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Notes on *fk* analysis of surface waves

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Abstract
The dispersive nature of Rayleigh waves is the basis of many identification procedures to infer the site stiffness profile from surface measurements. This article presents some important aspects related to *fk* analysis of seismic gathers, which is one of the procedures commonly used to obtain the experimental dispersion curve, focusing on the great influence that the scale of the survey has in the global process. At a short distance from the source, the seismic signal contains information related to different modes of propagation in a composite form. As the wave travels away, the different modal group velocities produce a separation of such information. Hence only if the testing array is sufficiently long is it possible to assume mode separation and to invert the dispersion curve for modal velocities. Otherwise the effects of mode superposition need to be carefully accounted for. To clarify this concept, the results of some numerical simulations are reported, together with some experimental results.

Key words *Rayleigh waves – soil stiffness – shear wave profiles – shallow geophysics – shear modulus*

1. Introduction

The dispersive behaviour of Rayleigh waves has been widely used in seismology for geometrical and mechanical characterisation of the Earth's crust (Dorman and Ewing, 1962; Aki and Richards, 1980). Recently much interest has focused on the use of surface waves for shallow geophysics and several applications have been proposed. The interest is mainly related to the need for a proper estimate of soil stiffness, that is the necessary information for a correct assessment of dynamical site response to seismic events or to induced vibrations.

Basically the identification process based on surface waves propagation can be divided into two main steps: the evaluation of the experimental dispersion curve from field data and the inversion process necessary to estimate the soil stiffness profile from the dispersion curve itself.

Many different procedures have been adopted for the estimation of the experimental dispersion curve, either based on the detection of microtremors (Horike, 1985) or with the use of artificial sources acting on the ground surface (Gabriels *et al.*, 1987; McMechan and Yedlin, 1981).

The inversion process is commonly based on the numerical simulation of surface waves propagation in layered linear-elastic media. The unidimensional model implies the basic assumption of a regular horizontally stratified soil deposit, which must always be kept in mind when interpreting the test results.

A main consequence of the soil heterogeneity is the existence of several different modes of propagation. Only for certain subsoil conditions the fundamental mode is predominant over the other ones, while in general, the propagation is associated with several modes. Most of the inversion procedures are based on the possibility of having distinct experimental dispersion curves for the fundamental and for the higher

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modes (Gabriels *et al.*, 1987) or on the use of the fundamental mode only (Xia *et al.*, 1999).

In geotechnical engineering, seismic tests based on surface waves propagation have become quite popular with the SASW (Spectral Analysis of Surface Waves) method, based on the use of a single pair of geophones (Nazarian and Stokoe, 1984). Cost and time effectiveness have made this seismic test often preferred to borehole methods, such as Cross-Hole and Down Hole tests.

It has been shown that the interpretation of SASW data requires simulations of the forward problem of surface wave propagation that account for mode superposition effects (Gucunski and Woods, 1992). Hence consistent numerical procedures are needed for the inversion process (Ganji *et al.*, 1998).

The outlines sketched above, concerning different applications of surface wave based characterisation tools, have shown that the multimodal nature of Rayleigh waves needs to be carefully accounted for. A very important aspect in the interpretation of field measurements is related to scale effects. Indeed, the length of the planned survey can have major effects, considering the dispersive nature of Rayleigh waves and the consequent difference between phase and group velocities. This paper is mainly devoted to exploring some of these peculiar aspects, both with results of numerical analysis and with real case histories of field measurements.

2. Frequency-wavenumber analysis of surface waves

Rayleigh waves typically dominate the wavefield generated by vertical point sources acting on the ground surface (Richart *et al.*, 1970). This is due to several properties of this kind of waves.

First of all, it is important to note that the energy attenuation associated to the geometrical spreading is lower for Rayleigh waves than for body waves. Body waves in an infinite elastic medium attenuate as $1/r$, while along a free surface compression and shear waves attenuate as $1/r^2$ (Ewing *et al.*, 1957). On the other hand, in a homogeneous elastic halfspace Rayleigh waves spread along a cylindrical wavefront and hence their attenuation is proportional to $1/\sqrt{r}$.

Moreover, it can be shown that much of the energy transmitted to the ground by a point source goes into Rayleigh waves. As an example for harmonically vibrating circular footing over an elastic homogeneous halfspace, it has been shown that 2/3 of the energy goes into Rayleigh waves and the remaining portion is divided between compression and shear waves (Richart *et al.*, 1970).

