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Preprint of:
Considering uncertainties in the determination of earthquake source parameters from seismic spectra

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SUMMARY

In this paper we present a method for handling uncertainties in the determination of the source parameters of earthquakes from spectral data. We propose a robust framework for estimating earthquake source parameters and relative uncertainties, which are propagated down to the estimation of basic seismic parameters of interest such as the seismic moment, the moment magnitude, the source size and the static stress drop. In practice, we put together a Bayesian approach for model parameter estimation and a weighted statistical mixing of multiple solutions obtained from a network of instruments, providing a useful framework for extracting meaningful data from intrinsically uncertain datasets. The Bayesian approach used to estimate the source spectra parameters is a simple but powerful mechanism for nonlinear model fitting, providing also the opportunity to naturally propagate uncertainties and to assess the quality and uniqueness of the solution. Another important added value of such an approach is the possibility of integrating information from the expertise of seismologists. Such data can be encoded in a prior state of information that is then updated with the information provided by seismological data. The performance of the proposed approach is demonstrated analysing data from the 1909 April 23 earthquake occurred near Benavente (Portugal).

Key words: Fourier analysis; Probability distributions; Earthquake source observations; Seismicity and tectonics.

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1 INTRODUCTION

Progress in instrumentation and seismic source theory have greatly improved the capabilities for determining seismic source parameters (SSP) from records of the radiated seismic waves. However, the determination of the basic SSP from ground-motion records is subject to high uncertainties of different nature such as the source model used, the source-site path parametrization, the input parameter values, the instrumental errors on records, and the procedures used for determining model parameter values.

The earthquake source parameters have been widely used for computing scaling laws applicable for seismic hazard assessments (e.g., Hanks & Wyss 1972; Hanks & Thatcher 1972; Abercrombie 1995). Earthquake source parameters as the seismic moment and the source dimension have been determined for earthquakes over a wide range of magnitudes (e.g., Abercrombie 1995); however, smaller earthquakes occur much more frequently than larger earthquakes (as described by the classical G-R magnitude-frequency relation, Gutenberg & Richter 1944), and for this reason a much greater portion of the data collected in seismic catalogues is composed of medium-to-low magnitude events. Therefore, determining SSP and relative uncertainties of both large and small events is equally important for gathering valuable information for seismic hazard studies.

Large earthquakes are however rare events and, for this reason, it is necessary to improve our capacities to analyse the records of old earthquakes (e.g., Kanamori 1988); nevertheless, the analysis of such data is often challenged by technical issues posed by the treatment of analogue records (e.g., Batlló et al. 2008). On the other hand, small events are associated to smaller source dimensions and therefore are characterized by higher frequencies that are most affected by attenuation along the path and by near-surface site effects (e.g., Abercrombie 1995). These two limiting cases put in evidence the importance of developing robust methods for data analysis and uncertainty treatment.

Many fundamental earthquake properties can be measured from the spectral content of P and S wave arrivals. For example, earthquake source parameters as the seismic moment (M_0), rupture length (r), and static stress drop ($\Delta\sigma$), are often obtained from measurements in the frequency domain (e.g., Brune 1970, 1971; Hanks & Wyss 1972; Hanks & Thatcher 1972; Molnar et al. 1973; Madariaga 1976; Hanks & Kanamori 1979). Furthermore, it can be found in literature that spectral analyses have been applied for determining SSP of medium to large earthquakes (as e.g., Hanks & Wyss 1972; Atkinson 1993; Sarkar et al. 2000; Ataeva et al. 2015), as well as for small sources and microseismicity considering both natural and induced events (as e.g., Abercrombie 1995; Prejean & Ellsworth 2001; Prieto et al. 2004; Abdulaziz 2014; Zollo et al. 2014; Hua et al. 2015; Ataeva et al. 2015). The seismic source model proposed by Brune (1970, 1971) is in particular one of the most frequently used to determine the main earthquake source parameters using spectral analysis.

M_0 is computed form the low frequencies of the spectra, whereas r and $\Delta\sigma$ are dependent of the corner frequency f_c (see Section 2 for details). Some authors however highlight the limitation of determining r and $\Delta\sigma$ from the corner frequency, pointing out other data (as the source rise time and the maximum slip velocity during fault rupture) as alternative parameters that can be accurately resolved from an observed corner frequency of the spectrum (e.g., Beresnev 2001, 2002).

Taking as reference widely accepted models for the determination of SSP using spectral data, in this paper we present a procedure for considering uncertainties in the estimation of the source parameters of earthquakes from spectral data. The procedure is based on Bayesian data analysis techniques and a weighted statistical mixing of multiple solutions obtained from a network of instruments. It is important to highlight that this study does not intend to discuss the validity or the physical significance of the source model implemented, but it is focused on the details of fitting a model to a dataset and to assess a rather large set of uncertainties that are finally propagated to the derived seismic information. The processing capabilities developed in this paper can be applied to any kind of digital seismic record; however, given the importance that large earthquakes have for seismic hazard studies, in this paper we stress in particular the added value that the proposed methodology has for analysing records of old (and generally large) earthquakes (i.e., events occurred between the end of the 19th century and the first decades of the 20th century that hereinafter are referred to as *historic events*). The performance of the proposed procedure is demonstrated through an illustrative application analysing seismic records from an event occurred on 1909 April 23 near Benavente, Portugal.

2 METHODOLOGY

2.1 Spectral model

The displacement spectrum $U(\omega)$ derived from recorded seismic waves can be described as (e.g., Zollo et al. 2014):

$$U(\omega) = S_0(\omega)G(\omega)R(\omega)I(\omega) \tag{1}$$

where $S_0(\omega)$ represents the spectrum of the source, $G(\omega)$ represents a set of factors related with wave propagation such as geometrical spreading, radiation pattern, and anelastic wave attenuation along the travel path; $R(\omega)$ encloses the characteristics of local site response and free surface, and $I(\omega)$ is the instrumental response in the frequency domain. Adopting a generalized version of the model originally proposed by Brune (1970), the source spectrum can be described as:

$$S_0(\omega) = \frac{\Omega_0}{1 + \left(\frac{\omega}{\omega_c}\right)^{\gamma}} \tag{2}$$

where Ω_0 is the spectral amplitude of the lower frequencies (much lower than the corner frequency), ω_c is the corner angular frequency, and γ is a parameter associated with the high-frequency spectral fall-off (e.g., Boatwright 1980; Zollo et al. 2014).

The term $G(\omega)$ can be expressed as:

$$G(\omega) = C_\xi e^{\frac{\omega t_\xi}{2Q_\xi}} \tag{3}$$

where t_ξ is the travel time, the subscript ξ denotes the wave type (P or S), and Q_ξ is the quality factor. C_ξ is a distance-dependent parameter that can be defined as:

$$C_\xi = \frac{R^\xi_{\theta\phi} F_s}{4\pi\rho\Gamma\nu^3_\xi} \tag{4}$$

where the parameters $R^\xi_{\theta\phi}$ and F_s are, respectively, the radiation pattern and the free surface coefficients, ρ is the average rock density, ν_ξ the ξ-type wave velocity, and Γ a geometrical spreading factor that is a function of the hypocentral distance (Aki & Richards 2002; Zollo et al. 2014).

