \partial \lambda = 0. \qquad (3.6)$$

These are six equations which can be solved for the six unknowns λ, A_i:

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ \lambda \end{bmatrix} = D^{-1} \begin{bmatrix} \frac{1}{K}\sum_{k=1}^{K}(h_{mk} - h_m)B_0 + \frac{1}{K}\sum_{k=1}^{K}\sum_{i=0}^{4} D_{0i}A_{ik} \\ \cdots \\ \cdots \\ \frac{1}{K}\sum_{k=1}^{K}(h_{mk} - h_m)B_4 + \frac{1}{K}\sum_{k=1}^{K}\sum_{i=0}^{4} D_{4i}A_{ik} \\ h_s - h_m \end{bmatrix} \qquad (3.7)$$

where

$$B_j = \int_0^l \sqrt{g} T_j^*(dg) \qquad (3.8)$$

$$D_{ij} = \int_0^l g T_i^*(g) T_j^*(g) \, dg = D_{ji} \qquad (3.9)$$

are constants, and D^{-1} is the inverse of the matrix D. The ARP function for the given profile set can now be calculated from (3.2). The representativeness of the ARP function is illustrated in figs. 5 and 6, where the thick lines are the ARP functions plotted *vs.* g and f_N, respectively. After applying this process to the E and F layers the complete ARP profile can be constructed.

We have «ARPed» the hourly profiles for January 1990 from Millstone Hill to analyze the diurnal variation of the electron distribution. Figure 7 shows selected ARPs at night, morning midday and evening. The hours on the curves are given in local time.

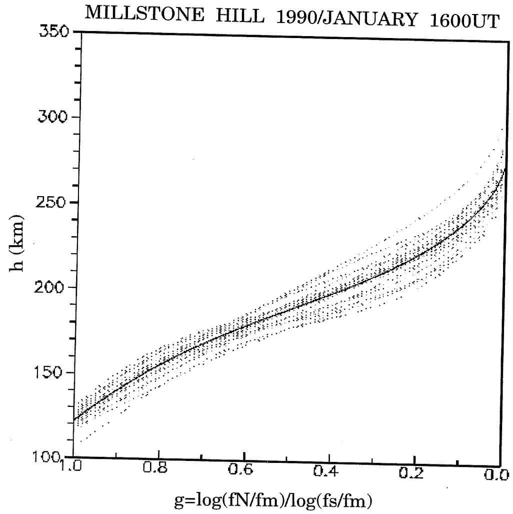

Fig. 5. Same as fig. 4 with ARP indicated by heavy line.

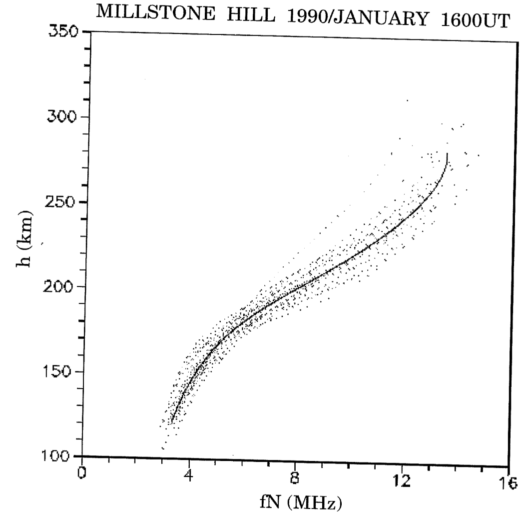

Fig. 6. Same as the F layers in fig. 3 with ARP indicated by heavy line.

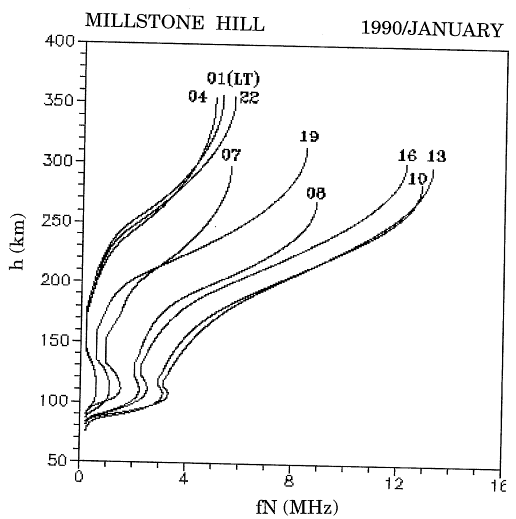

Fig. 7. Diurnal variation of the Monthly Average Representative Profiles (MARPs) for January 1990 at Millstone Hill.

4. Conclusions

An average representative profile can be determined for a set of profiles that are specified by their end and starting points (f_{mk}, h_{mk}), (f_{sk}, h_{sk}) and coefficients A_{ik}. The ARP function itself is defined by the boundary values (f_m, h_m), (f_s, h_s) and the coefficients A_0, ..., A_4, for each layer, offering a convenient yet well defined and robust method of representing average profiles. It would be very easy to calculate ARP functions for different levels of magnetic activity by dividing the set of input profiles into groups for different activity levels. This technique is applicable to ionogram data from all type ionosondes as long as the profile inversion uses the digisonde technique. Program NHPC, Version 3.02 accepts $h'(f)$ trace data from analog and digital ionosondes, the object code is available from the authors. The program CARP which calculates the ARP function can also be obtained from the authors.

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