For all the aforementioned reasons, Rayleigh waves are predominant in traces recorded on the ground surface. Usually they are regarded as a sort of coherent noise (ground roll) in seismic gathers collected for reflection surveys. For this reason, many procedures have been developed to recognise and cancel out such components (Yilmaz, 1987).

Nevertheless, it must be considered that surface waves implicitly carry the information related to the system they are travelling through and hence, if correctly analysed, their propagation can be a useful characterisation tool.

The velocity of propagation of Rayleigh waves in a homogenous elastic halfspace is not a function of frequency. In a heterogeneous medium they become dispersive, *i.e.* their velocity is a function of frequency and such variation can be associated with the variation of stiffness with depth. Hence, if the relationship existing between Rayleigh wave velocity and frequency (usually named dispersion curve) is experimentally determined, it can be used in a given framework to estimate the soil stiffness profile.

Several procedures have been proposed for this purpose (see Foti, 2000 for more detailed references). Most of them are based on the assumption of a horizontally layered linear elastic model for the soil (fig. 1). Such a model is characterised by four parameters for each layer (*e.g.*, thickness H_i, density ρ_i, shear modulus G_i and Poisson ratio υ_i), except for the lowermost halfspace which has no fixed thickness. Several procedures for the numerical simulation of Rayleigh wave propagation in such system can be found in the literature (see Lai, 1998 for a short review).

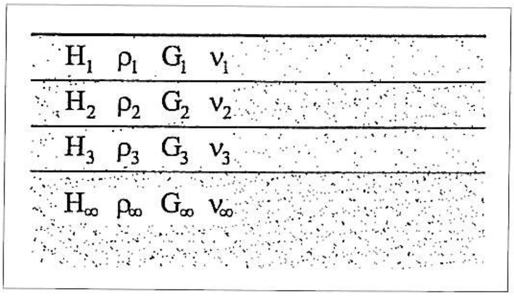

Fig. 1. Layered linear-elastic medium.

It is worthwhile noting that if this model is adopted, the number of unknowns to be determined is equal to $4n$-1, where n is the number of layers adopted for the inversion process. To alleviate the non-uniqueness of the solution and to simplify the heavy inversion procedure, density and Poisson ratio are usually assumed *a-priori* on the basis of former information and operator's experience. The goodness of such simplification has been shown through some parametric studies of Rayleigh waves propagation in layered media (Nazarian, 1984). For this reason, the results are usually reported in terms of shear modulus profile or, equivalently, of shear wave velocity profile.

In the frequency-wavenumber analysis, the experimental dispersion curve is obtained using a multistation shot gather. Assuming the aforementioned model, the ground motion associated to Rayleigh wave propagation can be evaluated using modal superposition (Aki and Richards, 1980)

$$s(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\sum_{m} S_m(\omega,x)\cdot e^{i(\omega t-k_m(\omega)x)}\,d\omega \tag{2.1}$$

where m is the mode number and the factor

$$S_m(\omega,x) = I(\omega)\cdot P_m(\omega)\cdot R_m(\omega)\cdot\frac{e^{-\alpha_m(\omega)x}}{\sqrt{x}} \tag{2.2}$$

is a combination of instrument response $I(\omega)$, source spectrum $P_m(\omega)$ and path response $R_m(\omega)$ with geometric (represented by the factor $\frac{1}{\sqrt{x}}$) and material ($e^{-\alpha_m}$) attenuation.

The modal wavenumber k_m is inversely proportional to the phase velocity V_{R_m}

$$k_m(\omega) = \frac{\omega}{V_{R_m}(\omega)} \tag{2.3}$$

and considering a spacing between geophones equal to Δx, the phase offset between any two geophones can be written as $k_m(\omega)\cdot\Delta x$.

The dependence of S_m on distance from the source is only related to the attenuation phenomenon. The influence of geometric attenuation can be easily removed by multiplying each contribution by the square root of the source-receiver distance and it will therefore be neglected in the following, assuming that such correction is applied on the original data. As far as the material attenuation is concerned, its contribution can be taken out from the expression of S_m, so that this last quantity becomes only a function of frequency.

Applying a discrete slant stack transform and subsequently a discrete Fourier transform the *fk* spectrum can be written as (Tselentis and Delis, 1998)

$$F(\omega,k) = \sum_{m} S_m(\omega)\cdot\left[\sum_{n=1}^{N} e^{-\alpha_m(\omega)\cdot x_n}\cdot e^{i(k-k_m(\omega))\cdot x_n}\right]. \tag{2.4}$$

If we neglect the material attenuation contribution, it is evident that differentiating the quantity in the square bracket with respect to k and setting the results equal to zero, the maximum of the energy spectra is obtained for $k = k_m(\omega)$. Furthermore, it can be shown that also if the above differentiation is conducted without neglecting the material attenuation the conclusion is the same, *i.e.* the accuracy is not conditioned by material attenuation (Tselentis and Delis, 1998).