Following the results of Keilis-Borok (1960), M_0 is related to the displacement spectra through:

$$M^\xi_0 = C^{-1}_\xi \Omega_0 \tag{5}$$

M_0 subsequently is used to calculate M_W using the relationship (Hanks & Kanamori 1979):

$$M_w = 2/3\log M_0 - 6 \tag{6}$$

The source size is related to the shear displacement spectral corner frequency f_c by (Brune 1970; Hanks & Wyss 1972):

$$r = \kappa_\xi \frac{\nu_\xi}{f_c^\xi} \tag{7}$$

where r in this case is associated with the radius of a circular fault; $f_c^\xi = \omega_c/2\pi$, is the corner frequency (of the ξ-type wave spectrum); κ_ξ is a coefficient depending on the rupture model adopted. For example, assuming the model proposed by Madariaga (1976), $\kappa_P = 0.32$ for P waves and $\kappa_S = 0.21$ for S waves; conversely, according to the model proposed by Brune (1970) (and corrected by Brune 1971), $\kappa_S = 0.37$ (originally not defined for P waves). To estimate the source dimension we adopt the Brune's model (Brune 1970, 1971) following the approach presented by Hanks & Wyss (1972). It is worth noting that although Brune (1970) does not attempt to relate the source dimension to a theoretical P-wave spectrum, different authors (see e.g., Hanks & Wyss 1972; Hanks & Thatcher 1972) have shown that this source model is applicable to the body waves in general, which can be done using the same coefficient ($\kappa_S = 0.37$), whereas the velocity (in Eq. 7) becomes the P or S wave velocity, depending on the analysed seismic phase spectrum.

The static stress drop is calculated using the following relationship (Keilis-Borok 1959):

$$\Delta\sigma = \mu \frac{7\pi\overline{u}}{16r} = \frac{7}{16}\frac{M_0}{r^3} \tag{8}$$

2.2 Determination of model parameters

We use the observed displacement spectra to determine the SSP values (and relative uncertainties). In literature, a number of methods are usually employed to estimate source parameters from spectral data, as for example by least-squared fitting (e.g., Boatwright 1980; Dineva et al. 2002; Sonley & Abercrombie 2006), using the L2 norm minimization (Prejean & Ellsworth 2001), or using more robust nonlinear best-fitting methods, as for example the Levenberg-Marquardt least squares algorithm used by Zollo et al. (2014). In this paper we present a methodology for the estimation of SSP values from a spectral model using a fully Bayesian approach. Beyond the information provided by the data, an approach based on Bayesian data analysis opens the way to incorporate other sources of information potentially available, which may be encoded in the prior density function of the model parameters. The prior is then a probability distribution reflecting all the knowledge that we have about a parameter before using the data.

The input data for estimating the source parameters are the displacement spectra of the wave train (P or S) at a given station, corrected for the instrumental response and seismic attenuation (see e.g., Julian & Anderson 1968). Regarding the geometrical ($R^\xi_{\theta\phi}, \Gamma$) and Earth model ($\nu_P, \nu_S, \rho$) parameters required in eqs. (4) and (7), they are usually considered as fixed values; however, we consider that determining their values is also subject to uncertainties. Therefore, we take into account uncertainties in these parameter values and rather than using constant numbers, we consider a range of likely values (defining maximum and minimum boundaries) with a central value as the best estimate parameter; such values are then used to set a truncated Gaussian distribution for representing such uncertainties. The central (best estimate) and uncertainty range for such parameters can be defined either by expert opinion, from literature or calculated for each specific case considering the respective uncertainties involved for their determination.

For the i-th instrument (or observation point) and the ξ wave type considered (P or S), the vector θ of model parameters for the Bayesian inference problem is defined with the three parameters of the source spectrum (eq. 2):

$$\theta^i_\xi = (\Omega_0^{i,\xi}, \omega_c^{i,\xi}, \gamma^{i,\xi}) \tag{9}$$

The Bayes theorem is used to update the defined prior probability density, $\pi(\theta^i_\xi)$, with the information provided by the likelihood of the data, $f(\mathbf{z}^i_\xi|\theta^i_\xi)$, to obtain the posterior distribution $p(\theta^i_\xi|\mathbf{z}^i_\xi)$:

$$p(\theta^i_\xi|\mathbf{z}^i_\xi) = \frac{f(\mathbf{z}^i_\xi|\theta^i_\xi)\pi(\theta^i_\xi)}{\int f(\mathbf{z}^i_\xi|\theta^i_\xi)\pi(\theta^i_\xi)\mathrm{d}\theta^i_\xi} \tag{10}$$

where $\mathbf{z}^i_\xi = (z_1, z_2, \ldots, z_n)$ is the vector of data (i.e., the spectrum of the ξ wave type at the i-th station), and $p(\theta^i_\xi|\mathbf{z}^i_\xi)$ is the conditional distribution of the parameters given the observed data. All the statistics about the model parameters are derived from the posterior distribution.

The opportunity of encoding other sources of information to build an informative prior is a particularly valuable tool to better constrain the results. Specifying $\pi(\theta)$ is a fundamental component of any Bayesian analysis and often one of the main practical problems. A possible way to define a prior state of information in this case is gathering information from the expertise of seismologist. With this aim, we use the displacement spectra plotted in a log-log plot, as shown in Fig. 1. From this plot, we select a range of likely values defining two bounds for Ω_0 and f_c; these bounds are assumed as representing the variance around a mean value, and this information is encoded in the prior as a normal distribution (in the log space). Regarding the γ parameter, we define an upper and a lower bound enclosing possible values as found in literature. In our applications, this range is set as the interval $(1, 3)$, and as for the other two parameters, this information is assumed as representing the variance around a mean and encoded in the prior as a normal distribution.

To set the likelihood function, we define the function:

$$f(\mathbf{z}^i_\xi|\theta^i_\xi) = \frac{p^{1-1/p}}{2\sigma_p\Gamma(1/p)} \exp\left\{-\frac{1}{p}\frac{|\log(\mathbf{z}) - \log(\mathbf{z}(\theta))|^p}{(\sigma_p)^p}\right\} \tag{11}$$

which represents the error distribution of the log difference between the observed ($\mathbf{z}$) and modeled [$\mathbf{z}(\theta)$] spectra. Eq. (11) represents a generalized family of error distributions, being p the parameter that defines which kind of distribution is used (e.g., Varanasi & Aazhang 1989). In our analyses we tested both the symmetric Exponential ($p = 1$) and the Gaussian ($p = 2$); except in specific cases in which the symmetric Exponential distribution may be preferred (e.g., noisy data with outlier points), the Gaussian distribution has been used.

To get samples from the posterior distribution we use a Markov chain Monte Carlo method (e.g., Gelman et al. 2004) using the Metropolis–Hastings algorithm (Metropolis & Ulam 1949; Metropolis et al. 1953). After running the

Markov chain, we remove the burn-in period and check the convergence of the simulated sequences using the Geweke-z-score (Geweke 1992).

