On the relationship between M_L and M_W in a broad range: an example from the Apennines (Italy)

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Online material: Description of the technique used to compute crustal attenuation, source excitation, and effective duration of the ground motion. A quantitative model is also provided. Figures show results calibrated over waveforms from the central and northern Apennines.

Abstract

Being tied to a physical quantity, moment magnitude (M_W) should be the reference estimate of earthquake size, and used whenever possible. Local magnitude (M_L) represents a simple alternative for a reliable estimate of size, its best use being either for quick outcomes or when the computation of M_W is difficult (e.g., for small earthquakes). However, M_L and M_W are profoundly different and not interchangeable.

Here we analyze a large set of 1509 $M_L - M_W$ data points from earthquakes of the Central and Northern Apennines (CNA), quantify the empirical scaling, and look for features of global validity. Our data set is made of 449 unpublished M_W's from moment tensor solutions of events from the AVN sequence, 170 published M_W's from moment tensor solutions of events from the L'Aquila seismic sequence (2009), and 890 published $M_L - M_W$ data points from earthquakes of the

Altotiberina fault (ATF, 2010-2014; M_W's from spectral correction). We integrate our empirical data set by computing the local magnitudes of the events from the AVN and L'Aquila sequences.

Our analysis of CNA earthquakes shows that, for earthquakes up to a crossover magnitude, $M_{L_{co}} \approx 4.3$:

$$M_W = \frac{2}{3} M_L + C'; \quad C' = 1.14 \,. \tag{1}$$

Moreover, for earthquakes with $M_L > M_{L_{co}}$, up to M_L 6.5, our data suggest:

$$M_W = b M_L + C''; \quad b = 1.28; \quad C'' = -1.50 \,, \tag{2}$$

where b depends on the combined effects of source scaling and crustal attenuation, and *C'* and *C"* on regional attenuation (*G(r), Q(f),* κ_0), focal depth, and rigidity at source.

Finally, a synthetic study calibrated on the crustal attenuation and the source characteristics of the AVN data set reproduces the observed scaling between M_L and M_W, predicting that M_L's in the analyzed region saturate above $M_L \sim 6.5$. Smooth transitions are predicted between the different regimes.

Keywords: *Magnitude, Earthquake.*

Introduction

Numerous studies compare either $\log_{10}(M_0)$ against M_L, or directly M_L against M_W, by using data from different regions of the world and ranges of magnitudes. Comparisons are generally performed without considering the details of source scaling, the role of the crustal attenuation, and which part of the seismic spectrum does the local magnitude depend on. As a consequence, most published empirical relationships do not highlight the fundamental characteristics of the M_L-M_W distributions.

However, a large number of studies may be found in the literature about issues related to estimating seismic magnitudes, and on the relationships between different kinds of these estimates. For example, Archuleta et al., (1982) used the spectral analysis of S-waves over several tens of events occurred at Mammoth Lakes CA ($2.9 \leq M_L \leq 6.2$), finding that M_L is roughly equal to $\log_{10}(M_0)$ plus a constant. Similar results, although with a less steep relationship between M_L and $\log_{10}(M_0)$, were found by Bakun and Lindh (1977) who found $M_L = 0.8 \log_{10}(M_0)$ plus a constant for earthquakes occurred near Oroville (CA) in the range $0 \leq M_L \leq 6$. Bakun and Lindh (1977) also investigated smaller Oroville earthquakes ($0 \leq M_L \leq 2$) and found similar results. Likewise, Bolt and Herraiz (1983) proposed a method for estimating the seismic moments of regional and local earthquakes based on simple measurements made directly on the Wood-Anderson (WA) seismograms of 16 earthquakes in central California. By using a set of earthquakes in the magnitude range $3.0 \leq M_L \leq 6.2$, they found that $M_L = 0.9 \log_{10}(M_0)$ plus a constant.

It is interesting to note that, on data sets of similar characteristics, different researchers obtained fundamentally different results. For example, both Drouet et al., (2008), in France, and Edwards et al. (2008), in the UK, simultaneously inverted source spectra, crustal attenuation parameters, and site responses. Whereas Drouet et al., (2008) found a similar scaling for moment and local magnitudes in the range: $3.0 \leq M_L \leq 5.3$ ($M_W=0.95M_L-0.27$), Edwards et al. (2008) found that $M_W=0.71M_L+0.58$ in the range: $2.0 \leq M_L \leq 4.7$ (that is, M_W and M_L are fundamentally different).

