

1 **Linear vs nonlinear methods for detecting magnetospheric and**
2 **ionospheric current systems patterns**

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6 **Key Points:**

- 7 • Both linear and nonlinear methods are used to investigate magnetic field patterns
8 obtained by Swarm data
9 • The EOF analysis does not allow to extract non-oscillating components
10 • The MEMD allows to detect both oscillating (of ionospheric origin) and non-oscillating
11 (of magnetospheric origin) contributions

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Abstract

There is a growing interest in the development of models and methods of analysis aimed to recognize in the geomagnetic field signals the different contributions coming from the various sources both internal and external to the Earth. Many models describing the geomagnetic field of internal and external origin have been developed. Here, we investigate the possibility to recognize in the magnetic field of external origin the different contributions coming from external sources. We consider the measurements of the vertical component of the geomagnetic field recorded by the ESA Swarm A and B satellites at low- and mid-latitude during a geomagnetically quiet period. We apply two different methods of analysis: a linear method, i.e., the Empirical Orthogonal Function (EOF), and a nonlinear one, i.e., the Multivariate Empirical Mode Decomposition (MEMD). Due to the high nonlinear behavior of the different external contributions to the magnetic signal the MEMD seems to recognize better than EOF the main intrinsic modes capable of describing the different magnetic spatial structures embedded in the analyzed signal. By applying the MEMD only 5 modes and a residue are necessary to recognize the different contributions coming from the external sources in the magnetic signal against the 26 modes that are necessary in the case of the EOF. This study is an example of the potential of the MEMD to give new insights into the analysis of the geomagnetic field of external origin and to separate the ionospheric signal from the magnetospheric one in a simple and rapid way.

1 Background

The Earth's magnetic field results from different sources, both internal and external with respect to the solid Earth. The largest part of the magnetic field is of internal origin (the so-called *main field*), being mainly due to a self-sustaining hydrodynamic dynamo operating in the Earth's fluid outer core, and only for a small part to the magnetized material in the crust. In addition to the internal field, there is the magnetic field generated by electric currents flowing in the ionosphere and the magnetosphere, called *external field*, whose strength ranges from less than one to some thousands of nT, according to different geomagnetic activity levels and latitudes. Lastly, in order to have an overall view of the different sources of the Earth's magnetic field we have to consider the magnetic fields generated by the electric currents in the crust and mantle, which are induced by the time-varying main and external fields. Similar induced currents can be also found within the salty waters of the oceans, which produce weak magnetic fields of the order of a few nanotesla at ground level [Baumjohann and Nakamura, 2009].

Of course when we make a measurement of the Earth's magnetic field on the ground or from a satellite in low Earth orbit it will collect the contributions from all the different examined sources, both internal and external to the solid Earth. For this reason, the recognition of individual contributions to the overall geomagnetic field is quite challenging. In recent years, there has been an increasing interest in the development of geomagnetic field models of increasing complexity and accuracy based on the combined analysis of both ground-based observatory magnetic measurements and data derived from several satellite missions. Among these models we mention GRIMM (it is an acronym for the GFZ Reference Internal Magnetic Model) [e.g., Lesur *et al.*, 2010], POMME (POTsdam Magnetic Model of the Earth) [e.g., Maus *et al.*, 2006], CHAOS (CHamp, Ørsted and Sac-C data) [e.g., Finlay *et al.*, 2017; Olsen *et al.*, 2014] and the well-known series of "Comprehensive Models" (CMs) [e.g., Sabaka *et al.*, 2002, 2004, 2015]. They are capable of adequately representing the different (internal and external) sources. In principle, these models were born with the goal of providing an accurate representation of the internal field, but very quickly it was clear that to push them to higher spatial and temporal resolution it was necessary to constrain at best also the magnetic field of external origin. Thus, the study of the external field is of cross-interest to the scientific community. For scientists working on

63 the core and crustal fields the contribution of the external field is unwanted, and represents
 64 essentially a source of noise which is useful to characterize [see, e.g., *Finlay et al.*, 2017;
 65 *Kunagu et al.*, 2013; *Maus and Lühr*, 2005]. At the same time, for scientists working on
 66 ionosphere and magnetosphere, the external field is of central interest, and permits the in-
 67 vestigation of processes involving small magnetic strengths but fast timescales with respect
 68 to the dominant contribution represented by the internal field. Different methods have
 69 been developed and used to study the spatial and temporal structure of the ionospheric
 70 and magnetospheric current systems at various latitudes, which are the sources of external
 71 fields. Standard methods, such as spherical harmonic analysis (SHA) or spherical elemen-
 72 tary current systems (SECs) [*Amm*, 1997; *Amm and Viljanen*, 1999], have been introduced
 73 to reconstruct the complex spatial and temporal features of these currents, but they have
 74 not often been capable of reproducing realistic current systems due to *a priori* constraints,
 75 the use of fixed basis functions, and intrinsic limitations caused by the unavailability of
 76 data.

77 In this paper we investigate the capabilities of two different methods of analysis to
 78 recognize and characterize the various sources responsible of the generation of the mag-
 79 netic field of external origin recorded at low and mid magnetic latitudes. To this aim, we
 80 analyzed the magnetic data acquired by two of the satellites of the Swarm constellation
 81 [see, e.g., *Friis-Christensen et al.*, 2006] spanning two years at 1 Hz cadence. We used
 82 the CHAOS-6 geomagnetic field model [*Finlay et al.*, 2017; *Olsen et al.*, 2014] to remove
 83 from the observed data the main field and its secular variation, so to obtain in the residual
 84 signal the geomagnetic field of external (magnetospheric and ionospheric) origin. We ap-
 85 plied to the obtained external magnetic field both the empirical orthogonal function (EOF)
 86 analysis [*Ghil et al.*, 2002] and the multivariate empirical mode decomposition (MEMD)
 87 method [*Rehman and Mandic*, 2010]. The aim is to extract from the analyzed signal the
 88 main intrinsic modes describing the different magnetic spatial features inside it. We recog-
 89 nize in the various intrinsic modes the different ionospheric and magnetospheric contribu-
 90 tions and compare the results from the two different methods in order to find the method
 91 that is capable of recognizing better the structures present in the analyzed signal.

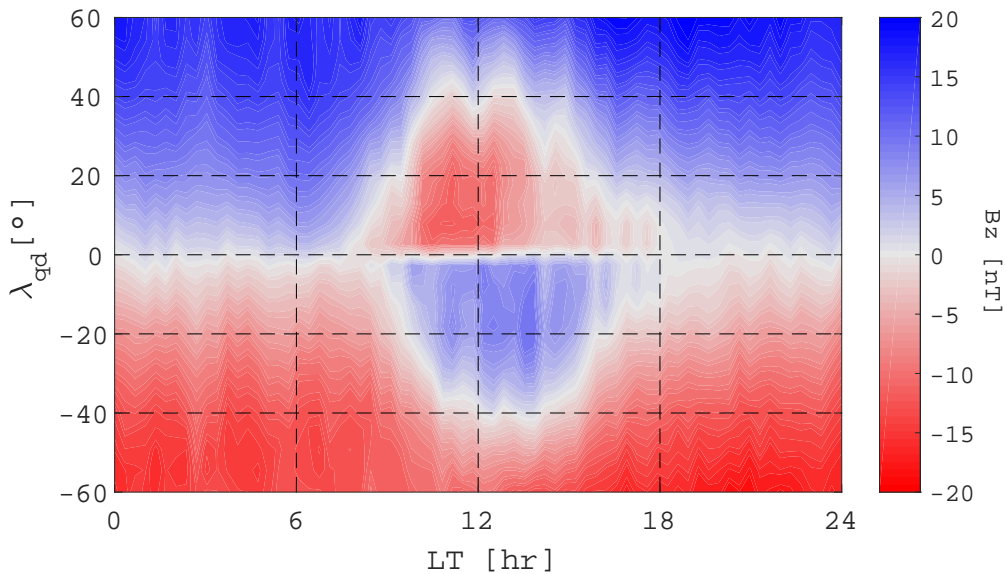
92 The paper is organized as follows. Section 2 is dedicated to the description of the
 93 analyzed dataset, while in Section 3 we illustrate the two different chosen methods (EOF
 94 and MEMD) and their applications. Finally, in the last Section we summarize the main
 95 findings and discuss the obtained results comparing the two different methods.

96 2 Data description

97 We used Level-1b low resolution (1 Hz) vector magnetic field data recorded on
 98 board of two of the three satellites of the Swarm constellation [see, e.g., *Friis-Christensen*
 99 *et al.*, 2006]. In detail, we considered data recorded by Swarm A satellite during a period
 100 of two years from 1 April 2014 to 31 March 2016, and, data recorded by Swarm B satel-
 101 lite for comparison. During this time interval the Swarm A (B) satellite flew around the
 102 Earth at an altitude of about 460 (510) km thus exploring the F-region of the ionosphere.
 103 Data are freely available at <ftp://swarm-diss.eo.esa.int> upon registration.

104 We analyzed the vertical component of the geomagnetic field (B_z , being measured
 105 inward to the Earth's surface) at low- and mid-latitudes (within $\pm 65^\circ$ magnetic latitude)
 106 recorded during periods characterized by very low geomagnetic activity levels, which were
 107 selected using simultaneously two different geomagnetic indices: *AE* [*Davis and Sugiura*,
 108 1966] and *SYM - H* [*Iyemori*, 1990]. In particular, we considered the following simulta-
 109 neous conditions: $AE < 80$ nT and $-10\text{nT} < SYM - H < 5\text{nT}$ that permitted us to select
 110 periods where the magnetic disturbances due to storm and substorm events were excluded.
 111 *AE* and *SYM - H* data with one minute time resolution were downloaded from the OMNI
 112 website (www.cdaweb.gsfc.nasa.gov/ftp-public/).
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113 As the main target of this work is to characterize the geomagnetic field of external
 114 origin and its spatial structure, we removed the internal geomagnetic field from the origi-
 115 nal data recorded by Swarm A (B) by using the CHAOS-6 model [Finlay *et al.*, 2017].
 116 It is the latest generation of the CHAOS series of global geomagnetic field models intro-
 117 duced by Olsen *et al.* [2006, 2010, 2014]. It is derived from Swarm, CHAMP, Østered
 118 and SAC-C satellite magnetic data and ground observatory data, respectively. It is able
 119 to estimate the internal geomagnetic field with high resolution in time and space. It in-
 120 cludes a parametrization of the quiet-time, near Earth magnetospheric field due to ring
 121 current, magnetotail, and magnetopause currents but it doesn't take into account the con-
 122 tribution coming from the ionospheric currents. In order words, CHAOS-6 does not model
 123 all the sources of external origin in representing the geomagnetic field potential, but only
 124 the magnetospheric ones. To remove from our data the internal field we have used the
 125 CHAOS-6 geomagnetic model up to the spherical harmonic degree $N=110$. We binned
 126 data into 5×5 degree-sized square bins across the Earth's surface after conversion to quasi-
 127 dipole (QD) latitude (λ_{qd}) and local time (LT). We used the QD coordinates reference
 128 system [Richmond, 1995] mainly for two reasons: i) with respect to orthogonal systems it
 129 captures the features (and the distortions) at all latitudes, and is well defined everywhere
 130 [Emmert *et al.*, 2010]; and ii) with respect to other nonorthogonal systems, due to its de-
 131 pendence on the geodetic altitude it is very useful for magnetically localized phenomena
 132 with a specific height distribution, such as the current systems confined in the conduct-
 133 ing layer of the ionosphere [Laundal and Richmond, 2016]. Moreover, we considered the
 134 LT to better visualize the effects on the geomagnetic field due to the dynamical processes
 135 affecting the magnetosphere-ionosphere system.