2.3 Merging measurements from multiple instruments

The source spectrum parameter values determined from the observed displacement spectra recorded in a network of stations are used to determine the earthquake's SSP of interest (i.e. M_0, M_W, r, and $\Delta\sigma$). This process is performed using a Monte Carlo (MC) procedure to sample the distributions defined for the source spectrum parameter values, the Earth model and the geometrical parameters (see Section 3). Using the samples drawn from the distributions (by wave type), a set of seismic parameters are first determined from each wave type recorded at each instrument (Fig. 2a). These solutions are then combined in order to obtain a set of parameters for the respective instrument in an observation point. The obtained distributions coming from different stations are then combined (Fig. 2b) in order to obtain a unique set of distributions for the source spectrum parameters derived from the available dataset. All these combinations are performed by statistically mixing the starting distributions (e.g., Ray & Lindsay 2005), calculating in this way a weighted average of the distributions, that is

$$f(\mathbf{x}) = \sum_i w_i \cdot f_i(\mathbf{x}) \tag{12}$$

where $\sum_i w_i = 1$, $f_i(\mathbf{x})$ is the ith probability density function with weight w_i, and $f(\mathbf{x})$ is the mixed probability density function. The weights w_i are defined at each merging stage (i.e., instrument components, seismic phases (P, S) and instrument/station; for details see the Appendix).

The weights w_i for the mixing in eq. (12) depend essentially on the quality and availability of the input data. The quantification of these weights is somehow subjective, and it is largely based on expert judgements. However, we argue that it is more subjective *a priori* assigning equal weights, in which case it is assumed that all the available records are equally informative, independently from their quality. For example, the seismic records should be first rotated in the direction of the incident ray before calculating the spectra. However, using records of historical events it is often the case that the orientation of the instrument is unknown, or only one or two components are available. To overcome this limitation, we usually perform the vector sum between the available components of the records. Of course, all of these practical problems should be reflected in the weighting schemes adopted, that is, the better the data and information available, the larger the weight. It is worth noting that the adoption of a weighting scheme is a transparent formalization that allows for accounting for data with different quality. In common practice, sources of data are often disregarded based on some asserted quality criterion. Here, the weighting scheme forces for an explicit discussion of such a process through a transparent evaluation of the quality of the input data, recalling in this the issue of weighting alternative models in Logic Trees or in Ensemble modeling (Bommer & Scherbaum 2008; Marzocchi et al. 2015) for which different techniques based on pre-defined criteria have been discussed in literature (e.g., Bommer et al. 2005; Cotton et al. 2006; Bommer et al. 2010).

3 CASE STUDY AND DATA: THE 1909 APRIL 23 EARTHQUAKE NEAR BENAVENTE, PORTUGAL

To implement and demonstrate the performance of the proposed methodology, we analyse the available data from an earthquake near Benavente (Portugal) occurred the 1909 April 23 (Fig. 3). The event is considered the largest crustal earthquake occurred in the Iberian Peninsula during the 20th century, and it caused considerable damages in the central part of Portugal. In particular, the village of Benavente (about 40 km NE from Lisbon) was completely destroyed (e.g., Teves-Costa & Batlló 2011). A set of values of SSP found in literature for this event (Dineva et al. 2002; Kárník 1969; Teves-Costa et al. 1999; Stich et al. 2005) are summarized in Table 1.

The spatial distribution of the seismic stations considered in this study is presented in Fig. 3. For our analyses, we have used the same dataset used by Stich et al. (2005). The records have been first corrected for the instrumental response. Information about the station network, available records, and the main characteristics of the instruments are summarized in Table A1 in the Appendix (more detailed information can be found in Stich et al. 2005). It is worth noting that verifying the instrumental response for such instruments is a difficult task (for a discussion see, e.g., Batlló et al. 2008; Palombo & Pino 2013); in some cases such a lack of information may induce systematic errors to this kind of analyses. This fact strengthens the need of taking into account the uncertainties in the information obtained from such a dataset. Likewise, it makes evident the importance of using a large dataset from a number of observation sites in order to compensate possible biases in the amplitude estimation.

The merging process requires assigning weights to the available records. In this work we have adopted a simple scheme of weights based on the quality of the seismic signal respect to the background noise, assigning a zero weight to highly noisy records, and equal weights to all the remaining records. The weights used in this work are summarized in Table A2 in the Appendix.

Regarding the parameters for amplitude correction, in the analysis of this dataset we have considered the effects of geometrical spreading, the radiation pattern, and the free surface coefficient, whereas we have neglected the site response and the anelastic attenuation. Neglecting anelastic attenuation may produce systematic biases in the model parameter values determined from the observed spectra; however, the earthquake considered in this example is big enough so that $f_c << 1$Hz and, therefore, the anelastic attenuation hardly affect the amplitude in this frequency range (e.g., Hanks & Wyss 1972). In practice, neglecting the effect of anelastic attenuation in this case have almost negligible effect in the determination of M_o (and as a consequence in M_W); conversely, it might tend to produce higher values of γ and a systematic underestimation of f_c (mainly for the longer travel times to the farer stations), which can affect our results for r and $\Delta\sigma$. Descriptions of such possible effects of under-correction for attenuation can be found, for example, in Hanks (1982), Anderson & Hough (1984) and Prejean & Ellsworth (2001).

As described in Section 2, our approach for SSP estimation may take into account possible uncertainties in the required input Earth model and geometrical parameters. For this example in particular, we determine a 'best estimate' and maximum/minimum boundary values from literature (most of them from previous literature analysing this event, as e.g., Teves-Costa et al. 1999; Dineva et al. 2002; Stich et al. 2005). For example, an average radiation pattern has been assumed as the best value for $R^{\xi}_{\theta\phi}$ (i.e., 0.40 and 0.63 for the P and S waves, respectively, Wyss & Brune 1968; Boore & Boatwright 1984), with an uncertainty range based on considering the likely uncertainty of the source orientation and the considerations in Boore & Boatwright (1984). The range of values for the geometrical spreading have been determined considering possible uncertainties in the source location. Finally, the free surface coefficient is the only coefficient that has been assumed constant, assuming vertical ray incidence at all the stations (e.g., Dineva et al. 2002; Moskvina 1987).

Examples of parameter values defined for an observation point (CRT station) are shown in Table 2. While the first four parameters in Table 2 are general and assumed constant for all the instruments, the last three parameters are dependent on the distance and orientation of the observation point respect to the earthquake source geometry.

4 RESULTS

The seismic parameters of interest for this case study have been determined from the full dataset available. Fig. 4 shows an example of the solution obtained for the source spectral model of the P-wave data recorded using a Wiechert instrument in the CRT station (Cartuja, Spain, see Table A1 in the Appendix). Fig. 4(a) shows the spectra data and the model solution fitted to the observations. The solid line of the model represents the median of the solution, whereas the discontinuous lines are the 10th and 90th percentiles of the solutions representing uncertainty bounds. The uncertainties in the source spectrum parameter values are shown in Fig. 4(b), where the empirical distributions obtained for the three source spectrum parameters are shown. Fig. 5 shows an example of the samples drawn from the distributions used to model uncertainties in Earth model and geometrical parameters, which were used to perform the calculations following an MC approach.