Fletcher et al. (1984) used 14 aftershocks of the Oroville, CA, earthquake ($M_L = 5.7$, 1 August 1975) that range in local magnitude from 2.8 to 5.2. They obtained estimates of seismic moment, and a relationship between M_L and $\log_{10}(M_0)$: below $M_L = 4.1$ they found a slope of 0.9, whereas using earthquakes larger than $M_L = 4.3$ the slope decreased to 0.74.

A shallow slope around 0.7 in the relationship between M_W and M_L for the small earthquakes was found by numerous other studies. For example, Sargeant and Ottemoller (2009), with 64 earthquakes from Britain in the magnitude range $2.7 \leq M_L \leq 4.7$, obtained $M_W=0.70M_L+0.70$. Moreover, Zollo et al. (2014) studied 717 micro-earthquakes in the moment range 4×10^9 - 2×10^{14} Nm from the southern Apennines (Italy), and found: $M_W=(0.74\pm0.01)M_L+(0.66\pm0.02)$ for a magnitude range $0.1 \leq M_L \leq 3.4$. For M_L smaller than about 2.5, they observed a systematic underestimation of moment magnitude by local magnitude, which they interpreted as due to inadequate corrections for wave propagation effects in the technique used for estimating M_L.

Deichmann (2006) found a deviation from the 1:1 scaling for earthquakes with magnitudes below about 3 that could be due to frequency-dependent attenuation along the propagation path. More recently, Deichmann (2017) presented a detailed analysis of a sequence of natural earthquakes, as well as events from induced seismicity. Based on the observations and simulations, he found that $M_L \propto 1.5\ M_W$ for small events ($M_W < 3$), in agreement with the results of other studies (Hanks and Boore, 1984; Edwards et al., 2010, 2015; and Munafò et al., 2016).

The results of other researchers leaned more towards a 1:1 dependence between M_L and M_W. Grünthal et al. (2009) used a catalogue of earthquakes occurred in central, northern, and northwestern Europe with $M_W \geq 3.5$, and found: $M_W=0.906M_L+0.65$. Johnson and McEvilly (1974) used 13 earthquakes with magnitudes between 2.4 and 5.1 located near the San Andreas fault in central California. They found: $\log_{10}(M_0)=(17.60\pm0.28)+(1.16\pm0.06)M_L$. Finally, Margaris and Papazachos (1999) calibrated a relationship based on data from Greek earthquakes ($3.9 \leq M_L \leq 6.6$); they used the half peak-to-peak WA amplitudes and found a tight equivalence between M_W and M_L: $M_W=1.0M_L+0.06$.

Ristau (2009) investigated the M_L-M_W relationship in different ranges of hypocentral depths. He used New Zealand earthquakes with $Mw \geq 3.5–4.0$ and focal depths < 33km, and found: $M_L=(0.88\pm0.03)M_W+(0.73\pm0.20)$. Ritsau (2009) also showed that shallow earthquakes in the region had values of M_L that were fairly consistent with the corresponding M_W's, particularly for events with $M_W \geq 4.5$, whereas deep earthquakes (depths > 33 km) had estimates of M_L that were consistently larger than the corresponding M_W's: $M_L=(1.09\pm0.10)M_W+(0.05\pm0.06)$.

Differently from the approaches just listed, Munafò et al. (2016, hereafter M2016) took into consideration the scaling of the observed seismic spectra, coupled to the filtering actions due to combined effects of the WA seismometer and of the Earth's crust. They demonstrated how such a combined effect would result in a relatively narrow bandpass filter centered roughly at the WA natural frequency (f_{cWA}=1.25 Hz). Such a filter effectively samples the spectral plateau (i.e., the seismic moment) of small earthquakes, up to a crossover magnitude $M_{L_{CO}} \approx 4$. For earthquakes beyond $M_{L_{CO}}$, the WA-Earth filter samples the part of the spectrum beyond the corner frequency, where the spectral roll-off takes place. As a consequence, the scaling between the two magnitudes changes, and the $M_W - M_L$ relationship becomes steeper.

M2016 analyzed a large data set of small earthquakes occurred on the fault plane and in the hanging wall of the Altotiberina Fault (ATF) in the northern Apennines, and inverted for excitation and site terms (coupled together), and for a fully decoupled regional attenuation term. M2016 computed precise seismic moments (and thus M_W) of very small earthquakes with a technique defined by Malagnini and Dreger (2016). M2016 also provided precise estimates of local magnitude for all the events in their data set.

