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On the relationship between M_L and M_W in a broad range: an example from the Apennines (Italy)

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We quickly summarize the procedure used in this study, for more details see Malagnini and Dreger (2016). In order to obtain the scaling relationships for the high-frequency ground motion in the region of central-northern Apennines (Italy) that was hit by the recent seismic sequence (2016-2017), regressions were carried out over 78,727 selected waveforms recorded during 659 events with magnitude ranging from Mw 3.0 to Mw 6.33. Digital data were corrected for instrument response to actual ground motion, and the peak ground velocities were measured in selected narrow-frequency bands, between 0.1 and 22.5 Hz. Ground motion attenuation with distance (Figure S1) and the variation of excitation with magnitude (Figure S2) were parameterized for this area to define a consistent model that describes peak ground motion. Regression results for peak velocities were used to define a piecewise linear (in the *log-log* space) continuous geometrical spreading function, a frequency-dependent attenuation parameter, $Q(f) = Q_0 \left(f / f_{ref}\right)^{\eta}$, and a distance-and-frequency-dependent duration function (Figure S3).

A general form for a predictive relationship for observed ground motion may be written as:

$$A_n\left(f_i, r_{jk}\right) = SRC_j\left(f_i, r_0\right) + SITE_k\left(f_i\right) + D\left(r_{jk}, r_0, f_i\right) \tag{S1}$$

where $A_n\left(f_i, r_{jk}\right) = \log_{10} a\left(f_i\right)$ represent the logarithm of peak amplitude observed, in the time domain, on a narrow bandpass-filtered version of the *n-th* time history, $SRC_j\left(f_i, r_0\right)$ is the excitation term for the ground motion at an arbitrary reference hypocentral distance r_0, $SITE_k\left(f_i\right)$ represents the distortion of the seismic spectra induced by the shallow geology at the recording site, the propagation term is indicated as $D\left(r_{jk},r_0,f_i\right)$ and represents an estimate of the average crustal response for the region at the hypocentral distance r_0, at the central frequency f_i. In the *log-log* space, it is determined as a piecewise linear function (Yazd, 1993; Anderson and Lei, 1994; Harmsen, 1997), allowing to consider complex behavior of the regional attenuation. Finally, r_0 is an arbitrary hypocentral distance at which all source terms are referred (for the regressions run in this paper we use 80 km); this is achieved by forcing the constraint $D\left(r_{jk},r_0,f_i\right)=0$ to the *i-th* regression (e.g. see Malagnini et al., 2000).

Crustal attenuation for the central-northern Apennines region

The regional attenuation term $D\left(r_{jk},r_0,f_i\right)$ obtained for the central and northern Apennines is shown in Figure S1. We modeled the empirical estimates of the peak amplitudes, as a function of hypocentral distance, at different sampling frequencies. Colored curves represent deviations from the 1/r trend for the normalized attenuation functions. Black curves in the background represent our theoretical predictions of the attenuation functions obtained for each central frequency with the following equation:

$$D\left(r_{jk},r_0,f_i\right)=\log g\left(r\right)-\log g\left(r_0\right)-\frac{\pi f_i\left(r-r_0\right)}{\beta Q_0\left(f/f_{ref}\right)^{\eta}}\log e \qquad \text{(S2)}$$

The crustal attenuation is described as a combination of the effects of the geometrical spreading *g(r)*, and of the anelastic attenuation represented by the quality factor *Q(f)*. The best fit is obtained with the following values, where $Q\left(f\right)=160\left(f/f_{ref}\right)^{0.33}$ ($f_{ref}=1.0$ Hz), and the geometrical spreading function at all distances are:

$$g\left(r\right)=\begin{cases} r^{-1} & r<r_1=30\ km \\ \left(\frac{1}{r_1}\right)\left(\frac{r_1}{r}\right)^{-0.5} & r\geq r_1 \end{cases} \qquad \text{(S3)}$$