Once the modal wavenumbers have been estimated for each frequency, they can be used to evaluate the dispersion curve from eq. (2.3).

An interesting application of this procedure for shallow soil characterisation has been reported by Gabriels *et al.* (1987). They were able to evaluate the dispersion curves of the first six modes of propagation using an FFT algorithm on a 256-trace record with receiver spacing 1 m. The inversion of the obtained dispersion curves (covering the frequency range 5-30 Hz) led to the reconstruction of the stiffness profile over a depth of about 50 m.

3. Numerical simulations

A series of numerical simulations have been planned to investigate the influence of the survey scale on the information obtained using frequency-wavenumber analysis (Foti, 2000). Synthetic seismograms related to different soil models have been created using the computer programs developed by R.B. Herrmann and his co-workers at S. Louis University. The impact source has been modeled as an impulsive vertical point source and the corresponding seismograms have been evaluated at a given number of detection points placed on the ground surface along a straight line passing through the application point of the source. The program used for this purpose is based on frequency-wavenumber modal superposition.

Several relevant profiles have been analysed, reproducing typical subsoil conditions. In the following, two cases are reported: case A represents a normally dispersive medium in which soil stiffness always increases with depth and strong impedance contrasts are not present; in case B a stiff top layer is placed above a normally dispersive medium, generating an inversely dispersive medium. Both the analysed cases are relevant for engineering purposes.

The synthetic seismograms were computed at 256 locations spaced 1 m along a straight line passing through the source location. This large amount of data was then used for different application of the *fk* analysis, selecting only part of all the available data. The results presented in the following are relative to the use of either all the 256 virtual receivers or only 24 receivers spaced 1 m. These choices are related respectively to the ideal case of a very long receiver array and to the engineering case of 24 receivers, which represents a reasonable number of channels for most commercial seismographs. The results obtained from the two virtual testing configurations have been compared to assess the effects of survey length.

3.1. *Case A*

The geometrical and mechanical parameters are reported in table I and the shear velocity profile is represented in fig. 2. This case corresponds to a normally dispersive medium constituted by a two-layer over halfspace system.

The *fk* spectrum has been estimated considering two different sets of receivers: the complete 256 traces and the first 24 traces, corresponding to an array of length equal to 23 meters starting at 1 m from the source. The two spectra are reported for comparison in figs. 3 and 4.

First of all it is important to point out that the spectral estimate using 1m spaced traces is not

Table I. Case A: layer geometry and mechanical properties.

Thickness (m)	V_S (m/s)	V_P (m/s)	Density (kg/m^3)
5	350	600	1800
10	400	700	1800
∞	450	800	1800

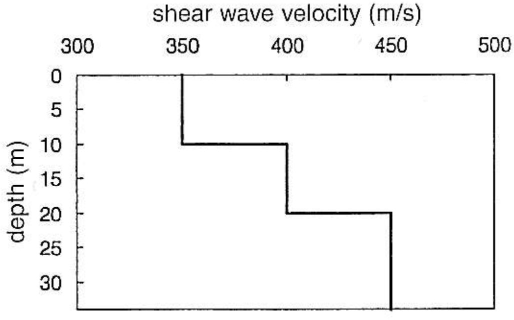

Fig. 2. Case A: shear wave velocity profile.

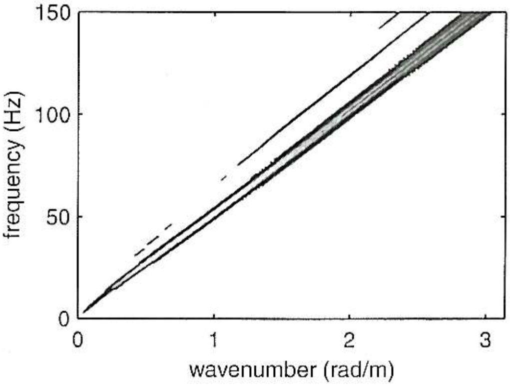

Fig. 3. Case A: *fk* spectrum estimated using 256 receivers.