136 **Figure 1.** Global map of the vertical to surface component of the geomagnetic field in the λ_{qd} -LT plane
 137 as computed from Swarm A observations during a period of two years from 1 April 2014 to 31 March 2016.
 138 Data refers to a geomagnetically quiet period ($AE < 80$ nT and $-10\text{nT} < SYM - H < 5\text{nT}$).

139 Figure 1 shows the λ_{qd} vs. LT global map of the geomagnetic field of external
 140 origin along the \hat{z} (vertical) component computed from Swarm A observations. The mapped
 141 values are the average values falling within each bin (5×5 degree-sized square bin). The
 142 minimum bin population is 3009, the maximum is 10025, and less than 5% of all the bins
 143 is populated with less than 4000 data points. Thus, each bin of our two years-long obser-

144 vations is adequately populated, and the statistics is robust enough to make the average as
 145 representative of each data bin, allowing us in describing the mean geometry of the cur-
 146 rents in the near-Earth space, i.e., these patterns are clearly invariant with time, although
 147 seasonal variations are present, which will be reported in a forthcoming paper. As shown
 148 in Figure 1, the bin-average external vertical field ranges between -20 and 20 nT and a
 149 two-lobe structure is clearly visible. It is consistent with the solar quiet (S_q) daily vari-
 150 ation of the geomagnetic field, a regular variation due to electric currents flowing in the
 151 ionosphere [e.g., *Campbell, 2003*]. The basic pattern of the equivalent S_q current system
 152 consists in a near-two-dimensional current circuit centered around noon at ~ 110 km alti-
 153 tude fixed with respect to the Earth-Sun line, and flowing in counter-clockwise direction in
 154 the Northern Hemisphere and clockwise direction in the Southern Hemisphere. This cur-
 155 rent system generates an induced magnetic field along \hat{z} directed outward in the Northern
 156 Hemisphere and inward in the Southern Hemisphere, in both cases opposite to the main
 157 geomagnetic field vertical component, and thus it is revealed by Swarm observations as a
 158 decrease of the geomagnetic field in the \hat{z} direction in the Northern Hemisphere and an in-
 159 crease in the Southern Hemisphere [e.g., *Campbell, 2003*]. The regular magnetic variation
 160 associated with this ionospheric system is visible mainly when solar-wind driven distur-
 161 bances are absent. During geomagnetically disturbed periods, associated with the occur-
 162 rence of storms and substorms, the S_q signal tends to be easily masked. At low and mid
 163 latitudes others magnetic signatures can be detectable such as the magnetospheric ring
 164 current, magnetotail, and magnetopause currents. All these currents become stronger dur-
 165 ing times of enhanced geomagnetic activity and for this reason their magnetic signatures
 166 become visible during geomagnetic disturbed periods. Nevertheless, a certain amount of
 167 ring current, which is the nearest magnetospheric current to the Earth, is always flowing
 168 even during quiet times. This current, centered in the magnetic equatorial plane, pro-
 169 vides at Earth a uniform magnetic field which is aligned with the magnetic dipole axis
 170 and pointing southward. Thus, on our global map of the geomagnetic field of external ori-
 171 gin along the vertical component, the field associated with the ring current appears as a
 172 positive contribute to B_z in the Northern Hemisphere and a negative in the Southern one.

173 3 Methods and Analysis

174 Usually, both univariate and multivariate analysis methods are based on a priori
 175 fixed decomposition basis, obtained by exploiting linearity and stationarity conditions
 176 [*Chatfield, 2016*]. The above requirements, strictly assumed to satisfy mathematical prop-
 177 erties, are not generally verified when natural signals are analyzed, requiring adaptive
 178 analysis methods [*Huang et al., 1998*]. In the following, we describe two different de-
 179 composition methods based on clearly different requirements: a linear method, i.e., the
 180 Empirical Orthogonal Function (EOF) analysis [see, e.g., *Lorenz, 1956; Ghil et al., 2002;*
 181 *Chatfield, 2016*]; and a nonlinear one, i.e., the Empirical Mode Decomposition (EMD) and
 182 its extensions [*Huang and Wu, 2008; Rehman and Mandic, 2010*].

183 3.1 Empirical Orthogonal Function (EOF) analysis

184 The Empirical Orthogonal Function (EOF) analysis, often called Principal Com-
 185 ponent Analysis (PCA) in Earth sciences [see, e.g., *Ghil et al., 2002; Chatfield, 2016*], is
 186 a decomposition technique for both univariate and multivariate data. Generally, the uni-
 187 variate method is used for decomposing data into a sum of (orthogonal) components ob-
 188 tained by the diagonalization of the covariance matrix of the data based on embedding
 189 a given series of discrete data $x(n)$ (of length N) in a matrix M of dimension $m \times N$,
 190 being m the embedding dimension [see, e.g., *Takens, 1981; Ghil et al., 2002; Chatfield,*
 191 *2016*]. In the multivariate case, the data set is described by a data matrix $\{\mathbf{s}(n)\}_{n \in N} =$
 192 $\{s_1(n), s_2(n), \dots, s_k(n)\}$, assumed to be related to k observations for a given length N .
 193 Then, the set of observations is converted into a set of values of linearly uncorrelated vari-

ables, i.e., the PCs $\Phi_l(n)$, as

$$\{\mathbf{s}(n)\}_{n \in N} = \sum_{l=1}^k \Phi_l(n) \mathbf{L}_l^T \quad (1)$$

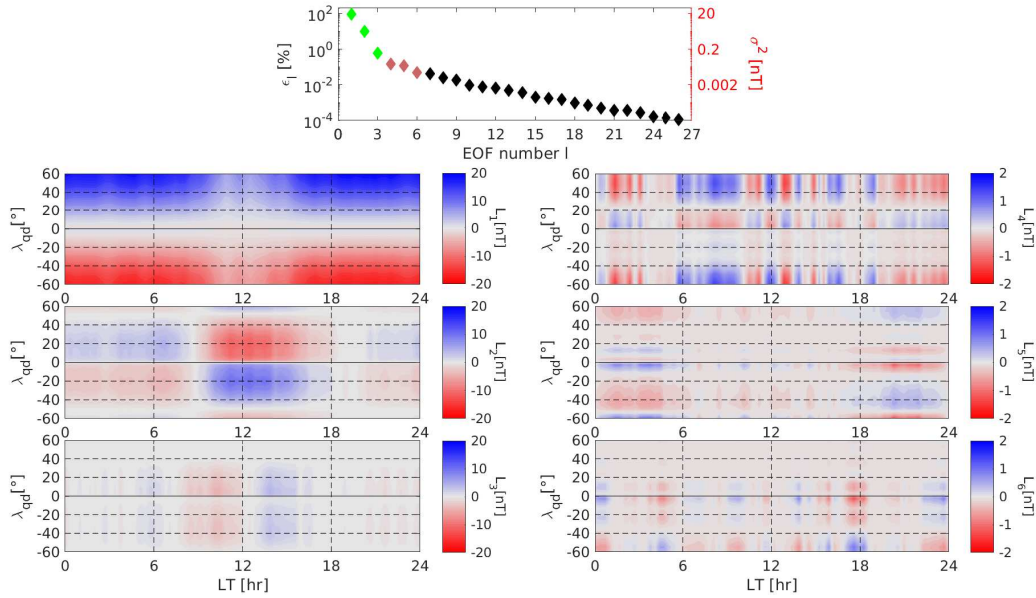
being \mathbf{L}_l^T the transpose of the l -th eigenvector of the covariance matrix of the data obtained as $\mathbf{C} = \{\mathbf{s}\}^T \{\mathbf{s}\}$. Both EOFs and PCs can be also retrieved by applying the Singular Value Decomposition (SVD) on the matrix of the data $\{\mathbf{s}(n)\}_{n \in N} = \mathbf{S}$ under investigation as $\mathbf{S} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$, where \mathbf{U} and \mathbf{V} are orthonormal matrices, and $\mathbf{\Sigma}$ is a diagonal matrix. The columns of \mathbf{U} are called left singular vectors, the rows of \mathbf{V}^T contain the elements of the right singular vectors, and the elements of $\mathbf{\Sigma}$ are called the singular values [Ghil *et al.*, 2002]. The right singular vectors are equivalent to the eigenvectors of the covariance matrix \mathbf{C} , while the singular values σ_l are equal to the square-root of the eigenvalues ϵ_l of \mathbf{C} [Ghil *et al.*, 2002; Chatfield, 2016]. Thus, the decomposition is complete and orthogonal (by construction), the normalized eigenvalue ϵ_l captures the partial variance (i.e., the energy content) of the l -th principal component, and their sum exploits the total energy content [Ghil *et al.*, 2002]. Summarizing, the main steps of the EOF method are:

1. to organize data as a matrix (by using the embedding theorem for univariate data [see, e.g., Takens, 1981]);
2. to evaluate the covariance matrix of data (embedded data for univariate data);
3. to diagonalize the covariance matrix to find eigenvectors and eigenvalues;
4. to project data on eigenvector directions to find the uncorrelated variables, i.e., the principal components.