Source excitation terms

The empirical excitation terms, $SRC_j(f, r)$ are modeled using the Brune (1970; 1971) spectral model:

$$SRC(f,r) = S\frac{M_0}{\left[1+\left(\frac{f}{f_0}\right)^2\right]} g(r)\exp\left[-\frac{\pi f}{\beta Q_0\left(\frac{f}{f_{REF}}\right)^{\eta}} r\right]\exp\left[-\pi\kappa_0 f\right]v(f)$$

$$\begin{aligned} &v(f) = Generic\ Rock\ Site \\ &\kappa_0 = 0.035\sec \\ &f_{REF} = 1.0\ Hz \\ &\Delta\sigma_B = \frac{7}{16}\frac{M_0}{r_{crack}^3} \\ &r_{crack} = \frac{2.34\beta}{2\pi f_0} \end{aligned} \qquad \text{(S4)}$$

They describe the horizontal peak ground velocity as a function of frequency at the reference hypocentral distance (Figure S2). We fit the empirical excitation curves using the Random Vibration Theory (RVT, Cartwright and Longuet-Higgins, 1956), with the spectral model defined in eqs. (S3) and (S4), and a duration function at the reference distance r_0 that is the result of a regression ($T=T(r=r_0, f_i)$, see Figure S3). The RVT model has been shown to be quite robust in predicting the standard engineering ground-motion parameters. The Parseval and the convolution theorems, together with the RVT, can be used to completely switch from peak values in the time domain to Fourier spectral amplitudes. Equation (S1) is solved in the time domain, from multiple narrow band-pass signals.

To predict the seismic spectra we used a single corner frequency Brune spectral model, $s(f, \Delta\sigma_B)$, with a stress drop $\Delta\sigma_B = \Delta\sigma_B(M_0)$ varying as a function of the seismic moment (or moment magnitude, see Figure 2 of the main text). The generic rock amplification function $v(f)$ by Boore and Joyner (1997) is used to obtain the excitation terms of Figure S2, coupled with a parameter

k_0 = 0.035 s. The $\Delta\sigma_B$ parameter is an effective stress parameter, which does not necessarily represent the stress drop relaxed coseismically across the fault plane, but which is needed in order to define, with a single corner frequency Brune spectrum, the spectral shapes of the empirical excitation terms.

Duration of the ground motion

The quantification of an effective duration of the seismograms as a function of hypocentral distance and frequency is critical for a correct use of RVT, and the reader is referred to Raoof et al., (1999) and to Malagnini et al. (2000) for a thorough discussion on this aspect of the technique. The definition of duration of ground motion is given as the width of the time window that limits the 5% - 75% portion of the seismic energy following the *S*–wave arrival. The computation of the duration of the seismic signals is preformed independently for each seismogram at each central frequency. Figure S3 shows the computed durations for the recordings available at 6 sampling frequencies.

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Figure S1. The empirical regional attenuation functional $D\left(r_{jk}, r_0, f_i\right)$ obtained for central and northern Apennines from the regression on the peak amplitudes of the band-pass-filtered ground velocities at the sampling frequencies (roughly, 0.1 – 19 Hz) shown by colored lines. Black lines in the background represent the theoretical predictions at the same sampling frequencies. The attenuation function is normalized to zero at the arbitrary reference hypocentral distance of 80 km. All lines in the Figure are normalized to a 1/r decay.

Figure S2. Filtered ground velocity excitation terms (black lines) of 659 events from central and northern Apennines data set. Red thick lines indicate the theoretical prediction at the indicated levels of moment magnitude from Brune source spectra coupled to the generic-anelastic attenuation model.

Figure S3. The 5-75% duration distribution for the regional data at different sampling frequencies, as a function of hypocentral distance. Small gray diamonds indicate individual measurements of effective duration, in the sense indicated by Raoof et al. (1999). White diamonds are the results of piece-wise linear regressions in the distance-duration space: they form L_2-norm sets of points where the effective durations are averaged.

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