Merging all the solutions from the available phases in a single station, a set of SSP solutions are determined for that observation point. As example, Fig. 6 shows the histograms and statistics of seismic parameters determined using all the data obtained from the CRT station. In each panel, the median and two percentiles (10th and 90th) of the obtained values are indicated.

The final SSP values are obtained merging the solutions from all the observation points. It is important to highlight that the main purpose of applying a statistical method for data analysis is primarily to estimate parameter statistics, not only the parameter values themselves. The results obtained with the proposed methodology represent our best estimate of the model parameters according with the data available from a network of observation points. Such solutions are represented as empirical distributions over each parameter value space, and from these distributions it is possible to extract summary statistics (mean, median, mode and percentiles) in order to obtain *best estimate* values and uncertainty bounds to express the results. For example, Fig. 7 illustrates the output of the merging process for the M_W parameter. The curves in Fig. 7(a) show the frequency distribution of M_W values obtained at each single station (each curve represents the frequency distribution of solutions in a single observation point), whereas Fig. 7(b) shows the histogram of the final solution obtained after the merging procedure. Fig. 7(b) shows also the summary statistics of the M_W values, and they represent both the best estimate of the parameter (e.g. the median) and the uncertainty bounds (10th and 90th percentiles). Similarly, histograms and summary statistics are also generated for the other source parameters considered; for the example at hand we get, for M_o, a median value of 1.8×10^{18} N m (2.2×10^{17} and 3.1×10^{20} Nm for the 10th and 90th percentiles, respectively); for r, a median value of 1.8×10^4 m (2.8×10^3 and 8.7×10^5 m for the 10th and 90th percentiles, respectively) was obtained, and for $\Delta\sigma$, a median value of 8.2×10^{-2} MPa (3.6×10^{-6} and 45 MPa for the 10th and 90th percentiles, respectively). Such results have been summarized in Table 1 for comparison with previously published data. The results in both r and $\Delta\sigma$ parameters exhibit particularly wide uncertainty ranges (especially the static stress drop, which spans by many orders of magnitude). Such a high variability is a consequence of the uncertainty in the parameters used for their calculation (M_0, ν_ξ, f_c); from these parameters, the uncertainty in f_c is probably the one mostly influencing the wide uncertainty in $\Delta\sigma$, first because of its dependence on corner frequency cubed, and second because of the effect in f_c of neglecting the anelastic attenuation.

5 CONCLUSIONS

In this paper, we present and implement a procedure for the estimation of SSP from spectra based on Bayesian data analysis. The main characteristics of the proposed approach are: (i) it provides a robust estimation of the source spectral model and relative uncertainties; (ii) it accounts also for uncertainties related to Earth model and geometrical parameters; and (iii) uncertainties are propagated down to the estimation of basic SSP of interest such as the M_0, M_W, r and $\Delta\sigma$.

As can be found in literature, the source spectral model considered in this study is still widely used for determining SSP (especially the seismic moment). The procedure for model parameter estimation here presented can be used for analysing any kind of digital seismic records. However, we consider that the proposed approach is particularly useful in two limiting cases, namely: (1) for the analysis of records from historic events, and (2) for analysing source properties of small events and microseismicity. Large uncertainties is the common denominator in these two limiting cases. On the one hand, determining basic source parameters from ground motion records of historic events is subject to high uncertainties derived from limited knowledge of different required data as the source location and mechanisms and the instrumental characteristics (response, orientation, etc.). On the other hand, small events are generally noisy (i.e. characterized by a low signal-to-noise ratio); furthermore, the small source dimensions characterizing microseismicity are associated with higher frequencies which, in turn, strongly suffer the path and site effects. Such characteristics pose objective difficulties and uncertainties for modeling the source properties.

Given the important role that the large, old, earthquakes play in any hazard assessment, especially in constraining the tails of frequency–magnitude distributions, our effort in this paper have been oriented towards demonstrating the performance of the proposed method for handling data from these old records. The same approach can be straightforwardly used for analysing modern records. It is worth noting however that for the analysis of small events and microseismicity, the anelastic attenuation is an important parameter to be considered and that could be introduced as a further parameter to be estimated from the data.

In conclusion, the Bayesian data analysis approach used to estimate the source parameters from spectral data is a simple but powerful mechanism for nonlinear model fitting, providing also the opportunity to naturally propagate uncertainties and to assess the quality and uniqueness of the obtained solution. Another important added value of such an approach is the possibility of integrating information from the expertise of seismologists; such data can be encoded in a prior state of information that is then updated with the information provided by seismological data. Regarding the specific case study presented in this work, the possibility of encoding information from experts is of key importance given the uncertainties generally involved when analysing digitized analogue records of historical earthquakes.

On the other hand, a merging procedure to statistically mix different solutions obtained from different stations is used to calculate a weighted average solution, where the weights for the mixing depend essentially on the quality of the input data. Putting together a Bayesian approach for model parameter estimation and a weighted mixing of multiple solutions provides a valid framework for extracting meaningful data from intrinsically uncertain datasets. In the case study we demonstrate the resolution capacity of such an approach. For instance, comparing the final results with the solutions found in literature (see e.g., Table 1), it can be seen that all these solutions fall well inside the distributions describing our results. Regarding in particular the results obtained for r and $\Delta\sigma$, it can be seen that solutions for these parameters show high variability and large uncertainties. Such variability in this case is a consequence, on the one hand, of the uncertainties resulting in the basic spectral parameters used for their determination, and on the other hand, an indirect effect of neglecting the anelastic attenuation, which as a consequence may have a systematic lowering of the corner frequency with increasing the travel time.

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Table 1. Main seismic parameters of 1909 Benavente earthquake obtained by previous authors. Our results are shown in the last row for comparison.

Reference	M_0 (Nm) $\times 10^{18}$	M_W	M_S	r (m) $\times 10^3$	$\Delta\sigma$ (MPa)
Kárník (1969)	–	–	6.6	–	–
Teves-Costa et al. (1999)	1.03	6.0	–	11.5	30.5
Dineva et al. (2002)	$2.3(\pm 0.93)$	$6.08(\pm 0.21)$	$6.3(\pm 0.25)$	$4.3(\pm 1.6)$	10.0
				$2.3(\pm 0.9)$	79.0
Stich et al. (2005)	1.08	$6.0(\pm 0.1)$	–	–	–
this work: median	1.8	6.2	–	18	0.1
(10th−90th perc)	(0.22 − 310)	(5.6 − 7.7)	–	(2.8 − 870)	(3.6×10^{-6} − 45.0)

Table 2. Example of parameter values defined for the observation point where the CRT station is located.