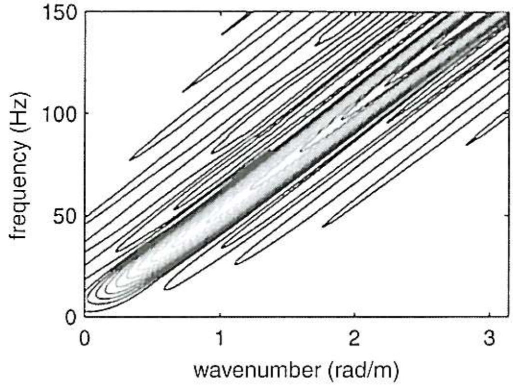

Fig. 4. Case A: *fk* spectrum estimated using 24 receivers.

affected by spatial aliasing in the frequency range of interest (up to 150 Hz). Indeed, for the highest frequency the phase velocity should be equal to the asymptotic values, *i.e.* to the Rayleigh wave phase velocity of a homogeneous halfspace having the same mechanical characteristics of the top layer ($V_S = 350$ m/s and $\nu = 0.25$). Hence the corresponding wavenumber can be estimated as

$$V_R = \frac{0.87 + 1.12\upsilon}{1+\upsilon} \cdot V_S = 0.92 \cdot V_S = 320 \text{ m/s}$$

$$k = \frac{2\pi \cdot f}{V_R} = \frac{2\pi \cdot 150}{320} = 2.95 \text{ 1/m}.$$

On the other hand, the maximum wavenumber that can be obtained from the 2D Fourier transform is the equivalent of the Nyquist frequency in the wavenumber domain, hence, recalling that the inter-receiver distance is 1 m

$$k_{\text{Nyquist}} = \frac{2\pi}{2 \cdot \Delta x} = \frac{2\pi}{1 \cdot 2} = \pi \text{ 1/m}.$$

Moreover spatial aliasing would have been revealed by the wrap-around in the wavenumber domain of the *fk* spectrum, that in this case is, in fact, not present.

Some important considerations can be made regarding the *fk* spectra (figs. 3 and 4). The difference in resolution caused by the different number of receivers is very strong. In the spectrum obtained using 256 traces, the peaks are sharper and it is possible to locate also higher modes, while the poor resolution prevents such possibility for the shorter receiver array. In both cases anyway the *fk* spectrum is dominated by the fundamental mode component, indeed at each frequency the global maximum is associated to it. This is a typical feature of normally dispersive profiles, in which the fundamental mode is always predominant over higher modes. The dispersion curves obtained from global maxima are in both cases coincident with the fundamental mode dispersion curve (fig. 5). In

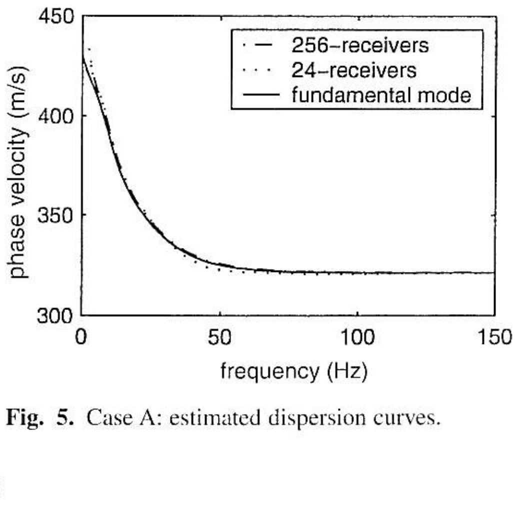

Fig. 5. Case A: estimated dispersion curves.

conclusion, for a normally dispersive medium the survey length affects the number of modes that can be resolved in the experimental dispersion curve. As a consequence, for a very long receiver array it is possible to use a multimodal inversion code with mode separation, whereas for a short receiver array the analysis is restricted to the fundamental mode only. Obviously more information supplied to the inversion program involves a better-conditioned solution, reducing the non-uniqueness of the problem.

3.2. *Case B*

To investigate the effects of a stiff top layer, the same stiffness profile of case A has been used, with the addition of a top layer as stiff as the halfspace. The same procedure of creating and analysing the synthetic seismograms has also been used for this case. No spatial aliasing problems are expected since the situation is similar to the previous one and a similar test can be done. The profile is quantitatively described in table II and graphically represented in fig. 6.

Table II. Case B: layer geometry and mechanical properties.