By convolving the theoretical moment-rate spectra expected in this region with the crustal attenuation, and with the WA transfer function, M2016 calculated the spectra that would come out of the WA, at two sampling hypocentral distances: 20 and 40 km. Such an operation demonstrated that the synthetic WA spectra, once filtered through the Earth's crust, are characterized by a fairly stable dominant frequency that is around the natural frequency of the instrument (f_{WA}).

In their study, M2016 used point-source Brune spectra that scale realistically as a function of magnitude in the range: $0.84 \le M_w \le 3.50$, and explicitly demonstrated that the local magnitude of small earthquakes scales as:

$$M_L = \log_{10}(M_0) + C; \tag{3}$$

or, which is the same:

$$M_W = \frac{2}{3} M_L + C'. \tag{4}$$

Goal of the present study is to investigate the relationship between M_L and M_W in a much broader range of sizes.

Local Magnitudes and Random Vibration Theory

The definition of local magnitude is (Richter, 1935):

$$M_L = \log_{10} A - \log_{10} A_0\left(\delta\right) = \log_{10}\left[A / A_0\left(\delta\right)\right], \tag{5}$$

where A is the largest peak-to-peak value observed on the WA seismogram (adjacent peaks), and $A_0\left(\delta\right)$ is an empirical correction (for a study about the calibration of M_L in Italy, see Di Bona, 2016).

Random Vibration Theory (RVT, see Cartwright and Longuet-Higgins, 1956) relates the peak value of a stationary time history of infinite length to the moments of its spectrum:

$$Peak\left(a\left(t\right)\right) \approx \eta\left(a\left(t\right)\right)_{RMS} \tag{6}$$

where:

$$\eta = \eta\left(m_0, m_2, m_4\right) \tag{7}$$

and:

$$m_n = \frac{1}{\pi}\int_0^{\infty} \omega^n \left|\hat{a}\left(\omega\right)\right|^2 d\omega . \tag{8}$$

By invoking the Parseval equality we can switch from peak values to Fourier amplitudes:

$$\int_{-\infty}^{+\infty} \left|a\left(t\right)\right|^2 dt = \int_{-\infty}^{+\infty} \left|a\left(f\right)\right|^2 df . \tag{9}$$

If we deal with a band-limited time history (≠0 only between f_1 and f_2) that is ≠0 only in the time window [0,T]:

$$a_{RMS} = \sqrt{\frac{\int_0^T \left|a\left(t\right)\right|^2 dt}{T}} = \sqrt{\frac{\int_{-\infty}^{+\infty} \left|\hat{a}\left(f\right)\right|^2 df}{T}} = \sqrt{\frac{2\int_{f_1}^{f_2} \left|\hat{a}\left(f\right)\right|^2 df}{T}} . \tag{10}$$

By using the previous equations, and keeping in mind the characteristics of the WA coupled to the regional crustal attenuation, Munafò et al. (2016) showed that M_L of small quakes directly samples their spectral plateau. Equations (3) and (4) directly follow from the previous statement, and are valid up to a crossover magnitude $M_{L_{CO}} \approx 4$. Because of the complications in the interaction between the sampling frequency and the corner frequencies rof larger seismic sources, for

$M_L > M_{L_{CO}}$ we expect a steeper relationship between M_W and M_L (see Di Bona, 2016). For large earthquakes, say beyond $M \sim 6.5$, M_L is expected to saturate.

In order to define the scaling relationship between M_L and M_W in a magnitude range as broad as possible, we compute precise M_W's and M_L's for a large set of earthquakes, and calculate a simple relationship between the two quantities. We extend the empirical data set by producing a synthetic set of M_L's starting from a given set of M_W's that overlaps the empirical data set and broaden it to smaller and larger magnitudes. We perform such task by modeling source spectra and crustal attenuation, taking into account the observed scaling of source spectra and dispersion of seismic waves (i.e., the effective duration of the ground motion, see Figure S3 available in the Electronic Supplement to this article).