This method has been applied to different fields as solar physics [see, e.g., Vecchio *et al.*, 2005; Consolini *et al.*, 2009], geomagnetic variations [see, e.g., Rotanova *et al.*, 1982; Xu and Kamide, 2004; De Michelis *et al.*, 2010; Balasis and Egbert, 2006; Shore *et al.*, 2016], and extensively in climate research [see, e.g., Lorenz, 1956; Ghil *et al.*, 2002; Lovejoy and Schertzer, 2013]. Here, we apply it to our dataset. Having binned data into 5×5 degree bins across the Earth's surface, the data matrix has a dimension $(m \times T) = (26 \times 72)$ and consequently the method extracts a set of $m = 26$ components (L_l). However, to correctly deal with boundary effects we show our results between $\pm 60^\circ$, without considering the boundary latitudinal bins. Since our dataset consists of spatial measurements we obtain eigenfunctions (i.e., EOFs and PCs) that depend on geomagnetic latitude and longitude, the latter expressed in terms of local time variations. Thus, we are investigating spatial variations at different scales by exploiting the local properties of the covariance matrix of the external geomagnetic field measurements. This means that we are able to detect the different spatial structures of the external components of the geomagnetic field.

Figure 2 reports the results obtained by applying the EOF method to our data. The partial variance of each eigenvalue is shown in the upper panel while some components (L_l) resulting from the analysis are reported in the other panels of the figure. From the values of the variance we notice that L_1 captures the most variance of the signal ($\epsilon_1 \sim 90\%$) and contributes with L_2 and L_3 to the reconstruction of the $\sim 98\%$ of the total variance. $L_4 - L_6$ capture $\sim 1\%$ of the variance and the remaining components are below the noise level [Ghil *et al.*, 2002].

The first three components (from L_1 to L_3), shown in the left column of Figure 2, are characterized by large scale spatial patterns. Interestingly, the most energetic contribution given by L_1 does not reproduce the main spatial pattern that is visible in the original data associated with the Sq daily variation. This structure is captured by L_2 . Indeed, L_1 is characterized by a symmetric spatial pattern both in latitude and in LT, which remembers the magnetic signature of the ring current. Conversely, L_2 is characterized by a two vortex-like structure centered around noon and symmetric with respect to the geomagnetic equator, in agreement with the Sq main pattern structure. On the contrary, L_3 seems to be characterized by a symmetric pattern in λ_{geom} , with no evidence of LT symmetry. The right



234 **Figure 2.** Empirical Orthogonal Function analysis of Swarm A data. (on the top) Percentage contribution
 235 and variance of EOFs. To the left, the first three EOFs corresponding with green diamonds in the top panel,
 236 and to the right EOFs 4-6 corresponding to the orange diamonds in the top panel.

246 column panels of Figure 2 present some of the main characteristics of components $L_4 - L_6$
 247 which show striped patterns, characterized by latitudinal ribbons of alternate positive and
 248 negative amplitudes. Finally, the remaining components (not shown) can be attributed to
 249 the noise, due to the low variance they account for [see, e.g., *Ghil et al.*, 2002].

250 3.2 Empirical Mode Decomposition (EMD) and its multivariate extension (MEMD)

251 3.2.1 Empirical Mode Decomposition (EMD)

252 The Empirical Mode Decomposition (EMD), differently from traditional data anal-
 253 ysis techniques (like Fourier analysis or Wavelets) [see, e.g., *Chatfield*, 2016], works di-
 254 rectly in the data domain rather than in a conjugate one to extract the so-called Intrinsic
 255 Mode Functions (IMFs) which satisfy two requirements: i) the number of extrema and
 256 the number of zero crossings must be either equal or differ at most by one, ii) at any data
 257 point, the mean value of the envelope defined using the local maxima and that obtained
 258 from the local minima is zero [*Huang et al.*, 1998]. They are derived through a direct and
 259 adaptive process, called sifting process [*Huang et al.*, 1998], which acts on a series $x(t)$ as
 260 follows:

- 261 1. the local extrema are identified (i.e., local maxima and minima, corresponding to
- 262 data points where abrupt changes are observed);
- 263 2. both local maxima and minima are separately interpolated by using a cubic spline,
- 264 in order to have continuous (and smoothed) functions with smaller error than other
- 265 polynomial interpolation, also avoiding the Runge's phenomenon [see, e.g., *Prenter*,
- 266 1975];
- 267 3. the spline interpolation produce the so-called upper $u(t)$ and lower $\ell(t)$ envelopes;
- 268 4. the mean envelope $m(t)$ is obtained as $m(t) = \frac{u(t)+\ell(t)}{2}$;
- 269 5. the so-called detail or candidate IMF is evaluated as $n(t) = x(t) - m(t)$.

270 The previous steps are iterated n times until the obtained detail $h(t)$ can be identified as
 271 an Intrinsic Mode Function (often called empirical mode) [Huang *et al.*, 1998], while the
 272 complete sifting process stops when no more empirical modes, e.g., IMFs $c_i(t)$, can be
 273 extracted from data such that

$$x(t) = \sum_{i=1}^{N_i} c_i(t) + r(t), \quad (2)$$

274 where $r(t)$ is the residue of the decomposition, which can be a constant function, a mono-
 275 tonic function, or a function with only one extremum not containing an oscillatory compo-
 276 nent physically meaningful [Huang *et al.*, 1998].

277 Analytically, the mathematical requirements for detecting an IMF are satisfied only
 278 when $n \rightarrow \infty$; numerically, the sifting process is stopped after n^* iterations according to a
 279 defined stopping criterion [Huang and Wu, 2008]. The first criterion has been proposed by
 280 Huang *et al.* [1998] such that, being

$$\sigma_{n^*} = \sum_{j=1}^T \frac{|h_{n^*}(t_j) - h_{n^*-1}(t_j)|^2}{h_{n^*-1}^2(t_j)}, \quad (3)$$

281 the sifting algorithm stops at the step n^* when $\sigma_{n^*} < \sigma_0$, being σ_0 between 0.2 and 0.3
 282 [Huang *et al.*, 1998]. Another stopping criterion, e.g., the so-called threshold method pro-
 283 posed by Rilling *et al.* [2003], sets two thresholds, i.e., θ_1 and θ_2 , to guarantee globally
 284 small fluctuations (as in Huang *et al.* [1998]) and, in the meanwhile, to take into account
 285 locally large excursions [see, e.g., Rilling *et al.*, 2003; Flandrin *et al.*, 2004, for more de-
 286 tails].

287 The decomposition procedure is completely adaptive, exclusively based on the local
 288 characteristic of the data, and highly efficient for processing nonlinear and/or nonstationary
 289 data [Huang and Wu, 2008]. From a mathematical point of view, convergence is assured
 290 by construction while orthogonality of the basis is satisfied in all practical senses, unless
 291 it is not theoretically guaranteed. However, by construction all empirical modes are locally
 292 orthogonal, since they are obtained by local maxima and minima properties (i.e., by the
 293 zeros of the first derivative), and also a posteriori globally orthogonal [e.g., Huang and
 294 Wu, 2008].

295 One of the novelties introduced by the EMD, beyond its adaptive character, is the
 296 concept of instantaneous amplitude and instantaneous phase [Huang *et al.*, 1998]. Indeed,
 297 once the decomposition is completed, by applying the Hilbert transform to each empirical
 298 mode it is possible to construct a complex analytical signal described by an amplitude-
 299 wave modulation model. In this way, assuming to consider a time series, each empirical
 300 mode can be seen as an oscillating function with both time-dependent amplitude $a_i(t)$ and
 301 phase $\phi_i(t)$ as

$$c_i(t) = a_i(t) \cos[\phi_i(t)]. \quad (4)$$

302 Both $a_i(t)$ and $\phi_i(t)$ can be obtained by the Hilbert transform of the i -th empirical mode,
 303 which is defined as

$$H[c_i](t) = \frac{1}{\pi} \mathbb{P} \int_{-\infty}^{\infty} \frac{c_i(t')}{t - t'} dt', \quad (5)$$

304 being \mathbb{P} the Cauchy principal value, such that from the complex analytical signal $z_i(t) =$
 305 $c_i(t) + iH[c_i](t)$ we obtain

$$a_i(t) = \sqrt{c_i^2(t) + H[c_i]^2(t)}, \quad (6)$$

$$\phi_i(t) = \tan^{-1} \frac{H[c_i](t)}{c_i(t)}. \quad (7)$$

306 From the above concepts of instantaneous amplitude and phase, the mean energy content
 307 of each empirical mode can be simply derived as $e_i = \frac{1}{T} \sum_{j=1}^T a_i^2(t_j)$, the instantaneous
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308 frequency as $\omega_i(t) = \frac{d\phi_i(t)}{dt}$, the mean frequency as $\langle\omega_i(t)\rangle_t = \frac{1}{T} \sum_{j=1}^T \omega_i(t_j)$, and the mean
 309 timescale as $\tau_i = \frac{2\pi}{\langle\omega_i(t)\rangle_t}$ [see, e.g., *Alberti et al.*, 2017].

310 Based on numerical experiments on white noise data, *Wu and Huang* [2004] found
 311 that the EMD acts a dyadic filter, being the empirical modes all normally distributed and
 312 covering the same area on a semi-logarithmic scale [see also *Flandrin et al.*, 2004]. This
 313 means that the product between the energy density of the i th empirical mode, defined as
 314 $E_i = \frac{1}{N} \sum_{j=1}^N [c_i(j)]^2$ with being N the length of the data, and its corresponding mean
 315 timescale τ_i is constant such that the energy density is chi-squared distributed [*Wu and*
 316 *Huang*, 2004]. This method can be used to assess the significance of empirical modes
 317 with respect to those derived from purely white noise processes, giving us theoretical
 318 spread function values on different confidence levels.

319 Being direct and intuitive, the EMD method is one of the most used adaptive meth-
 320 ods, which is able to carefully analyze all those data resulting from nonlinear and/or non-
 321 stationary processes [see, for example, *Guhathakurta et al.*, 2008; *Consolini et al.*, 2017;
 322 *Piersanti et al.*, 2017]. It is capable of overcoming some limitations of different decom-
 323 position techniques (as for example a required fixed decomposition basis), also avoiding
 324 misleading results (as for fixed eigenfunction analysis) when complex and chaotic time
 325 series are analyzed [see, e.g., *Consolini et al.*, 2018; *Alberti et al.*, 2019]. However, some
 326 outstanding problems, mostly dealing with end effects and/or stopping criteria need to be
 327 outlined [see, e.g., *Huang and Wu*, 2008; *Wu and Huang*, 2009; *Alberti et al.*, 2018], al-
 328 though various methods have proposed to avoid and/or mitigate these effects, as mirror
 329 and data extending methods [see, e.g., *Huang and Wu*, 2008; *Yang et al.*, 2014].