Parameter	Best guess	Lower bound	Upper bound
ρ (kg m^{-3})	2.7×10^3	2.5×10^3	3.0×10^3
ν_P (m s^{-1})	6.78×10^3	6.0×10^3	7.0×10^3
ν_S (m s^{-1})	3.9×10^3	3.7×10^3	4.1×10^3
F_s	2.0	2.0	2.0
Γ (1/m)	1.09×10^{-6}	0.9×10^{-6}	1.19×10^{-6}
$R^P_{\theta\phi}$	4.0×10^{-1}	2.0×10^{-1}	5.0×10^{-1}
$R^S_{\theta\phi}$	6.3×10^{-1}	5.0×10^{-1}	7.0×10^{-1}

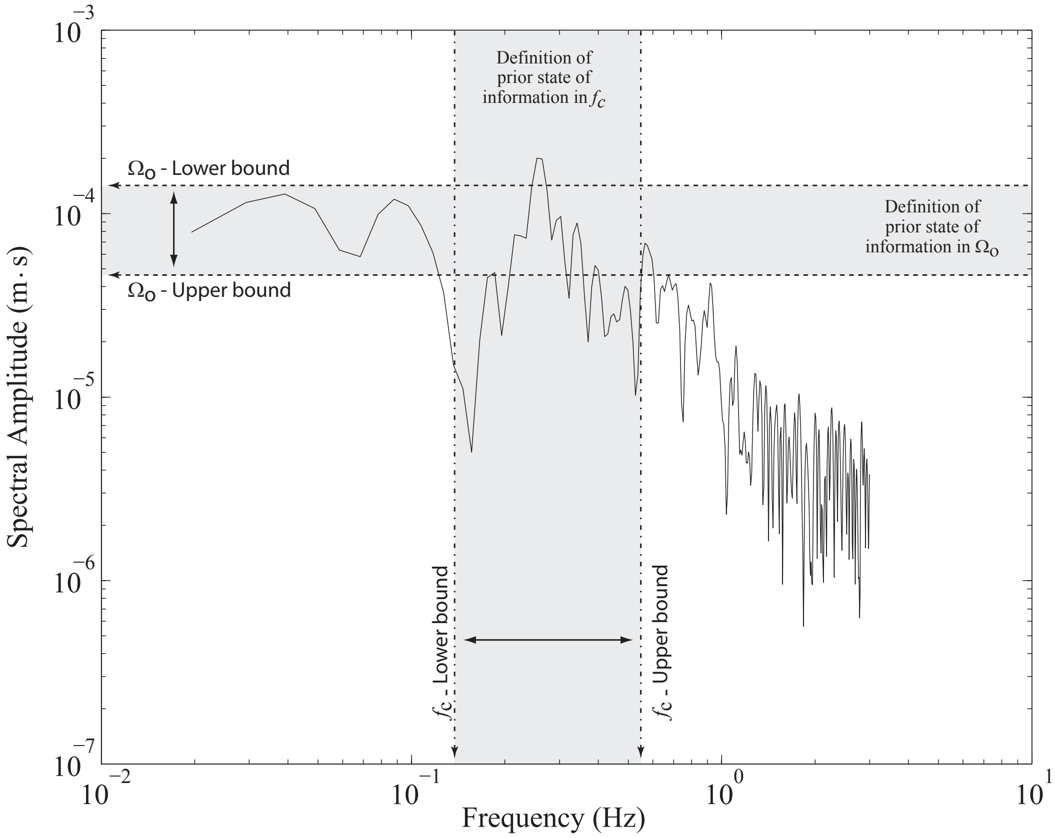

Figure 1. Example of a displacement spectra and the upper and lower bounds identified to set the prior information for Ω_0, f_c and γ parameters.

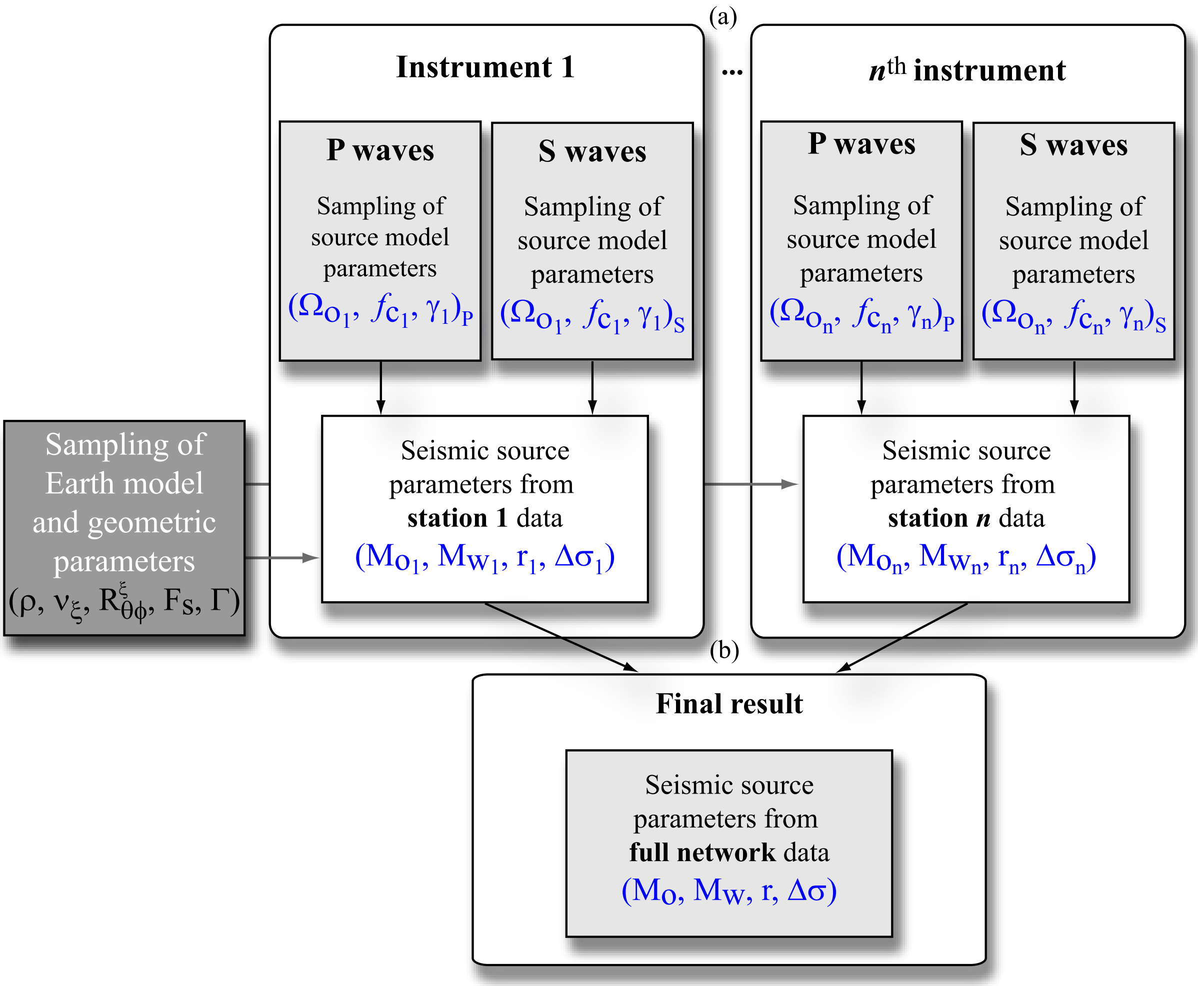

Figure 2. Sampling procedure for the quantification of SSP from the observed displacement spectra obtained from a seismic network. For details see the text.