Whereas all the mentioned quantitative models could be taken from the literature (Malagnini et al., 2011), we prefer to investigate the recent Amatrice-Visso-Norcia seismic sequence of the central and northern Apennines (hereafter CNA), which represents the most important part of our data set. Once we obtain the Earth's attenuation for the region illuminated by the recent sequence, and a scaling relationship for the source parameters, we extrapolate our results to larger and smaller events, generate stochastic time histories between 10 and 200 km of hypocentral distance, and compute estimates of M_L on the synthetic data.

In order to quantify the scaling of the Brune stress drop (Brune, 1970, 1971), we use a source spectral ratio approach based on the observed seismic spectra of the sequence: source spectra of the recent earthquakes are obtained using a regression technique outlined by Malagnini et al. (2011); the details of the spectral ratio approach are explained in Malagnini et al. (2008), who used coda waves, and extended to direct S- waves by Malagnini et al. (2014). An extension of the technique to the calculation of seismic moments using a hybrid approach on both Fourier and peak amplitudes was described by Malagnini and Dreger (2016).

It is important to emphasize that this work does not rely on the results obtained by Di Bona (2016). Rather, we use the original magnitude correction proposed by Richter (1935). We do so in order to build upon the results by Munafò et al. (2016), our ultimate goal being to indicate the scaling to be expected between M_L and M_W anywhere in the world. In any case, Di Bona showed that there are minor differences between his correction, calibrated over Italy, and the Richter's correction.

Data Set

Here we analyze 659 events of the 2016-2017 Amatrice-Visso-Norcia seismic sequence (AVN, $3.0 \leq M_w \leq 6.33$; 78,727 individual waveforms). For 449 of these events we find a moment tensor solution on R.B. Herrmann's web page (see Data and Resources section), and for each event of the sequence we calculate an estimate of M_L.

From Munafò et al. (2016) we got the M_W's and M_L's of the 890 events of the Altotiberina Fault (ATF, 2010-2014; $0.84 \leq M_w \leq 3.50$; 53,160 individual waveforms). The M_W's of 170 events of the 2009 L'Aquila sequence were obtained by Herrmann et al. (2011, $2.8 \leq M_w \leq 6.13$). Their M_L's were calculated in this study from the data set of 13,098 individual waveforms analyzed by Malagnini et al. (2011). Figure 1 shows the location of all the 1719 events of our data set. For 1509 of these earthquakes we have coherent estimates of M_L and M_W, either from this study or from the cited, published works.

From the inversion of 78,727 individual seismograms that were collected during the AVN seismic sequence, we obtain the crustal attenuation model that is applied in a synthetic study on the relationship between M_W and M_L (see the Electronic Supplement to this article).

Source scaling

We study the source scaling in our data set with the source spectral ratios technique by Malagnini et al. (2014), which is based on the analysis of direct S-waves. Signal-to-noise ratios for the source spectra have been maximized with the use of RVT (see Malagnini and Dreger, 2016 for more details). Figure 2 contains the corner frequencies of a subset of 341 events of the AVN seismic sequence of 2016-2017.

From the visual inspection of Figure 2 we see that some variability characterizes the Brune stress drop of the earthquakes (the slanted dashed lines in Figure 2 indicate constant values of Brune stress drop). Specifically, the Brune stress drops are in the following ranges, at different values of moment magnitude:

$$\Delta\sigma\left(M_W\right)=\begin{cases}0.5-10\ MPa\ for\ M_W\sim 3\\ 1-8\ MPa\ \ for\ M_W\ 4\\ \sim 5\ MPa\ \ for\ M_W\ 5\\ \sim 10\ MPa\ \ for\ M_W\ 6\\ \sim 20\ MPa\ \ for\ M_W\ 6.5\\ 20\ MPa\ \ for\ M_W\geq 7\end{cases} \tag{11}$$

The observed variability in the stress drop was used to produce the synthetic data points shown in Figure 3.

Producing synthetic waveforms and estimates of M_L as a function of M_W

We produce synthetic spectra at a suite of moment magnitudes, from M_W -1 to M_W 8, using the Brune (1970, 1971) spectral model. About the effective validity of such a spectral model for modeling the spectra of large earthquakes, Mayeda and Malagnini (2009) demonstrated that the Brune model could successfully be used to satisfactorily fit the spectral features of even the Chi-Chi earthquake. Specifically, Mayeda and Malagnini (2009) used the Brune spectrum to implement a fitting procedure that was applied to source spectral ratios calculated on the Chi-Chi mainshock (M_W 7.8) and some of its aftershocks.