330 The usefulness of this method is demonstrated by several papers on different fields
 331 and with different time series analyzed [see, e.g., *De Michelis et al.*, 2013; *Alberti et al.*,
 332 2014; *Vecchio et al.*, 2017; *Bengulescu et al.*, 2018], including applications in geophysical
 333 research [see, e.g., *De Michelis et al.*, 2012; *Alberti et al.*, 2016], in signal denoising [see,
 334 e.g., *Wu and Huang*, 2004; *Flandrin et al.*, 2004], and also in financial studies [see, e.g.,
 335 *Nava et al.*, 2018; *Zhu et al.*, 2018].

336 3.2.2 Multivariate Empirical Mode Decomposition (MEMD)

337 Although the EMD allows us to overcome some limitations when univariate signals
 338 are analyzed, it cannot be directly applied to multivariate data. The problem is that local
 339 extrema cannot be well defined on a n -dimensional space and, consequently, the compu-
 340 tation of the local mean is not possible and the concept of empirical mode is rather un-
 341 known [*Rehman and Mandic*, 2010]. First attempts to approach to multivariate signals by
 342 using EMD were based on channel-wise processing by applying univariate EMD to each
 343 channel [*Huang and Wu*, 2008]. The algorithm idea was to generate a pseudo-multivariate
 344 EMD by translating the univariate algorithm on n directions, grouping modes on similar
 345 scale by processing ensemble EMD over each direction [*Huang and Wu*, 2008].

346 To extend the concept of local extrema on k -dimensional space and to produce more
 347 suitable multivariate decompositions, *Rehman and Mandic* [2010] proposed to consider the
 348 k -variate signal as formed by k -dimensional datasets, each of which was projected to ap-
 349 propriate directions over the k -dimensional space. In this way for each projected signal
 350 the envelopes can be calculated for each direction and, by averaging over the k -dimensional
 351 space, the local mean of the multivariate signal can be obtained using two different meth-
 352 ods able to create a suitable set of direction vectors in the k -dimensional space. They are:
 353 i) the uniform angular sampling coordinates method and ii) quasi-Monte Carlo-based low-
 354 discrepancy sequences. These methods provide a uniform distribution of direction vectors
 355 and more accurate local mean estimates in k -dimensional spaces [see, e.g., *Rehman and*
 356 *Mandic*, 2010, for more details].

357 Then, the usual steps (e.g., multivariate spline interpolation and Intrinsic Mode
 358 Function properties check) of the standard EMD are used to evaluate the multivariate
 359 IMFs such that a k -variate signal $\{\mathbf{s}(n)\}_{n \in N} = \{s_1(n), s_2(n), \dots, s_k(n)\}$ can be written
 360 as

$$\{\mathbf{s}(n)\}_{n \in N} = \sum_{i=1}^{N_i} \{\mathbf{c}_i(n)\}_{n \in N} + \{\mathbf{r}(n)\}_{n \in N} \quad (8)$$

361 where the set of k -dimensional embedded patterns $\{\mathbf{c}_i(n)\}_{n \in N}$ is affine to the univari-
 362 ate decomposition basis formed by the IMFs and $\{\mathbf{r}(n)\}_{n \in N}$ is affine to the univariate
 363 residue. This process decomposes a multivariate signal in several local mono-component
 364 k -dimensional functions, each of which containing the same frequency distribution.

365 A characteristic scale for each MEMD mode can be obtained as

$$\tau_i = \frac{1}{N} \int_0^N n' \langle \{\mathbf{c}_i(n')\}_{n' \in N} \rangle_k dn', \quad (9)$$

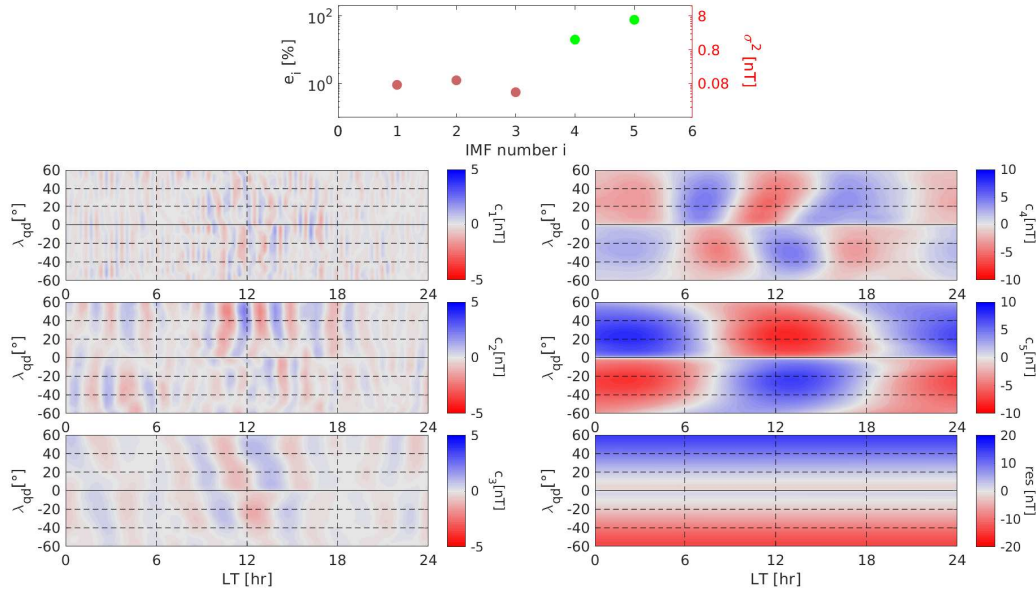
366 being $\langle \dots \rangle_k$ an ensemble average over the k -dimensional space. Moreover, as for EMD,
 367 instantaneous amplitudes $\{\mathbf{a}_i(t)\}_{n \in N}$ and phases $\{\phi_i(t)\}_{n \in N}$ of each MEMD mode can
 368 be retrieved by applying the Hilbert Transform over the projection of the multivariate sig-
 369 nal along different directions of the k -dimensional spaces. From instantaneous amplitudes
 370 we can derive the instantaneous energy contents $\{\mathbf{E}_i(n)\}_{n \in N}$. By averaging over the k -
 371 directions, we obtain the mean energy associated with each MEMD mode, through which
 372 the relative contribution can be derived as

$$e_i = \frac{\frac{1}{N} \int_0^N n' \langle \{\mathbf{E}_i(n')\}_{n' \in N} \rangle_k dn'}{\sum_{i=1}^{N_i} \frac{1}{N} \int_0^N n' \langle \{\mathbf{E}_i(n')\}_{n' \in N} \rangle_k dn'}. \quad (10)$$

373 Finally, as for EMD modes [Huang *et al.*, 1998], also MEMD modes empirically and lo-
 374 cally satisfy orthogonal and completeness properties [Rehman and Mandic, 2010] in the
 375 k -dimensional space such that partial sums of eq. (8) can be obtained.

376 When spatio-temporal signals are analyzed, MEMD is able to extract intrinsic spatio-
 377 temporal components with different characteristic spatial and temporal scales that can be
 378 used to investigate spatial patterns evolving in time without any a priori fixed assump-
 379 tion on linearity and stationarity of the signal. This means that MEMD is able to describe
 380 local (in terms of space) nonstationary (in terms of time) variations due to nonlinear com-
 381 ponents (in terms of amplitude variations in space and time). In our case, we applied the
 382 MEMD to spatial measurements such that the MEMD modes depend only on spatial co-
 383 ordinates (i.e., geomagnetic latitude and local time). In this way, we are able to detect
 384 variations of the external components of the geomagnetic field measurements at different
 385 spatial scales, which can be used to investigate the different spatial patterns of both iono-
 386 spheric and magnetospheric current systems. We chose the threshold method proposed
 387 by Rilling *et al.* [2003] to stop the sifting process and we used the improved characteristic
 388 wave algorithm to prolong the data series at the boundaries to deal with the edge effect
 389 [see, e.g., Huang *et al.*, 1998; Huang and Wu, 2008]. However, the results are not signifi-
 390 cantly sensitive to the chosen threshold parameters and/or boundary algorithms.

395 Figure 3 reports the results of the MEMD decomposition of B_z for the Swarm A
 396 satellite observations. In the top panel of the same figure we report the percentage en-
 397 ergy, calculated from eq. (10), associated with each IMF as a function of the correspond-
 398 ing number. The first three modes contain less than 3% of the total energy of the signal
 399 (brown dots); conversely, the modes with $i = 4, 5$ contain $\sim 97\%$ of total energy and conse-
 400 quently the signal obtained from the superposition of these modes represents the main part
 401 of the original one. In the other panels the IMFs (c_i), obtained applying the MEMD tech-
 402 nique, are shown and sorted in an increasing-scale order from 1 (the smallest spatial scale)
 403 to 5 (the largest spatial scale). At last, the residue is shown in the bottom right panel of



391 **Figure 3.** Multivariate Empirical Mode Decomposition analysis of Swarm A data. Relative contribution
 392 and variance of MEMD modes (top panel), first three MEMD modes (c_1 - c_3 , left panels) corresponding to the
 393 brown dots in the top panel, MEMD modes c_4 and c_5 (right panels) corresponding to the green dots in the top
 394 panel.