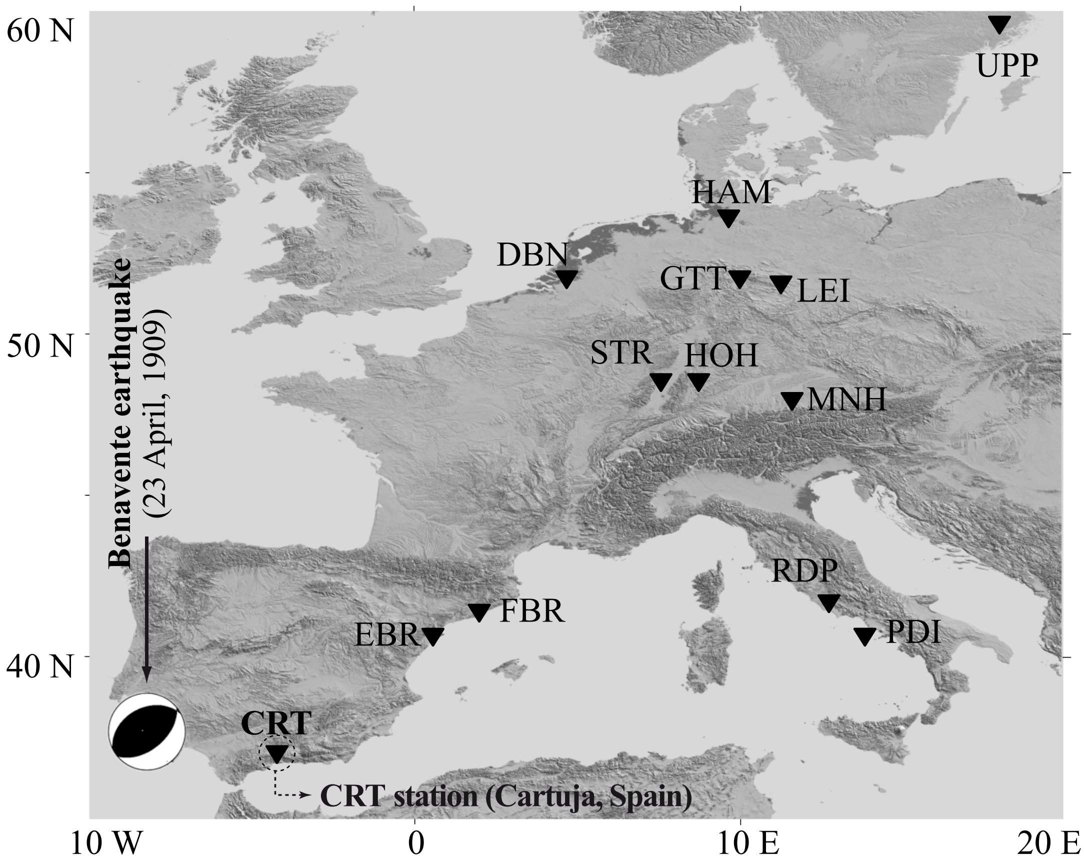

Figure 3. Location and focal mechanism of the 1909 Benavente earthquake (modified from Stich et al. 2005). Triangles show the seismic stations from which it has been possible to recover records (for details see the Appendix).

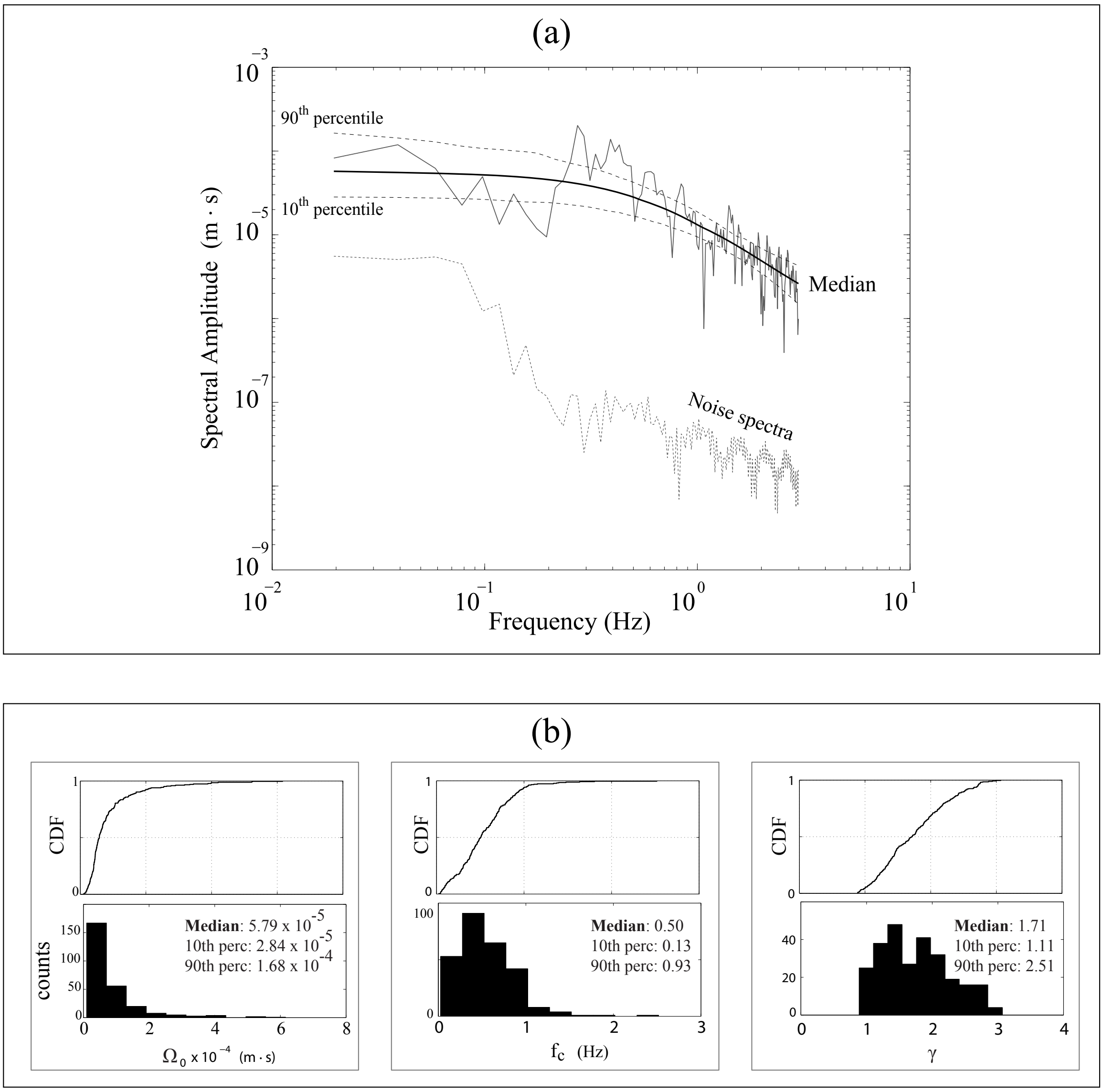

Figure 4. Example of the solution obtained from the spectra of a P wave (CRT- Wiechert station): (a) source spectral model fitted to the observed data and (b) empirical cumulative distribution obtained for the three SSP