Once we have reliable spectra, and the measure of the characteristics of dispersion (duration) as a function of hypocentral distance and frequency, we follow Boore (1983) and produce stochastic time histories at a suite of hypocentral distances between 10 and 200 km, for $-1 \leq M_W \leq 8$ every 0.1 M_W units. For each synthetic event we calculate an average local magnitude. In Figure 3, white diamonds indicate the synthetic data points.

Calculations are performed using the Generic Rock Site published by Boore and Joyner (1997), the duration functional form described in Figure S3, available in the Electronic Supplement to this article, and the regional attenuation calculated solely on the current data set from the AVN seismic sequence (Figure S1 of the Electronic Supplement to this article):

$$Q(f) = 160\left(\frac{f}{f_{ref}}\right)^{0.33} \;;\; f_{ref} = 1.0\ Hz \tag{12}$$

$$g(r) = \begin{cases} r^{-1} & r < r_0 = 30\ km \\ \left(\dfrac{1}{r_0}\right)\left(\dfrac{r_0}{r}\right)^{-0.5} & r \geq r_0 \end{cases} \tag{13}$$

$$S(f, r_{ref}) = C\frac{M_0}{\left[1+\left(\dfrac{f}{f_0}\right)^2\right]} g\left(r_{ref}\right)\exp\left[-\frac{\pi f}{\beta Q_0\left(f\right)^{\eta}} r_{ref}\right]\exp\left[-\pi\kappa_0 f\right] \tag{14}$$

$$r_{ref} = 80\ km \tag{15}$$

The relationship between the radius of the crack and its corner frequency is (Brune, 1970, 1971):

$$r_0 = \frac{2.34\beta}{2\pi f_0}\,. \tag{16}$$

The relationship between corner frequency and stress drop is that by Keilis-Borok (1959) or Kanamori and Anderson (1975):

$$\Delta\sigma = \frac{7}{16}\frac{M_0}{r_0^3}\,. \tag{17}$$

The source spectral scaling used for the fit shown in Figure S2, available in the Electronic Supplement to this article, is the following:

$$\Delta\sigma\left(M_W\right)=\begin{cases}1\ MPa\ \ for\ M_W\ 3\\3\ MPa\ \ for\ M_W\ 4\\7\ MPa\ \ for\ M_W\ 5\\12\ MPa\ \ for\ M_W\ 6\\18\ MPa\ \ for\ M_W\ 6.5\\20\ MPa\ \ for\ M_W\geq 7\end{cases}\tag{18}$$

The synthetic excitation terms of Figure S2, available in the Electronic Supplement to this article, are obtained using the Brune source model, the scaling described in eq. (18), the regional attenuation of eq. (12), the Generic Rock site amplification by Boore and Joyner (1997), duration as a function of frequency and hypocentral distance, and the following high-frequency cut-off filter

$$\exp\left(-\pi\kappa_0 f\right)\tag{19}$$

with

$$\kappa_0=0.035\,\mathrm{sec}\tag{20}$$

For the calculation of the stochastic synthetic seismograms we actually used the duration at around 2 Hz (see Figure S3, available in the Electronic Supplement to this article). However, if we tried durations at other frequencies, results would not change appreciably.

Putting together all data points from this study, and from the studies by Munafò et al. (2016), and Herrmann et al. (2011), we gathered the 1509 M_W - M_L data points plotted in Figure 3, whose epicenters are mapped in Figure 1. Gray squares in Figure 3 indicate data points from Munafò et al. (2016); dark triangles represent data points of the AVN and L'Aquila sequences. For the L'Aquila seismic sequence of 2009 we use the moment magnitudes calculated by Herrmann et al. (2011). Finally, for the recent seismic sequence of 2016-2017, moment magnitudes were taken from Robert B. Herrmann's web page (See Data and Resources section).

The variability observed in the Brune stress drop (Figure 2) was reproduced with the synthetic data. The latter were calculated using RVT and eqs. (11)-(15), with the information on duration given in Figure S3, available in the Electronic Supplement to this article. Synthetic data are plotted in Figure

3 using white diamonds, together with the distribution of the observed M_W - M_L data points, and a bi-linear fit obtained only on the empirical data points: one linear fit for small earthquakes, virtually identical to what found by M2016:

$$M_W = \frac{2}{3} M_L + 1.14 , \tag{21}$$

hinged to a second linear fit calculated for events larger than M_L 4.3:

$$M_W = 1.28\ M_L - 1.50 \tag{22}$$

(hinge is around M_L 4.3). The distribution of the synthetic data points, however, suggests a smooth transition between the two regimes (small events to large events). Finally, M_L saturates above M_L 6.5 with a smooth transition.