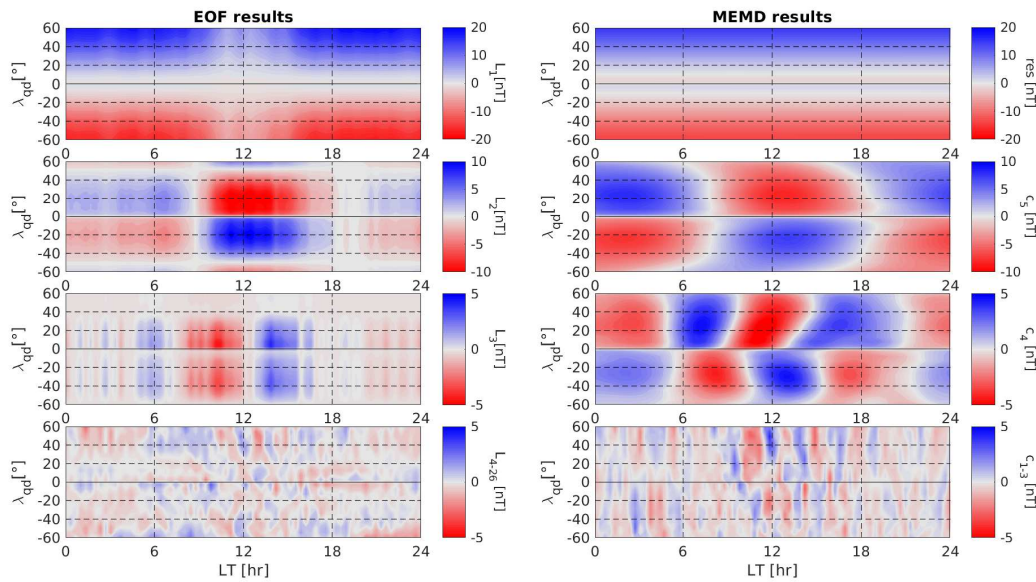
404 Figure 3. The number of detected IMFs and their characteristic spatial scales are automa-
 405 tically found by the algorithm according to the criteria described above, being the proce-
 406 dure completely adaptive (in contrast with EOF analysis, where the number of components
 407 depends on data matrix dimension). Moreover, due to the nonlinear behavior and spatial
 408 dependence of the different components, the adaptive nature of the MEMD method can
 409 be really helpful in detecting the different spatial features and variations of both magneto-
 410 spheric and ionospheric source processes and currents. The left column panels of Figure
 411 3 illustrate some of the main characteristics of the first three IMFs. These IMFs (c_1 , c_2
 412 and c_3) are characterized by an amplitude in the range ± 5 nT and their spatial structures
 413 are similar to latitudinal ribbons alternating positive and negative amplitudes. In the right
 414 column of Figure 3 the IMFs 4-5 are shown (c_4 and c_5). The large scale patterns in the
 415 maps have strengths spanning the range from $\sim \pm 5$ to $\sim \pm 10$ nT and represent the main
 416 structure originated by the S_q current in quietness, being c_5 the main component and c_4
 417 its spatial harmonics. In fact, the component with the largest spatial scale (c_5) contains
 418 patterns which have the right characteristics in order to represent the main contribution
 419 to the S_q : they are centered at noon, have a negative (positive) field variation in the Northern
 420 (Southern) Hemisphere in a background of opposite sign, and extend for about 12 hours,
 421 which is the time period marking the transition from the day- to the night-side and vice
 422 versa. On the other hand, the features appearing in c_4 may be considered as harmonics
 423 embedded in the main variation.

424 The MEMD technique provides also the residual of the original map (referred in Eq.
 425 (8) as $\mathbf{r}(t)$), i.e., the part of the original signal that cannot be decomposed into IMFs, as
 426 shown in the bottom right panel of Figure 3. It ranges between $\sim \pm 20$ nT, is positive in
 427 the Northern Hemisphere and negative in the Southern one. This implies that the MEMD
 428 residual of B_z is inward in the Northern Hemisphere and outward in the Southern Hemi-
 429 sphere. At λ_{qd} between $\sim \pm 20^\circ$ the residual assumes very small values, which increases
 430 at increasing λ_{qd} . We also note that the increase of the external field at higher latitudes is

431 a feature common to all longitudes, and no localized patterns appear, unlike it happens in
 432 all the detected IMFs (and also at high latitudes for the most energetic component L_1
 433 detected by EOF analysis, see Figure 2). Similar results have been found for the B_x and B_y
 434 components [see, e.g., *Alberti, 2018*].

435 4 Results and Conclusions

436 We applied two different methods of analysis to our data set consisting of the spatial
 437 measurements of the geomagnetic field vertical component at low and mid latitudes during
 438 a geomagnetically quiet period. The aim is to compare the results coming from the two
 439 methods, one linear (EOF) and one nonlinear (MEMD), in order to understand which one
 440 is the best to recognize the magnetic spatial structures of external origin embedded in the
 441 data.



442 **Figure 4.** Comparison between EOF (left panels) and MEMD (right panels) results. (From top to bottom)
 443 L_1 and the residue of the MEMD method can be attributed to the ring current contribution, L_2 and c_5 pat-
 444 terns can be related to the main S_q pattern, L_3 and c_4 can be attributed to a sub-harmonic structure of the S_q
 445 current, while short-scale reconstructions L_{4-26} and C_{1-3} could be related to different source mechanisms
 446 (external driver, magnetopause current).

447 Figure 4 reports a comparison between the results obtained from the two different
 448 methods of analysis. In detail, we report the results obtained from EOF decomposition
 449 method in the panels on the left of Figure 4, while the results obtained from MEMD are
 450 shown in the panels on the right of the same figure. From Figure 4 we notice that by ap-
 451 plying the MEMD method we are capable of separating the different modes that contribute
 452 to the magnetic field of external origin during quiet periods. We find that our patterns can
 453 be represented as a linear combination of five empirical modes and a residue. The first
 454 three modes, i.e., those characterized by the smallest spatial scales in LT, appear in form
 455 of spurious North-South patterns. The other two modes, i.e., those with the largest spatial
 456 scales, seem to describe the effects on the geomagnetic field of the electric currents flow-
 457 ing in the ionosphere, i.e., mainly the S_q ionospheric current pattern. Lastly the residual,
 458 which represents the long-term trend of the analyzed data, seems to be due to the electric

459 currents flowing in the magnetosphere and describes the effect on the geomagnetic field
 460 of the magnetospheric ring current. In fact, when considering only the \hat{z} component of
 461 the magnetic field, the presence of the magnetospheric ring current should add a contri-
 462 bution to the magnetic field which is basically null at and nearby the magnetic equator,
 463 and should increase with the latitude, like what can be observed looking at the residual
 464 map. It is important to notice indeed that the ring current, which is known to lead to a
 465 global-scale reduction in the horizontal component of the geomagnetic field during the
 466 geomagnetic storm, is a magnetospheric current which always exists, also during quiet pe-
 467 riods [Shore *et al.*, 2016]. Only its intensity and distance from the Earth change during
 468 the disturbed periods [De Michelis *et al.*, 1997], together with the partial ring current [Mi-
 469 lan *et al.*, 2017]. By applying the EOF analysis we are able, also in this case, to separate
 470 the different modes, which contribute to the magnetic field of external origin. However,
 471 in this case, the different magnetic spatial structures embedded in the analyzed signal are
 472 more difficult to recognize. We can recognize the magnetic field due to the ring current in
 473 the first EOF (L_1) and the magnetic field due to the S_q ionospheric current pattern in the
 474 second EOF (L_2). Conversely, the third EOF (L_3) does not seem to describe the effect on
 475 the magnetic field produced by a particular current system but it could be a sub-harmonic
 476 of the EOF L_2 and consequently to partially describe the effects on the magnetic field of
 477 the S_q ionospheric current pattern. However, all these three modes are contaminated by
 478 the solar quiet daily variation. Thus, the method does not seem to be capable of com-
 479 pletely separating the different spatial structures probably due to the nonlinear nature of
 480 the analyzed signal. Moreover, other 23 EOFs are necessary to completely reproduce the
 481 original data. To confirm our interpretation about the origin of the different contributions
 482 (ionospheric Sq or magnetospheric ring current) we have repeated our analysis on mag-
 483 netic data recorded by Swarm B satellite, which flows at an higher altitude than Swarm A
 484 (about 50 km). By analyzing the difference between the results obtained by the two satel-
 485 lites (data not shown here) we found that the residual magnetic field increases with the
 486 altitude, as it is expected in the case of a contribution due to the magnetospheric current
 487 systems, while the contribution due to the Sq current system decreases with the altitude.
 488 Furthermore, by analyzing the ionospheric field, obtained by removing from the original
 489 data the internal magnetic field and the magnetospheric one modelled by CHAOS-6, the
 490 contribution due to the ring current cannot be revealed (data not shown).

497 In order to show more clearly the differences between the two methods, we compare
 498 the longitudinal (i.e., local time) behavior of the ionospheric contribution obtained from
 499 the original data by using CHAOS-6 model at fixed latitudes with the signals describing
 500 the magnetic field due to sources localized in the ionosphere obtained from the two meth-
 501 ods. The results are reported in Figure 5. First, we notice that the behavior of B_z^{iono} (red
 502 asterisks) is that expected in quiet conditions, being a few nT from dusk to dawn, with
 503 a negative bump up to ≈ 10 -15 nT in the Northern Hemisphere and a positive bump in
 504 the Southern Hemisphere around noon. The comparison among the three signals shows
 505 that the MEMD analysis is able to reconstruct the magnetic signal of ionospheric origin
 506 better than the EOF analysis. This is clearly visible at mid-latitude where the trend re-
 507 produced by the combination of the IMFs c_4 and c_5 (green line) well describes the effect
 508 of S_q ionospheric pattern on the magnetic field. Conversely, the EOF reconstruction of the
 509 magnetic field of ionospheric origin (blue line, $L_2 + L_3$) is not very good as can be realized
 510 comparing it with the original data at mid-latitude, due to an incorrect estimation of the
 511 nonlinear residue (note that nor L_1 neither the residue of the MEMD have been included
 512 in reconstructions of EOFs and IMFs). To quantify the different fits to the B_z^{iono} data we
 513 have estimated the correlation coefficients between B_z^{iono} and both MEMD and EOF re-
 514 constructions of the Sq variability in the local time interval between 06:00 LT - 18:00 LT,
 515 where the Sq current systems are localized. The results, reported in Figure 5, confirm that
 516 a higher correlation is found between B_z^{iono} and MEMD reconstructions. Moreover, it is
 517 interesting to note that similar large-scale structures have been found by using both EOF
 518 and MEMD which is an indication of the robustness and significance of the detected spa-
 519 tial variability on these scales.

520 In general, therefore, it seems that MEMD method can help in the interpretation of
 521 the external magnetic field signals better than EOF method. Using MEMD analysis a few
 522 modes are necessary to recognize in the magnetic signal the different contributions com-
 523 ing from external sources. They are not the result of a model but can be directly extracted
 524 from the original signals with no *a priori* assumption on the nature of data. These modes,
 525 each associated with a characteristic spatial scale, describe the basis representing the data
 526 and are able to identify various dynamical components of the analyzed signals that can be
 527 related to different physical scales and sources. This study is an example of the potential
 528 of the MEMD method to give new insights into the analysis of the different sources re-
 529 sponsible for the geomagnetic field of external origin; and at the same time, it can be used
 530 as a good filter in the analysis of the geomagnetic field of external origin, permitting to
 531 separate the ionospheric signal from the magnetospheric one.