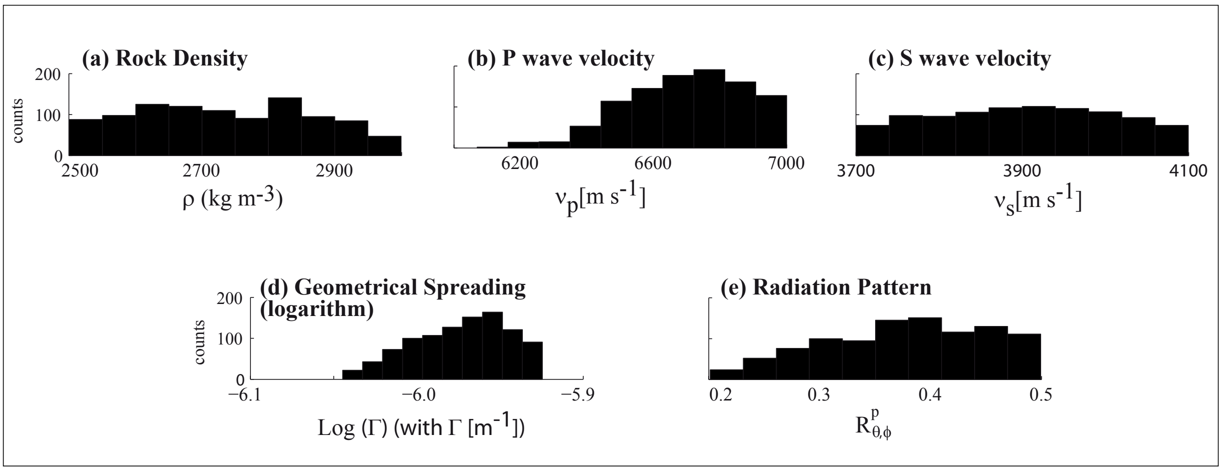

Figure 5. Histograms of the samples drawn from the Earth model and geometrical parameter distributions

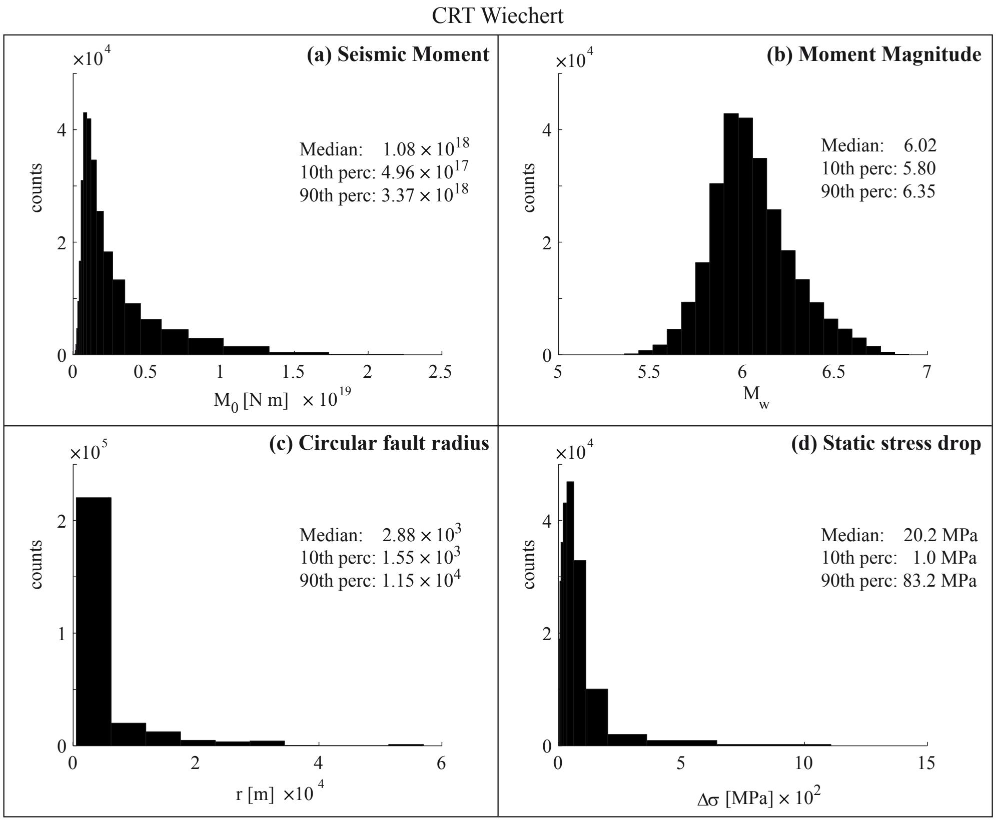

Figure 6. Histograms of the solutions of source parameters obtained for the CRT Wiechert station: (a) seismic moment; (b) moment magnitude; (c) radius of the circular source and (d) static stress drop.