Thickness (m)	V_S (m/s)	V_P (m/s)	Density (kg/m^3)
3	450	800	1800
5	350	600	1800
10	400	700	1800
∞	450	800	1800

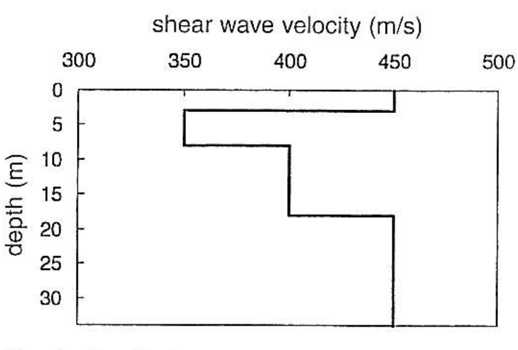

Fig. 6. Case B: shear wave velocity profile.

The spectra evaluated using respectively 256 and 24 traces, as for case A, are reported in figs. 7 and 8. Obviously also in this case the reduced number of observation points causes a strong reduction of the resolution, but in this case the consequences are more relevant. Indeed it is clear from the 256-receiver spectrum (fig. 7) that as the frequency increases there is a shift of dominant mode, so that only at low frequencies does the fundamental mode have the most relevant influence.

The way in which this aspect is reflected by the two spectra is significantly different. While in one case modal components remain separate showing different ridges (fig. 7), in the other the different mode contributions are combined in a single ridge (fig. 8). Obviously the implications on the dispersion curves obtained from the maxima location are very important.

If 256 traces are used to estimate the *fk* spectrum, at every single frequency the global maximum is associated to the pure dominant mode and following the local maxima it is possible to recover several distinct modal curves (fig. 9). When the estimate is based on 24 receivers, only a single dispersion curve can be recovered and it does not correspond to any of the modes, but is a combination of the modal phase velocities (fig. 10).

The implication of the above results on the inversion process must be carefully evaluated. First of all, it is important to point out that, independently of the length of the testing array and the number of receivers, a fundamental mode approach cannot be used. Indeed, for an inversely dispersive medium, the higher modes play a strong role in the propagation of surface waves and their influence cannot be neglected.

Using a long receiver array, several modal dispersion curves can be recovered, as for case A, and a multimodal inversion program can be profitably used for a better-conditioned inversion approach. Nevertheless, in the engineering situation in which a reduced number of receivers is used, modal superposition effects must be carefully evaluated and the inversions need to be performed estimating the effective (apparent) velocity, *i.e.* the quantity that is actually detected.

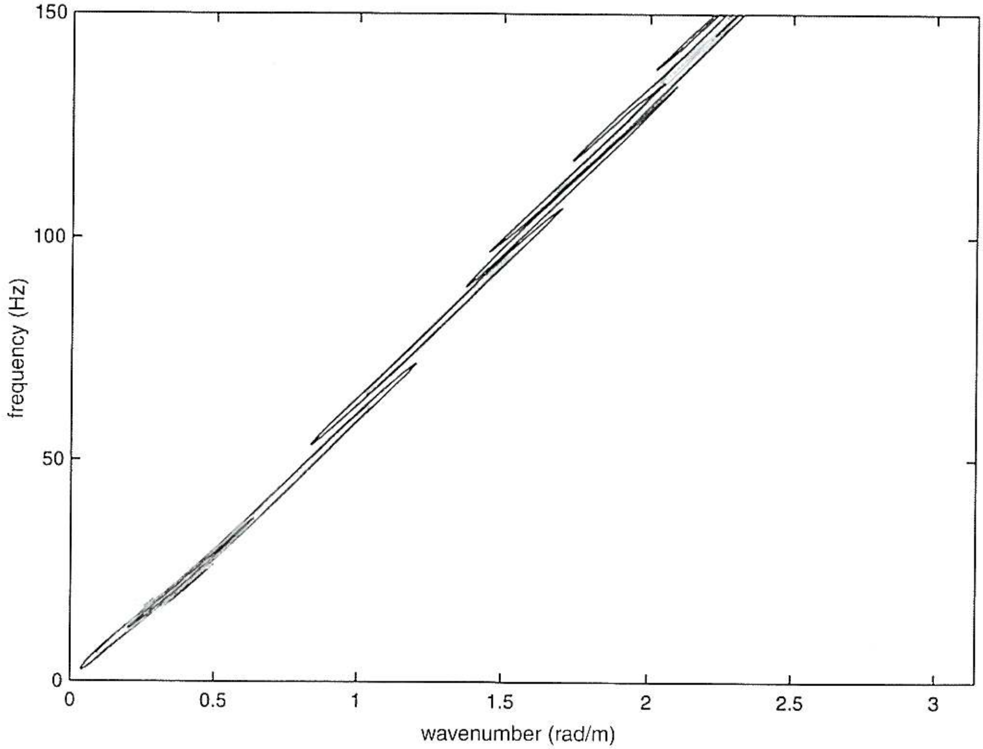

Fig. 7. Case B: *fk* spectrum estimated using 256 receivers.