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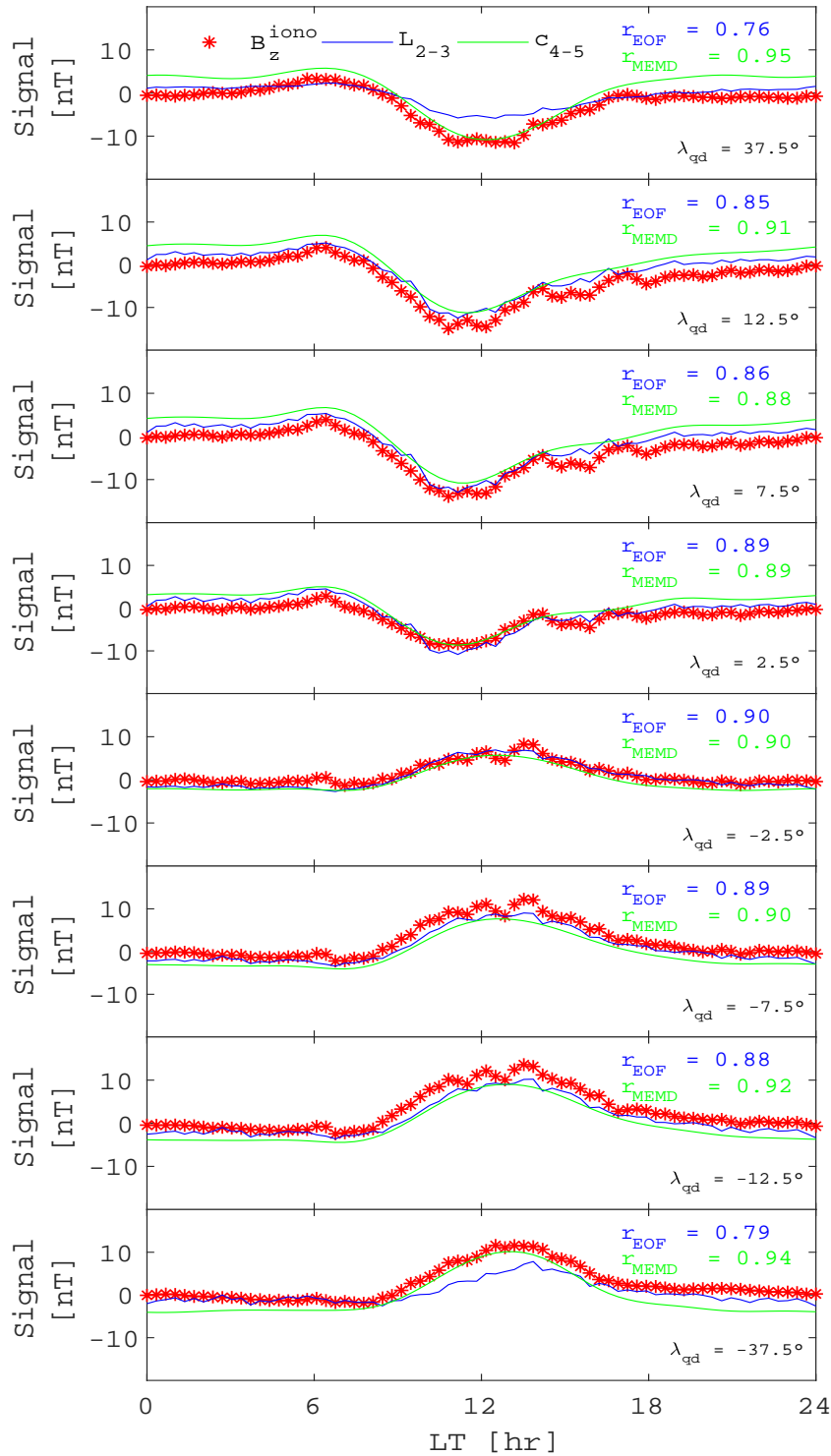
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491 **Figure 5.** Longitudinal (LT) behavior of B_z from Swarm A observations at different latitudes, from
 492 37.5 ± 2.5 (top panel) down to -37.5 ± 2.5 (bottom panel), respectively. Red asterisks mark the ionospheric
 493 contribution derived by CHAOS-6 (B_z^{iono}); the blue solid line represents the summed EOFs $L_2 + L_3$; the solid
 494 green line represents the summed IMFs $c_4 + c_5$. r_{EOF} and r_{MEMD} refer to the values of correlation coefficient between B_z^{iono} and Sq reconstructions by
 495 using EOF (blue text) and MEMD (green text), respectively.
 496

Figure 1.

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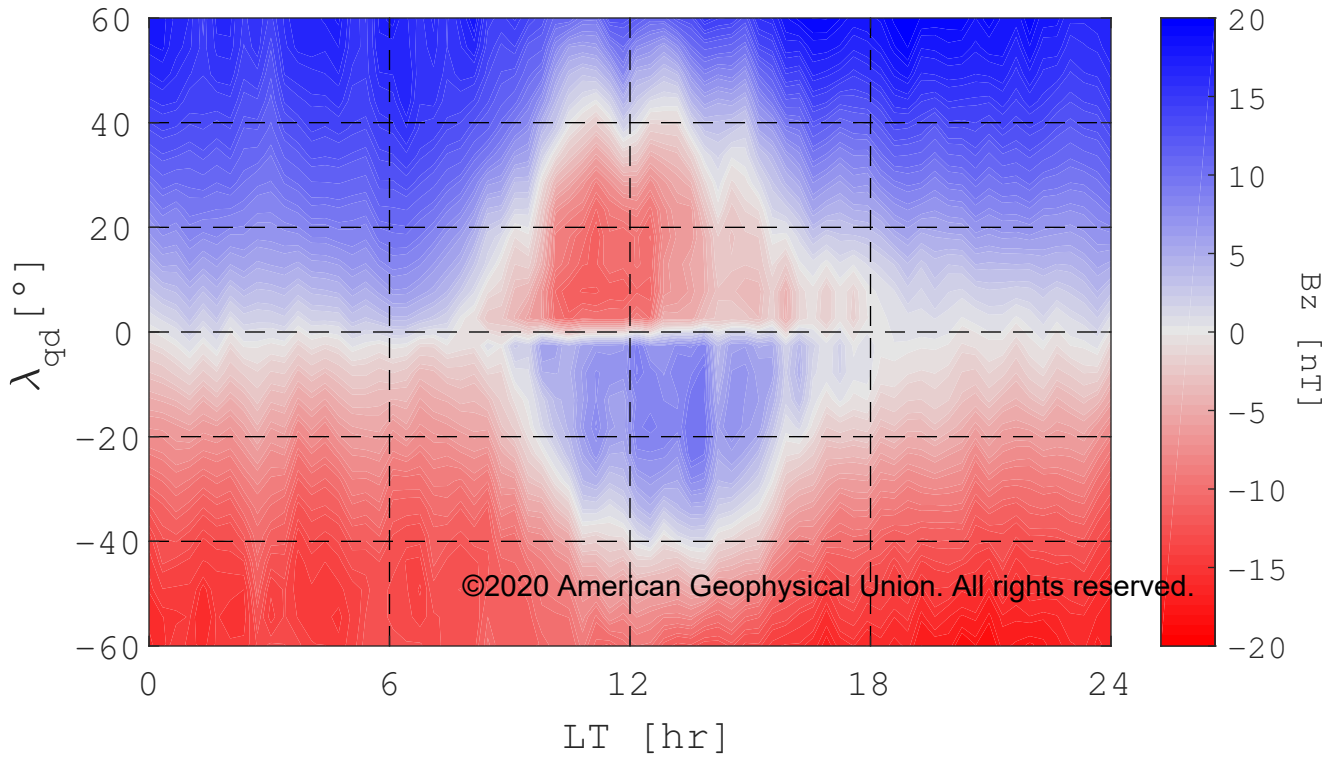


Figure 2.