Conclusions

Goal of this paper is not to propose a new correction for the central-northern Apennines, but rather to show, in a broad range of sizes, the details of the $M_L - M_W$ scaling that are due to the narrowband filtering action of the Wood-Anderson seismometer. For small earthquakes, up to a crossover magnitude ($M_{L_{co}} \approx 4.3$ for the CNA), we obtain:

$$M_W = \frac{2}{3} M_L + 1.14 , \tag{23}$$

a result that is virtually identical to what obtained by Munafò et al. (2016). For larger earthquakes in the CNA, beyond the crossover magnitude and up to M_L 6.5 or less, the slope becomes steeper:

$$M_W = 1.28\ M_L - 1.50. \tag{24}$$

From the results of a set of stochastic simulations we find that a smooth transition is to be expected between the two regimes that characterize the magnitude scaling for "*small*" and for "*moderate size*" earthquakes. Finally, we show how M_L smoothly saturates for large earthquakes, above M_L 6.5.

We stress that the general features obtained from our distributions of $M_W - M_L$ data points are valid globally.

Data and Resources

The earthquake catalogue used in this study was created using the INGV portal of the Centro Nazionale Terremoti (info.terremoti.ingv.it)

The raw waveforms of the Italian earthquakes that are used in this study may be downloaded from the European Integrated Data Archive (EIDA) repository at http://www.orfeus-eu.org/eida/eida.html (last accessed September 2017).

Moment magnitudes for the Amatrice-Visso-Norcia seismic sequence were taken from Robert B. Herrmann's web page http://eqinfo.eas.slu.edu/eqc/eqc_mt/MECH.IT/ (last accessed September 2017). Moment magnitudes of the events of the L'Aquila sequence were taken from Herrmann et al. (2011). Moment magnitudes of the ATF faults were taken from Munafò et al. (2016).

Some figures were made using the Generic Mapping Tools version 4.2.1 (www.soest.hawaii.edu/gmt; Wessel and Smith, 1998).

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Figure Captions

Figure 1. Red dots: 890 events of the Altotiberina Fault (ATF, 2010-2014; 0.84 ≤ M_W ≤3.50); **yellow dots:** 659 events of the Amatrice-Visso-Norcia seismic sequence (2016-2017; 3 ≤ M_W ≤ 6.33); **green dots:** 170 events of the 2009 L'Aquila sequence (2.8 ≤ M_W ≤ 6.13). Gray squares indicate the main cities of the area. Note that the scales in the legend are different.

Figure 2. Corner frequencies calculated for a subset of 341 events of the Amatrice-Visso-Norcia seismic sequence of 2016-2017, obtained with an approach based on source spectral ratios, in which the latter were modeled using Brune (1970, 1971). From the corner frequencies of Figure 2 we infer

the variability in the Brune stress drop that is used to obtain the synthetic set of M_W-M_L data points of Figure 3 (see eq. (11)).

Figure 3. M_W - M_L data points from: i) ATF events in the time window 2010-2014 (light gray squares, from Munafò et al., 2016); ii) central-northern Apennines seismic sequence (Amatrice, Visso, Norcia - AVN - 2016-2017, from R.B. Herrmann's web page, see Data and Resources section), and L'Aquila (2009, from Herrmann et al., 2011); iii) synthetic estimates of M_L for a given M_W, for 0.1 M_W units increase in seismic moments, obtained using the crustal attenuation parameters calibrated on the AVN sequence, and the source scaling described by eq. (11). Diamonds indicate synthetic M_L's obtained as follows: from a seismic model completely calibrated on a specific seismic sequence, we produce reliable seismic spectra at various distances to the fault. We then use a stochastic approach to compute synthetic waveforms, given the seismic spectrum and the empirical estimates of duration as a function of distance for the region. From the synthetic waveforms we compute M_L's. Slanted lines indicate the best-fit function up to a crossover local magnitude ($M_{L_{co}}$, slope 2/3), and beyond $M_{L_{co}}$ (slope 1.28). Vertical dotted lines indicate $M_{L_{co}} \approx 4.3$ and the saturation M_L.