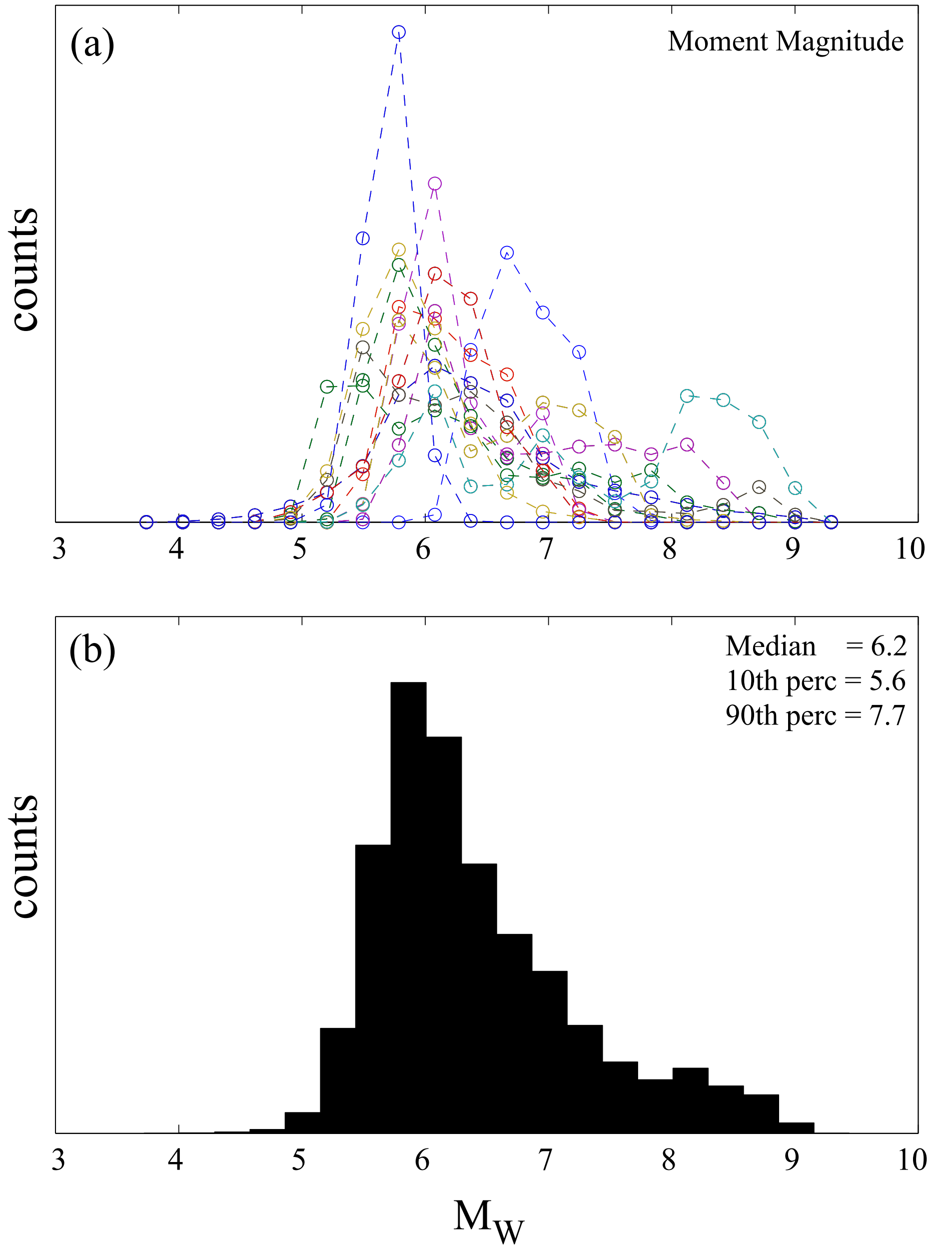

Figure 7. Solutions obtained for the M_w parameter after executing the merging procedure using all the data from the available network: (a) curves showing the frequency of solutions (each curve represents one station) and (b) final histogram after applying the merging procedure.

APPENDIX A: INSTRUMENTAL CHARACTERISTICS OF THE STATIONS

Table A1 summarizes the list of seismic stations whose digitized records where used in this study. The table shows also the main characteristics of the instruments (natural period, T_0, dumping, α and amplification gain) according to original sources (modified from Stich et al. 2005). The parameters in PDI station are not documented for the specific instrument in the station (values in brackets), and therefore these values were adopted from a similar instrument elsewhere. The * mark indicates those instruments in which a generic α-value has been adopted (for details, see Stich et al. 2005).

Table A2 shows the weights used in the merging process (Eq. 12). w_i^c represents the weight assigned to the i-th component of the respective instrument located in a given station; w_j^p and w_j^s are the weights assigned, respectively, to the P- and S-wave data in the jth instrument/station. Finally, w_j^s is the weight assigned to the jth instrument/station used.

Table A1. Main instrumental characteristics of the stations used in this work and from which waveforms for the 1909 Benavente earthquake are available (modified from Stich et al. 2005).

Station	Instrument	T_0 (s)	α	Gain	Component
CRT (Cartuja)	Mod. Omori	14.0	0.4	33	NNW-SSE (N340E)
	Wiechert	5.0	0.4	77	N-S
	Bifilar	6.3	0.45	80	E-W
DBN (De Bilt)	Bosch-Omori	18.0	0.4	20	1 unknown orientation
EBR (Ebre)	Grablovitz	13.0	*0.4	8	NE-SW; SE-NW
FBR (Fabra)	Cancani	4.0	*0.4	17.3	NE-SW; SE-NW
GTT (Gottingen)	Wiechert	11.7	0.35	147	N-S
		11.7	0.4	157	E-W
		5.7	0.55	159	Z
HAM (Hamburg)		19.5	0.48	32	N-S
		20.0	0.45	32	E-W
HOH (Hohenheim)	Bosch-Omori	9.0	0.33	23	N-S; E-W
	Schmith Trifilar	1.5	–	400	Z
LEI (Leipzig)	Wiechert	8.5	0.34	227	N-S
		8.5	0.27	241	E-W
MNH (Munich)	Wiechert	12.5	0.4	240	N-S; E-W
PDI (Porto d'Ischia)	Grablovitz	[13.0]	[*0.4]	[8]	NE-SW; NW-SE (?)
RDP (Rocca di Papa)	Agamennone	4.2	*0.4	60	NE-SW; NW-SE
STR (Strasbourg)	Wiechert	8.3	0.46	200	E-W
UPP (Uppsala)	Wiechert	9.8	0.38	189	N-S
		9.4	0.38	191	E-W

Table A2. Weights assigned for the merging process (Eq. 12) to each instrument component (w_i^c), to each phase (w_j^p for the P and w_j^s for the S wave), and to each instrument/station (w_j^s).

Station (Instrument)	Component	Component w_i^c	Phase weight w_j^p	Phase weight w_j^s	Station weight w_j^s
CRT (Omori)	NNW-SSE (N340E)	1	1	0^+	$1/14$
CRT (Wiechert)	N-S	1	1	0^+	$1/14$
CRT (Bifilar)	E-W	1	1	0^+	$1/14$
DBN	unknown orientation	1	$1/2$	$1/2$	$1/14$
EBR	NE-SW SE-NW	$1/2$ $1/2$	$1/2$	$1/2$	$1/14$
FBR	NE-SW SE-NW	$1/2$ $1/2$	$1/2$	$1/2$	0^{++}
GTT	N-S E-W Z	$1/3$ $1/3$ $1/3$	$1/2$	$1/2$	$1/14$
HAM	N-S E-W	$1/2$ $1/2$	$1/2$	$1/2$	$1/14$
HOH (Bosch-Omori)	N-S E-W	$1/2$ $1/2$	$1/2$	$1/2$	$1/14$
HOH (Schmith T.)	Z	1	$1/2$	$1/2$	0^{++}
LEI	N-S E-W	$1/2$ $1/2$	$1/2$	$1/2$	$1/14$
MNH	N-S E-W	$1/2$ $1/2$	$1/2$	$1/2$	$1/14$
PDI	NE-SW (?) NW-SE (?)	$1/2$ $1/2$	$1/2$	$1/2$	$1/14$
RDP	NE-SW NW-SE	$1/2$ $1/2$	$1/2$	$1/2$	$1/14$
STR	E-W	1	$1/2$	$1/2$	$1/14$
UPP	N-S E-W	$1/2$ $1/2$	0^+	1	$1/14$

0^+phase not considered. 0^{++}station (instrument) not considered.

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