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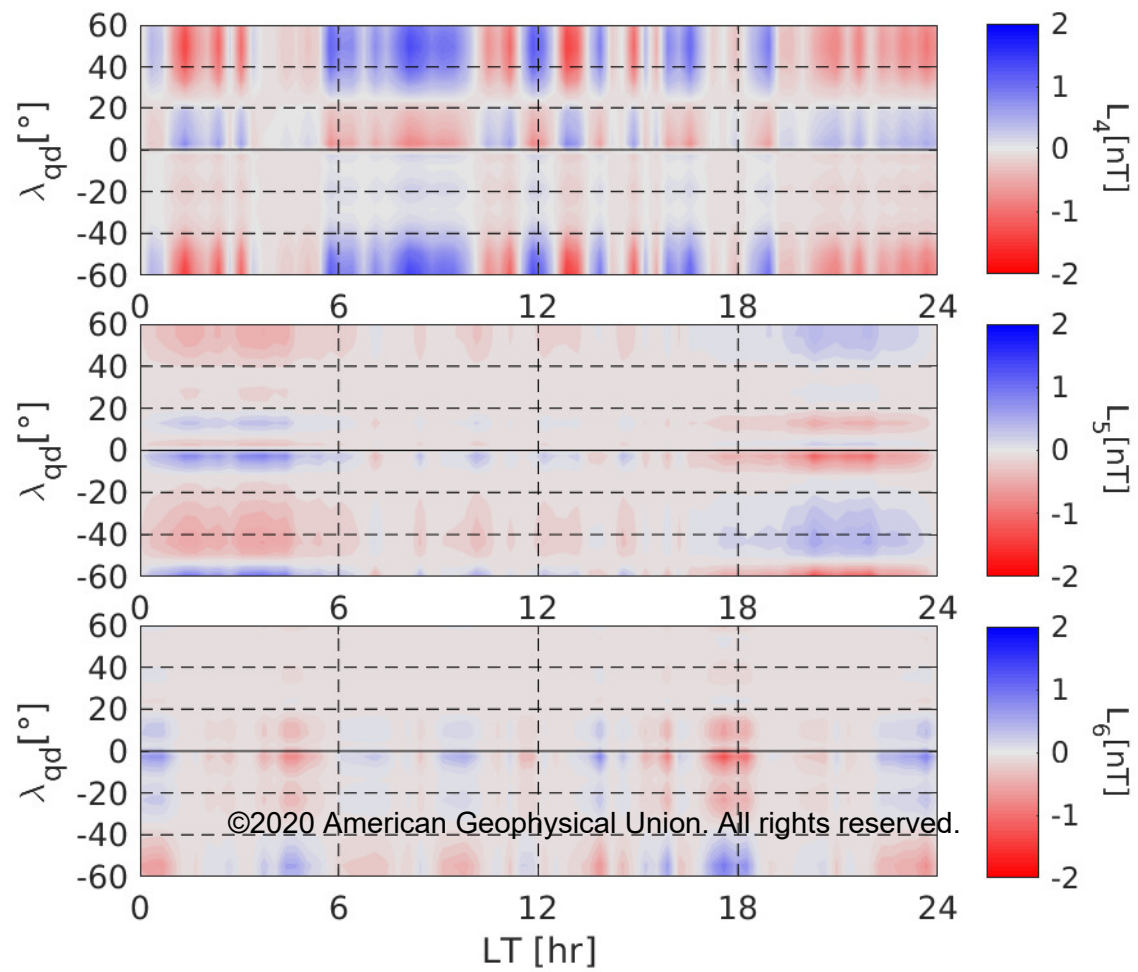
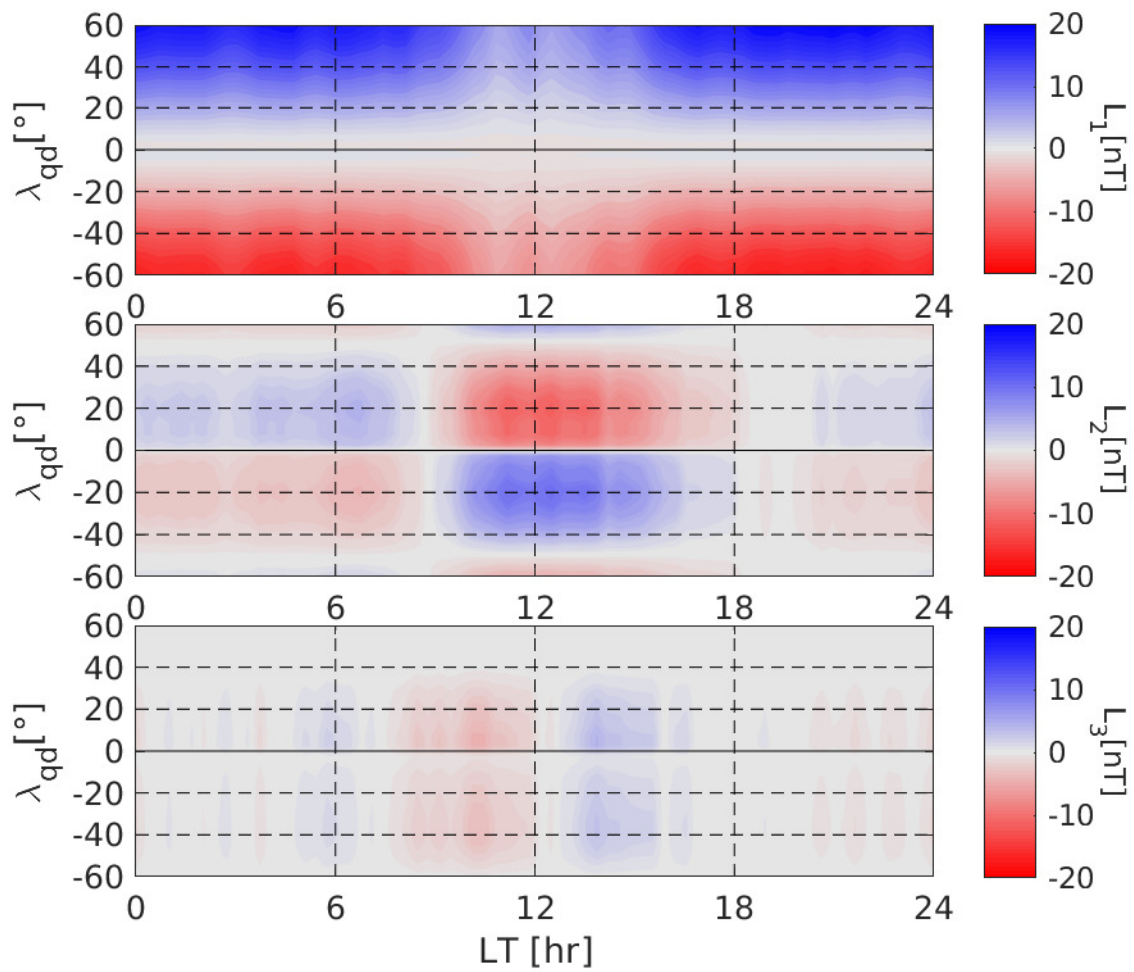
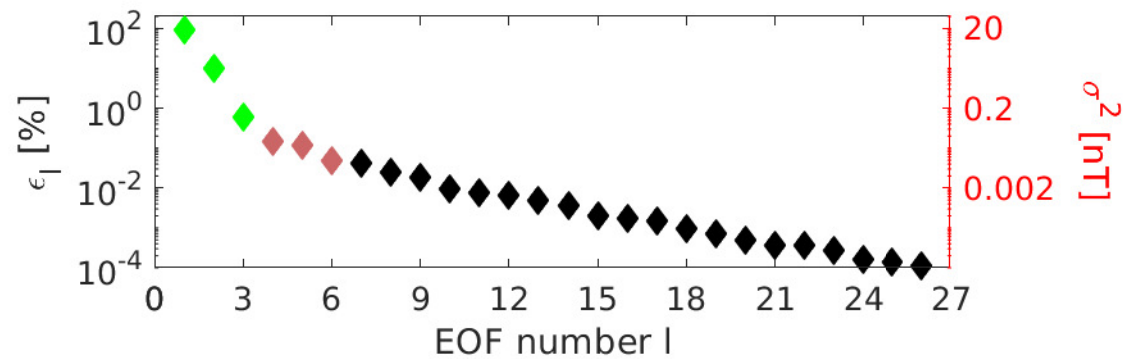


Figure 3.