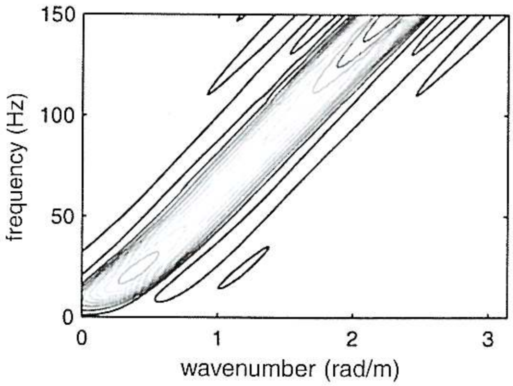

Fig. 8. Case B: *fk* spectrum estimated using 24 receivers.

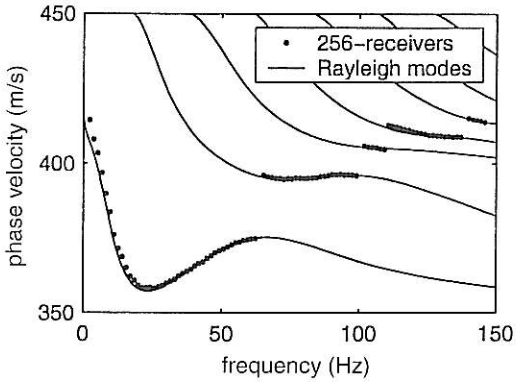

Fig. 9. Case B: estimated dispersion curves (256 receivers).

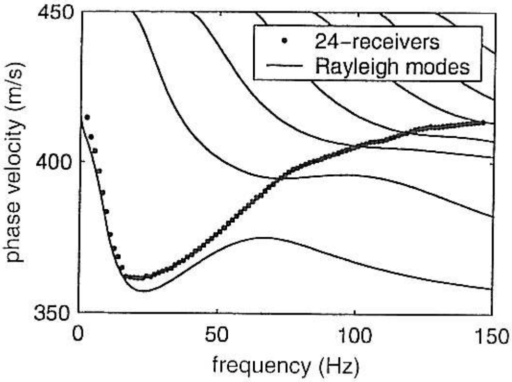

Fig. 10. Case B: estimated dispersion curves (24 receivers).

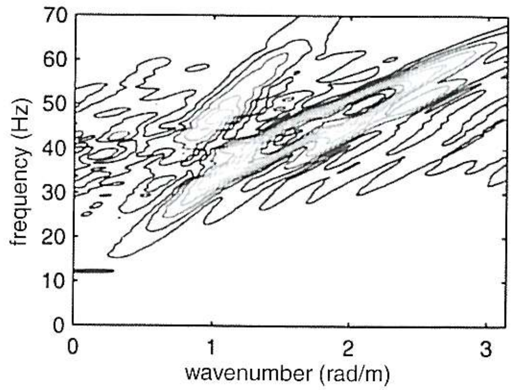

Fig. 11. Site A: *fk* spectrum (receiver spacing: 1 m; source: hammer).

4. Experimental results

Some results relative to short receiver arrays are presented in the following. The subsoil conditions are similar to the cases analysed with the numerical simulations: a normally dispersive profile and a stiff top layer profile.

Data were collected using a 24 channel seismograph Mark6 (by ABEM Ltd) and 24 vertical geophones, having 4.5 Hz natural frequency.

4.1. *Site A*

The testing site is located in Saluggia (VC) in the northern part of Italy, close to the Dora Baltea River and it is part of a large flat area of fluvial sediments. The soil is composed basically of gravels and gravelly sands, with the presence of fine sand and clayey silt, in the form of lenses. The water table is at very shallow depth, between 2 and 3 m below the ground surface. Some additional results from previous geotechnical testing surveys are available for the site.

The data were collected using two different test arrangements having respectively receiver spacing equal to 1 m and to 3 m and with the source-first geophone spacing equal to the inter-receiver spacing. The impact sources were respectively a 6 kg hammer and a 130 kg weight-drop system (height 3 m above the ground level). The two different configurations were chosen to investigate the high and the low frequency ranges respectively. The estimated *fk* spectra are reported in figs. 11 and 12.

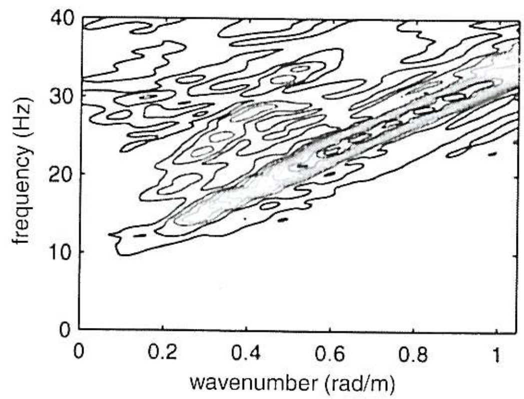

Fig. 12. Site A: *fk* spectrum (receiver spacing: 3 m; source: weight-drop).

The dispersion curve obtained by the *fk* analysis of the two shot gathers (fig. 13) shows that in the frequency range in which there is an overlap of information the consistency is very good, confirming that the estimate is stable. Globally the frequency range between 8 and 68 Hz is covered. For low frequencies (below 15 Hz)

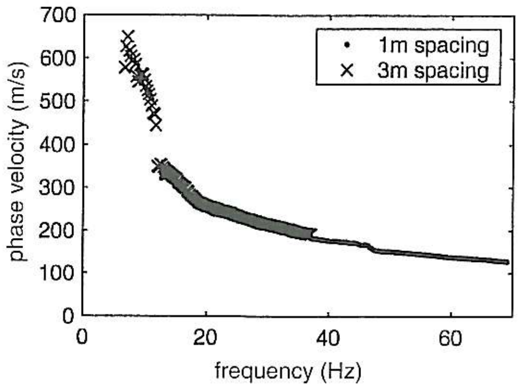

Fig. 13. Site A: experimental dispersion curves.

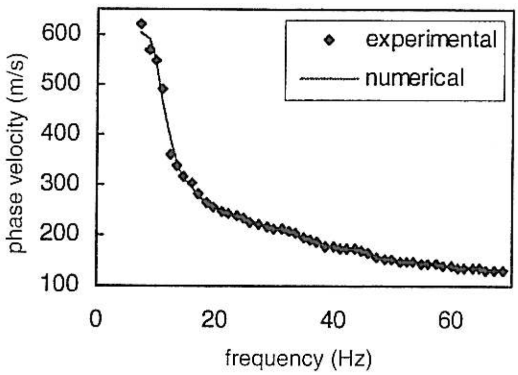

Fig. 14. Site A: experimental *versus* numerical dispersion curve.

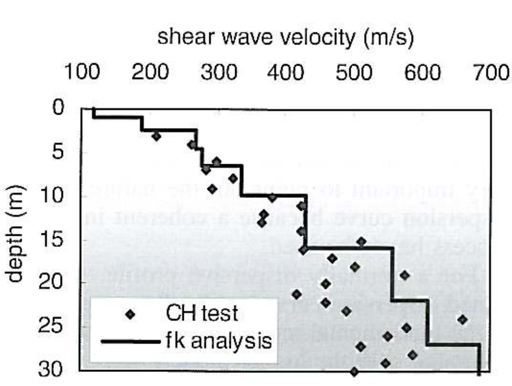

Fig. 15. Site A: shear wave velocity profile.

only information from the 3 m gather is available, while for high frequencies (above 35 Hz) the other gather supplies the information.

The experimental dispersion curve implicitly contains the information relative to the geometry and the mechanical parameter of the soil deposit. An inversion process based on the fundamental mode has been used in this case, because the site is clearly normally dispersive and only the fundamental mode has been recovered from the analysis of the seismic gathers. The starting profile for the inversion process was selected on the basis of an approximate procedure for the estimate of the shear wave velocity as a function of depth (Foti, 2000). The numerical dispersion curve, corresponding to the final iteration of the inversion procedure, shows a good agreement with the experimental one (fig. 14).

The resulting shear wave velocity profile, compared to the results of a cross-hole test, which was executed nearby, is represented in fig. 15.

4.2. *Site B*

In the second testing site a stiff top layer, composed by a common road pavement, is present over a natural soil. Unfortunately no detailed information from other tests is available for this site and hence a comparison is not possible. Only the presence of bedrock at a quite shallow depth is known by a later excavation.

Since in this case the depth of interest for the survey was shallower than in the previous case, a testing configuration having a short receiver spacing (0.5 m) was selected. The source used for this survey was the 6 kg hammer. The corresponding *fk* spectrum is reported in fig. 16.