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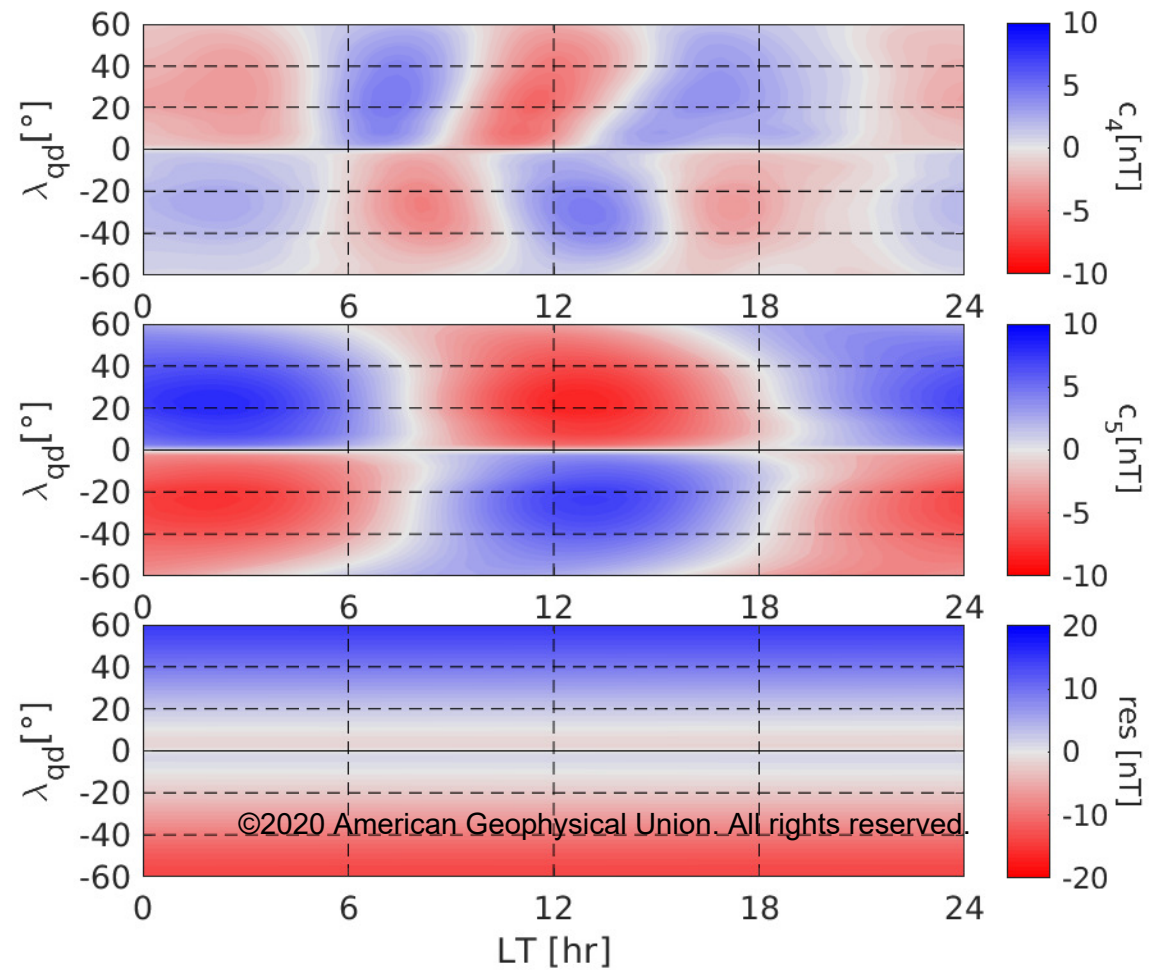
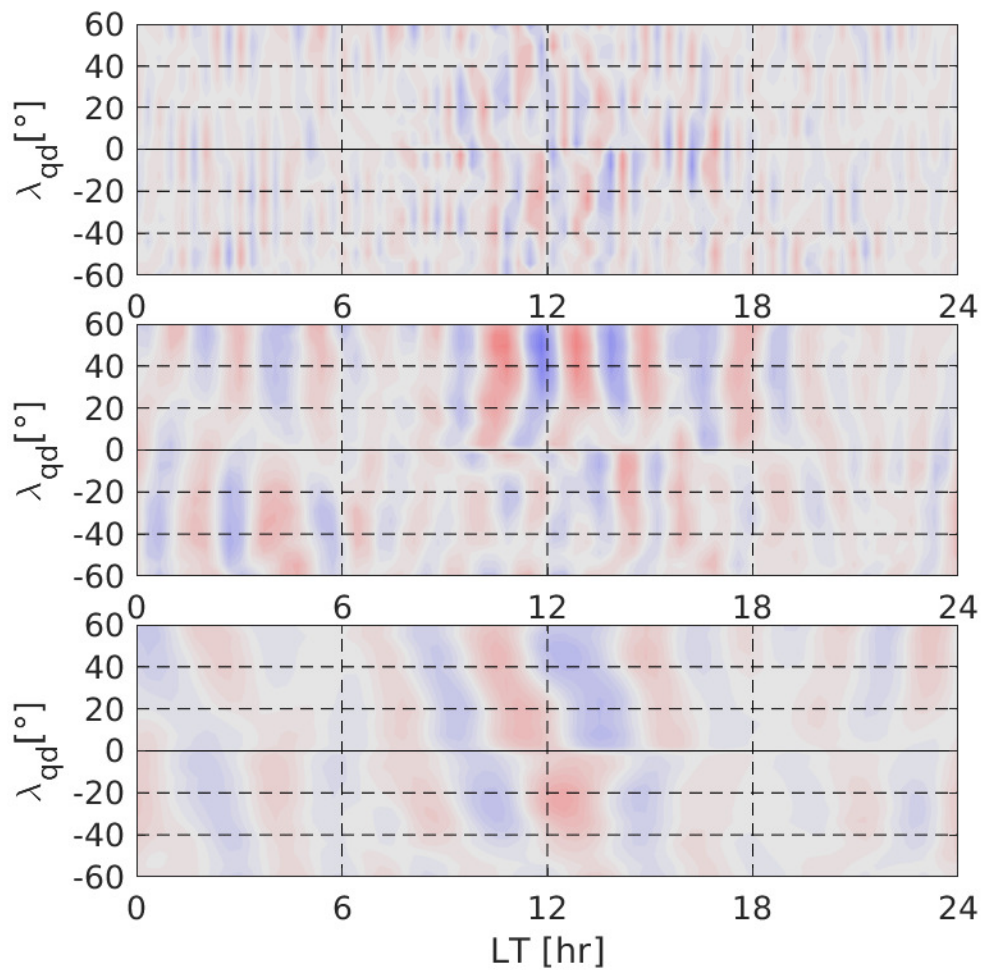
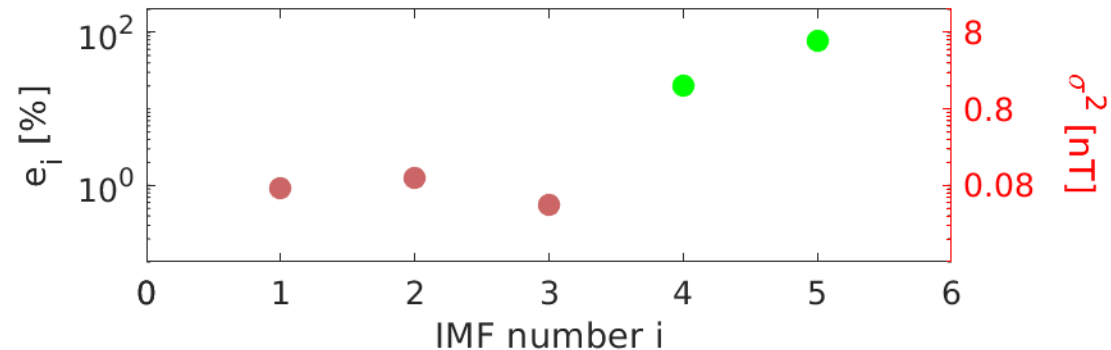
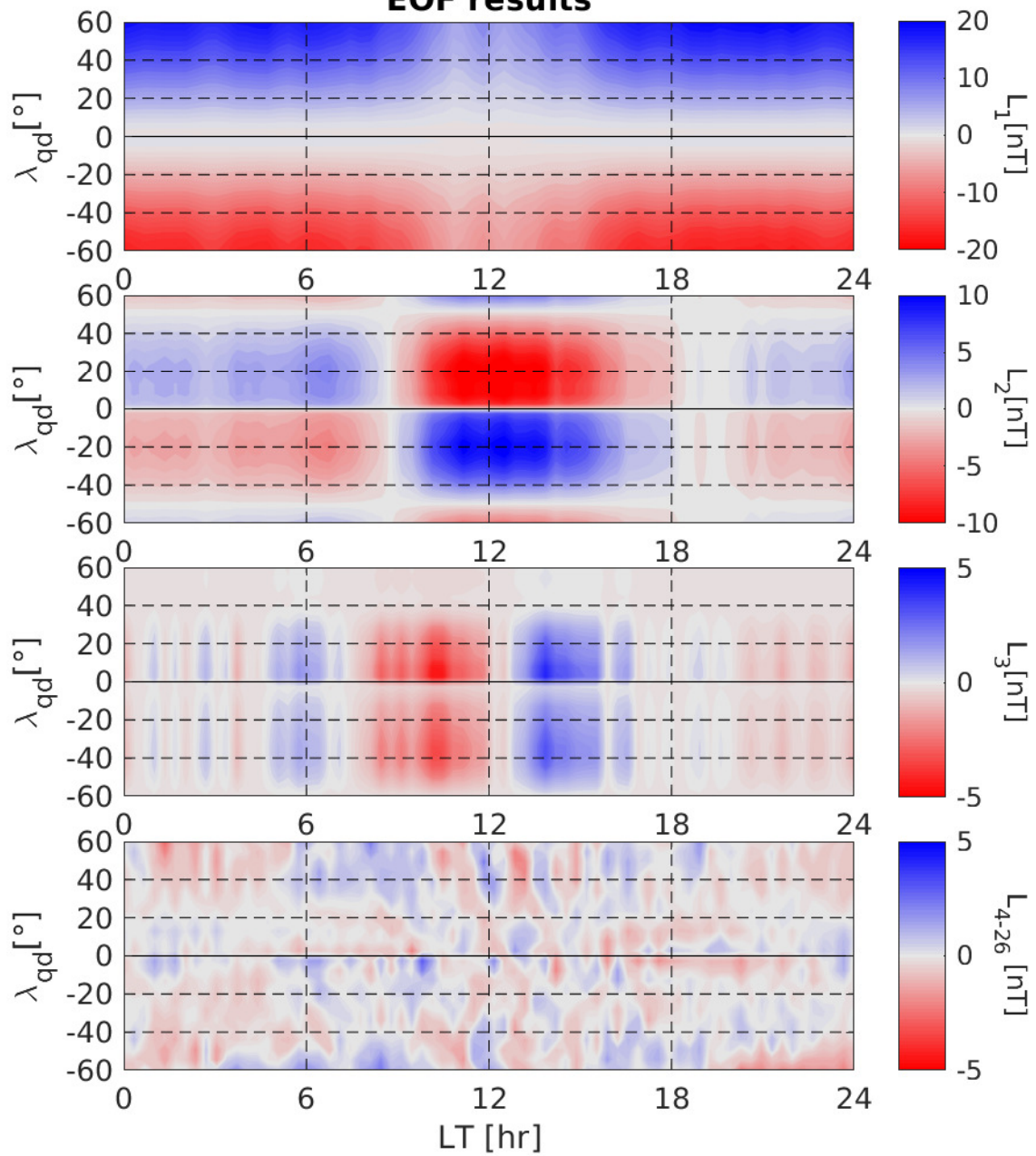


Figure 4.

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EOF results



MEMD results

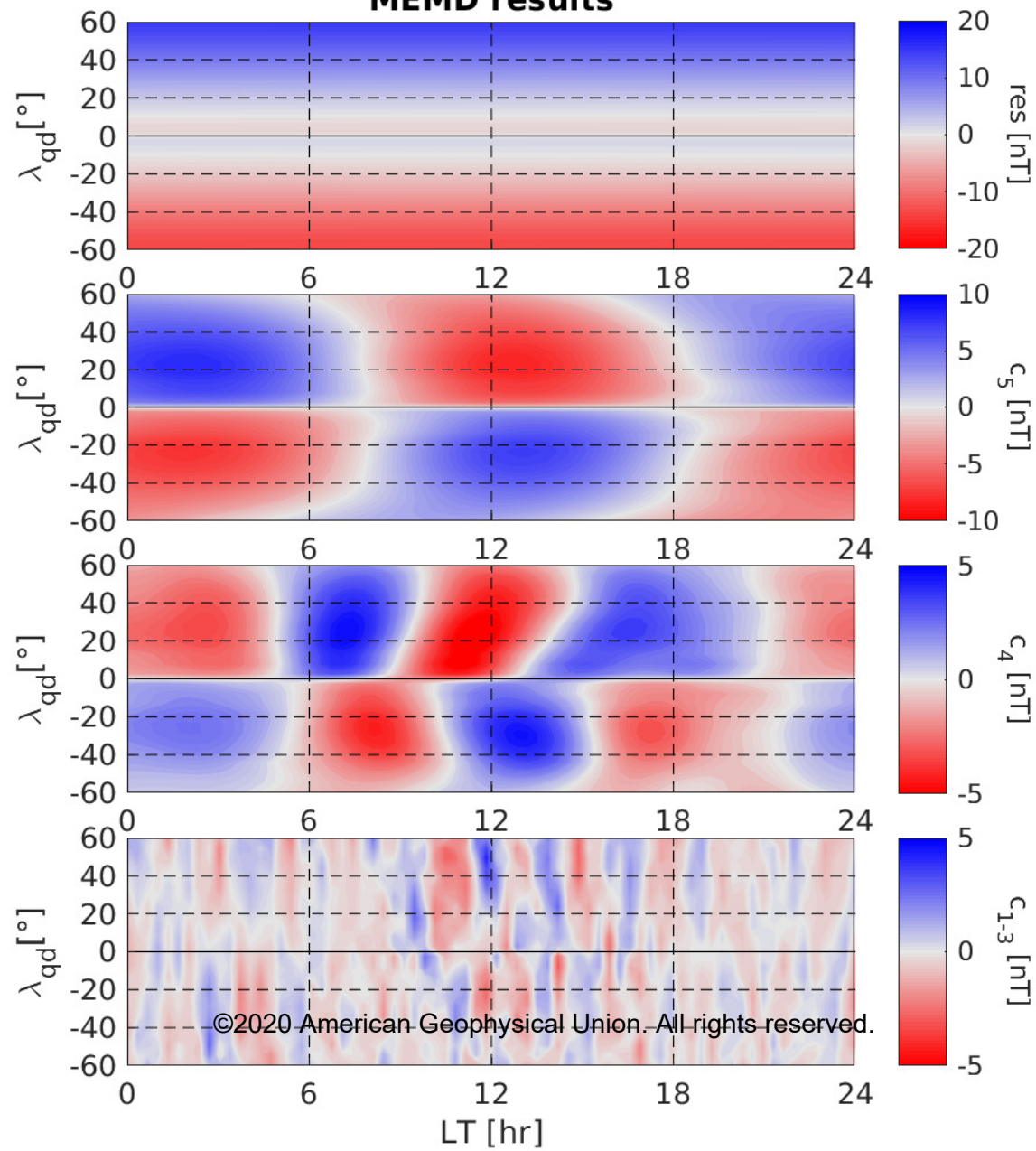


Figure 5.

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