The presence of the stiff top layer makes this profile inversely dispersive and the obtained experimental dispersion curve (fig. 17) must be considered as the superposition of several modes of propagation.

In this case a different approach must be used for the inversion process. A numerical estimate of the apparent velocity that is associated to mode superposition can be derived from an

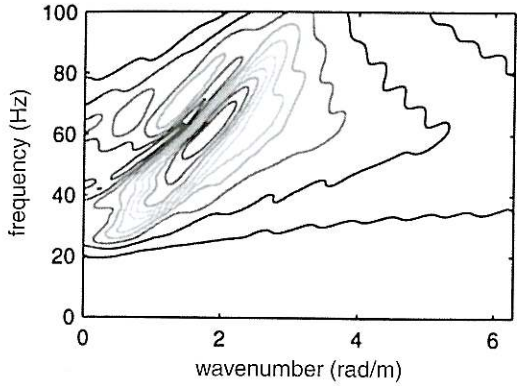

Fig. 16. Site B: *fk* spectrum (receiver spacing: 0.5 m; source: hammer).

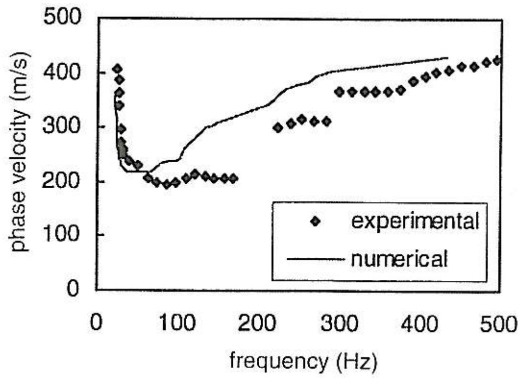

Fig. 17. Site B: experimental *versus* numerical dispersion curve.

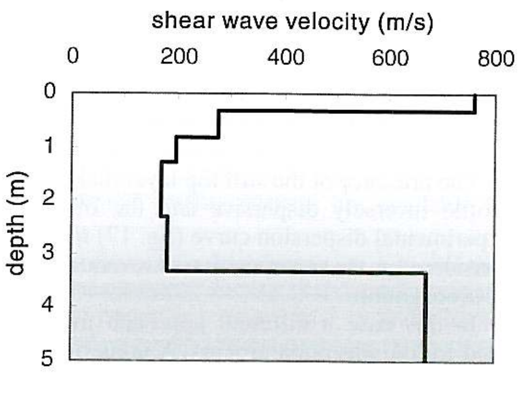

Fig. 18. Site B: shear wave velocity profile.

analysis of the numerical simulation of the wavefield generated by a point source at short distances from the source itself. An iterative inversion procedure based on such estimate was used to obtain the shear wave velocity profile (fig. 18). The numerical apparent phase velocity corresponding to the profile is compared with the experimental one in fig. 17.

5. Conclusions

The analysis of surface waves has proven to be a very powerful tool for soil characterisation when the stiffness of the medium is of interest. One main step in the test interpretation is the estimation of the experimental dispersion curve, which is subsequently used for an inversion process in view of obtaining the shear wave velocity profile of the site. Such an estimate can be obtained using several techniques. Among them one of the most effective is the analysis of seismic gathers in the frequency-wavenumber domain.

Using numerical simulations and experimental results, it has been shown that the length of the survey has many major consequences on the estimation of the dispersion curve. These consequences are strictly related to the multimodal nature of surface waves and to their dispersive behaviour. Indeed the different group velocity causes mode separation to take place over large distances, but at short distances from a point source the wave train is constituted by a composition of the different modes.

In general, using short receiver arrays, as is usual in engineering applications, the resolution of the *fk* spectrum is reduced and it is not possible to distinguish several modes of propagation. Hence a single dispersion curve is obtained, as in the two-receiver SASW test. It is very important to point out the nature of this dispersion curve because a coherent inversion process has to be used.

For a normally dispersive profile, the obtained dispersion curve practically corresponds to the fundamental mode, which is always predominant over the higher modes.

In inversely dispersive profiles, *i.e.* when the soil stiffness does not always increase with depth or when there are strong impedance contrasts, higher modes play a major role in the propagation of Rayleigh waves. In such cases, the dispersion curve obtained with the *fk* analysis regards an «apparent» phase velocity, that is related to a mode superposition effect. Nevertheless, it is possible to estimate the correct shear wave velocity profile using an appropriate numerical procedure, accounting for modal superposition.

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